Structural Descriptors for 3D Shapes

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Abstract

Assessing the similarity among 3D shapes is a challenging research topic, and effective shape descriptions have to be devised in order to support the matching process. There is a growing consensus that shapes are recognized and coded mentally in terms of relevant parts and their spatial configuration, or structure.

The presentation will discuss the definition and use of structural descriptions for assessing shape similarity. The idea is to define a shape description framework based on results of differential topology which deal with the description of shapes by means of the properties of one, or more, real-valued functions defined over the shape. Studying these properties, several topological descriptions of the shape can be defined, which may also encode different geometric and morphological attributes that globally and locally describe the shape. Examples and results will be discussed and ongoing work outlined.

1 Introduction

Assessing the similarity among 3D shapes is a very complex and challenging research topic. While human perception have been widely studied and produced theories that received a large consensus [7, 21], the computational aspects of 3D shape retrieval and matching have been only recently addressed. Due to the recent improvements to 3D object acquisition, visualization and modeling technologies, the number of 3D models available on the web is more and more growing, and there is an increasing demand for tools supporting the automatic search for 3D objects and their sub-parts in digital archives.

These considerations suggest that in the future a primary challenge in computer graphics will be how to find models having similar global and/or local appearance. The methods developed so far span from coarse filters suited to browse very large 3D repositories on the web, to domain-specific approaches to assessing similarity of part models containing semantic as well as structural information.

The majority of the methods proposed in the literature mainly focus on the geometry of shapes, in the sense of considering its spatial distribution or extent in

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the 3D space [34, 23, 24, 18]. From a practical point of view, the main advantage of these methods is that they do not make specific assumption on the topology of the digital models, usually triangle meshes or even triangle soups. Moreover, these methods are also computationally efficient.

Nevertheless, there is a growing consensus that shapes are recognized and coded mentally in terms of relevant parts and their spatial configuration, or structure. Methods approaching the problem from a geometric point of view do not take into account the structure of the shape and generally the similarity distance between two objects depends on their spatial embedding. However all of them could be necessary and useful in a multi-step approach which considers a series of filters to progressively refine the set of geometrically similar candidates and/or a multimodal query mechanism that could provide a combination of various measures of shape similarities, corresponding to function, form and structure analysis of 3D shapes.

The use of structural descriptions for shape similarity has been firstly addressed by [14] where the Reeb graph is proposed in a multi-resolution fashion to build a graph and perform shape similarity by means of graph-matching techniques. The importance of structural descriptions for measuring shape similarity has been also recently pointed out by [31] where a method for decomposing a shape into relevant surface patches has been presented. The decomposition is finalized to the definition of a structural description of the shape, which is coupled with an error-correcting subgraph isomorphism to provide in shape retrieval system [22].

The work herein presented is based on results of differential topology which deal with the description of shapes by means of the properties of one, or more, real-valued functions defined over the shape. Studying these properties, topological descriptions of the shape can be defined, namely the Reeb graphs, which can be embedded in the 3D space and augmented with different geometric and morphological attributes that globally and locally describe the shape [4].

We believe that by differentiating the geometric, structural and possibly the semantic level of description of shapes, an automatic retrieval system will be able to provide results closer to the human intuition of similarity [33]. Most importantly, since there is neither a single *best* shape characterization nor a single *best* similarity measure, we propose a *framework* for working on shape retrieval where different characterization methods can be plugged-in and tested, while keeping the same computational setting.

Based on these structural descriptions, the sub-part correspondence between two shapes is obtained by matching their corresponding directed attributed graphs, using a specialization of the method described in [20, 19]. Moreover, the graph matching framework makes it possible to plug in heuristics for tuning the algorithm to the specific application and for achieving different approximations to the optimal solution.

2 The Shape Structural Descriptor

The use of structures for shape description has been widely addressed in Computer Graphics and Vision. Probably the best-known structure is the one related to the medial axis transformation, that provides a decomposition of the shape in protrusions detected by spheres of different radius inscribed in the shape, [8]. The medial axis nicely simulates the human intuition and it is well-suited for shape matching especially for 2D shapes, like for example done in [27] using shock graphs, and more recently proposed for 3D shapes through thinning approaches [30, 29, 10, 36, 16]. Several geometric descriptors have been proposed for associating to the nodes of a skeletal graph the description of the related model sub-parts. A minimal solution consists in coding in a vector the *relevance* of the skeletal edges incident in a node (e.g. edge length, diameters and average circumference of the skeleton loops) as proposed in [30, 16]. Another strategy is to use geometric descriptors able to support global comparison of 3D shapes, like the mean curvature histogram [36], or associating a weight (for example the volume) to the centroids of a shape segmentation, like proposed in [11].

Methods based on the Reeb graph theory, [25] are an effective alternative to skeletal methods: intuitively, the Reeb graph describes the shape by storing the evolution of level sets of a given real-valued function associated to the shape. The shape can be represented by a graph which stores slices of the shape, possibly with some geometric attributes. [14] have used the Reeb graph in a multi-resolution fashion for shape matching, and have associated to each node, i.e. shape slice, the ratio of the area and the length of the model sub-part in the whole model. Similar criteria have been successively used in [2] and further enriched by [32], where for each slice the geometric attributes considered are the volume, a statistical measure of the extent and the orientation of the triangles, an estimation of the Koenderink shape index and a statistic of the texture.

2.1 Extended Reeb Graph

Given a surface S and a real, continuous function f defined on it, the Reeb graph of S with respect to the mapping function f is the quotient space of S with respect to the equivalence relationship that collapses each contour level of f into a single point. See [25, 28] for further references.

Our *Extended Reeb Graph (ERG)* representation generalizes the definition of Reeb graph to a surface on which a finite set of contour levels $C(S)$ is defined.

Since contours are supposed to be non degenerate (i.e. points or open lines), they subdivide S into a set of regions bordered by elements of $C(S)$. Then, we define two points $P, Q \in S$ as Reeb-equivalent in an extended sense if they belong to the same region or the same contour, see [3] for details. The quotient space obtained from this relation is a discrete space, which we call *Extended Reeb (ER)* quotient space. In figure 1, an example of the ER with respect to the distance from the center of mass is shown for a linkage model. Figure 1(a) highlights how the

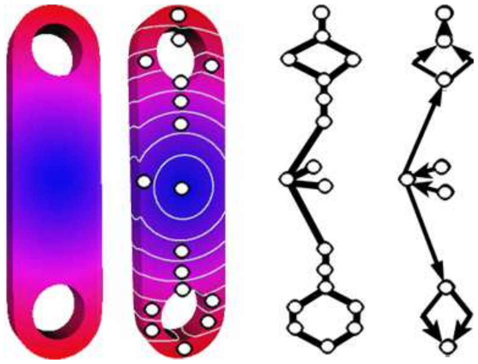

Figure 1: A model (a), graph nodes that correspond to the regions generated from the contour levels (b), the ERG before (c) and after (d) the simplification of nodes.

function f varies on the model: blue regions correspond to minima while red ones represent maxima.

This quotient space is coded in the Extended Reeb Graph as follows: first of all, each region is coded into a node of the graph; then, if two regions share a contour, the nodes corresponding to these regions are connected by an edge. In general, a node will be linked to as many nodes as the number of components of the border of the associated region. In figure 1(b-c) it is highlighted how the sequence of points of the quotient space ER represents the arcs and nodes of the ERG.

The edges of the ERG may be oriented according to the monotonicity of the function f , which implies that the ERG is directed and acyclic. Furthermore, each node of the graph identifies a sub-graph which is empty only in case of leaf nodes, that are nodes with out-degree zero.

Finally, the ERG is further simplified by collapsing all nodes whose number of incoming and outgoing edges is 1, without altering the topological correctness of the coding. After this merging step, the ERG simply consists of nodes representing the regions where the topology of the contour levels varies and the associated connecting edges, see figure 1(d).

The ERG naturally provides a graph representation of the shape that reflects into its coding the invariance properties of the mapping function [5]. In particular, when it is necessary that the shape description is invariant under some transformation, like rigid motions or affine transformations, it is sufficient to choose a mapping function which is invariant under those transformations. Therefore, the dependence of the graph on the mapping function provides a flexible shape characterization that can easily be tuned according to the user needs. Moreover, the Reeb graph is able to correctly code the topology of a closed surface [9].

2.2 The shape descriptor: coupling the ERG with geometric attributes

The functions used to validate the matching framework proposed in this paper are the distance from the center of mass of the object and the integral geodesic distance $f(v) = \int_{p \in M} g(v,p)dS$, where $g(v,p)$ represents the geodesic distance between v and p, when p varies on M. The latter function was originally proposed in [14], while variations may be found in [35, 17]. To be suitable for sub-part correspondence issues any other suitable f could be used, provided that it is invariant with respect to object rotation, translation and scaling [5, 6].

The most important aspect to evaluate when choosing the mapping function is the kind of features that we want to highlight in the description and the type of matching we would like to get. In our example, the distance from the center of mass (barycentre) naturally highlights the distribution of the object with respect to its barycentre, like shown in figure 1. Therefore this function is rotation invariant, but sensitive to pose variations. On the contrary, the function in [14] is pose invariant because it depends on the shape distribution with respect to the geodesic center of the surface. In both cases, the shape will be described as a configuration of protrusions and hollows, but the geodesic will not discriminate between objects in different poses while the distance from the center of mass will do. Therefore, the geodesic is best suited for retrieving articulated objects disregarding the pose, while the distance from the center of mass will allow to distinguish among articulated models in different poses.

Also computational aspects have to be taken into account when choosing a mapping function. For example, the center of mass may be computed in linear time with respect to the number of vertices and, since it depends on all surface vertices, it is robust to noise. On the contrary, the exact computation of the integral geodesic function may be performed only with $O(n^2 \log(n))$ operations, where n is the number of vertices of S; however, its approximation [14] runs in $O(kn \log(n))$ operations, where k is a constant that represent a number of basis for the function evaluation. The approximation of the geodesic distance using the Dijkstra algorithm makes this function sensitive to the vertex distribution. To avoid this problem a uniform remeshing is needed.

Moreover, the underlying slicing mechanisms has to be handled with care: for example, if only a too small number of contour levels is considered, holes completely contained in the interior of a single region are missed.

This problem is related to the slicing frequency, and allows the user to get rid of little features that are considered irrelevant. Nevertheless, if topological accuracy is required, the problems can be easily overcome with the insertion of an additional number of contours into regions having holes (these regions can be always detected locally in each slice using the Euler Characteristics, see [1] for details). Therefore, even if the contours may be non uniformly distributed on the domain of f, the ERG will correctly represent the topology of the surface.

The value of f and a geometric descriptor are associated to each node in the simplified ERG. The attributes used in the literature mainly associate a set of local

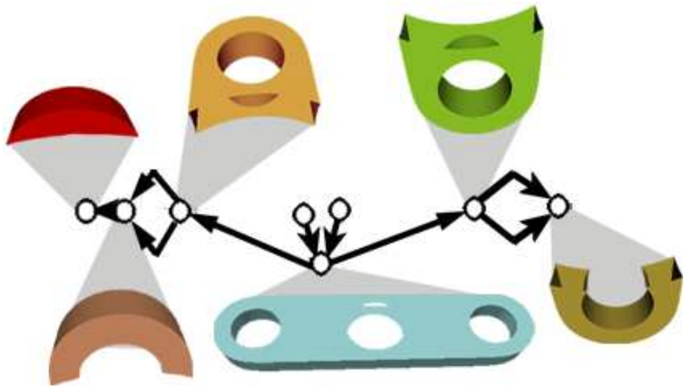

Figure 2: The ERG representation of the model in figure 1 and some of the model sub-parts associated to graph nodes.

geometric attributes to each shape slice. In our approach we move from a local description of the surface slice to a more general representation of the model sub-parts, based on the assumption that the larger the model portion associated to a node is, the more relevant the node should be. Since the ERG is directed, each node is associated to a sub-graph, and this sub-graph defines a sub-part of the shape. For the nodes whose out-degree is zero (leaf nodes), whose sub-graph is empty, we consider only the slice of the shape that correspond to them. Once sub-parts have been associated to each node, we use the spherical harmonic analysis of the sub-part to describe its geometry. Spherical harmonic analysis has been defined in [18], and this descriptor is rotation and scale invariant and stores the shape distribution of each shape sub-part. Therefore, each node is indexed using a matrix, whose values depend on the spherical harmonic values of the related sub-part.

In figure 2, we show the shape description with respect to the distance from the barycentre of the linkage model (see figure 1). The ERG structure is represented by the graph, and each node is depicted with the sub-part it generates. Since the holes in the model are not symmetric with respect to the center of mass, the sub-parts associated to the two leaf nodes slightly differ.

In addition, mainly for visualization purposes, we also store the coordinates of the centroid of the region corresponding to the node and the type of the node (i.e. maximum, minimum or saddle). Finally, the number of regions crossed during the edge construction is also associated to each edge of the ERG, and this value reflects the length of the edge before the simplification step.

The structural representation provided by our shape descriptor may differ from the intuitive notion of shape structure but, since it is related to the mathematical properties of the mapping function, it objectively reflects these properties and discard the other ones acting as a filter for the following matching operations.

The flexibility of the structural descriptor with respect to the choice of the mapping function is a characteristic quality that, in this sense, differs from the skeletal decomposition obtained from flow discretization [11] or thinning methods [10, 36] and may become an advantage when the sub-parts to be rocognized through the matching have well defined mathematical properties. Another distinctive feature of our descriptor is the use of a descriptor for each node sub-graph (the spherical harmonic transform, [18]) more complex that the usual ones.

3 Conclusions

In this paper we have presented a general and flexible framework for computing structural and geometrical shape descriptions.

The flexibility related to the choice of f may be regarded as an advantage over previous methods: the approach described in [15] works on an arbitrary object segmentation (the model is always split into three parts) independently of the shape object complexity; the method proposed in [12] produces an automatic and structural subdivision of the object surface but it works only on simple surfaces where shape discontinuities are present and easily recognized; the object segmentation in [26, 13] is obtained by user interaction. Although enriched of geometric attributes, the ERG is not a ”medial” structure in the usual sense (like meant in [30, 10, 36]). In fact, those skeletons are expected to lie on the middle of the shape and parameters are used to make them the most unique as possible. On the contrary, the edges of the ERG may connect regions that are geometrically far but that are (topologically) close with respect to the mapping function. Provided that the shape characterization is consistent with the user's needs, in principle, it is not yet necessary that the geometric embedding of the graph is intuitive. Therefore, the same framework may be used to highlight different features at time: in the future we are willing to investigate how to automatically combine different measuring functions on the same process.

The structural descriptor is particularly suitable for sub-part correspondence, as fully demonstrated in [19]. To define a complete framework able of automatically recognize sub-parts, a larger number of measuring functions that are independent of affine shape transforms must be still investigated.

In conclusion, the graph representation proposed is a first step for defining a tool able to select a set of shape characteristics that the user may combine with other information and vary according to his desiderata. As further development, we foresee to couple our approach with other methods, in order to contribute to the design of multistep, multimodal search engines for 3D models.

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