06401 Abstracts Collection
Complexity of Constraints
— Dagstuhl Seminar —

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Abstract. From 01.10.06 to 06.10.06, the Dagstuhl Seminar 06401 "Complexity of Constraints" was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Constraint satisfaction problems, computational complexity, universal algebra, mathematical logic, finite model theory

06401 Executive Summary – Complexity of Constraints

In this document we describe the original motivation and goals of the seminar as well as the sequence of talks given during the seminar.

Keywords: Constraint satisfaction problems, complexity

Joint work of: Creignou, Nadia; Kolaitis, Phokion; Vollmer, Heribert

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2006/800

On the power of k-consistency

Albert Atserias (TU of Catalonia - Barcelona, E)

It was known that if the treewidth of the core of the structure A is less than k, then CSP(A,-) is solvable by the k-consistency algorithm.

We prove the exact converse to this: if the treewidth of the core of A is at least k, then CSP(A,-) is not solvable by the k-consistency algorithm.

To build our counterexample, we use the characterization of treewidth in terms of brambles.

Keywords: Constraint satisfaction problem, treewidth, bramble, pebble game

Joint work of: Atserias, Albert; Bulatov, Andrei; Dalmau, Victor

Complexity of Temporal Constraint Satisfaction

Manuel Bodirsky (HU Berlin, D)

We present a new tractable temporal constraint language, which strictly contains the class of Ord-Horn constraints introduced by Bürkert and Nebel. We also prove that our language is *maximally* tractable, i.e., if we add a new temporal relation to our constraint language, the corresponding constraint satisfaction problem becomes NP-complete. For that we apply the so-called product Ramsey theorem, which we believe will be useful in similar contexts of constraint satisfaction complexity classification. Finally, we also plan to sketch the proof that the language cannot be solved by Datalog, or, equivalently, by local consistency techniques.

Keywords: Temporal reasoning

Joint work of: Bodirsky, Manauel; Kara, Jan

CSP, algebras, and varieties

Andrei A. Bulatov (Simon Fraser University, CDN)

We survey the connection between the complexity of constraint satisfaction problems and properties of the corresponding algebras and varieties. We also introduce certain local properties of algebras that are useful for the study of constraint satisfaction problems.

Keywords: CSP, complexity, universal algebra

Basic Universal Algebra

Ferdinand Börner (Universität Potsdam, D)

We give an introduction to the Galois connection Inv - Pol between sets of relations and sets of functions on a finite set A.

The Galois closed sets of functions are the clones and the closed sets of relations are the relational clones. For a set Γ of relations, the closure InvPol Γ is the set of all relations that can be defined from relations in Γ, using only existential quantification and conjunction. This yields the connection to the CSP: If Pol Γ_1 is contained in Pol Γ_2, then CSP(Γ_2) is polynomial-time reducible to CSP(Γ_1).

Keywords: Clones, relational clones, Galois connections

Some other Galois connections

Ferdinand Börner (Universität Potsdam, D)

In this talk we construct some Galois connections different from the usual Pol - Inv that are important for other problems, e.g. the connection sPol - Inv which is helpful for the investigation of the QCSP.

Keywords: Galois connections, surjective polymorphisms, QCSP

Expressibility of VCSPs

David Cohen (RHUL - London, GB)

In this talk we explain what a Valued Constraint Satisfaction Problem is and give a reasonable definition of what the expreesive power of a valued cosntraint language might be.

We show that with this definition we do not change the tractability or NP-hardness of the langauge. Furthermore we show that this language is exactly captured by the polymorphisms of the underlying feasibility relations and the fraction polymorphisms of the cost functions. A direct corollary is that the complexity of a constraint language (valued or otherwise) is completely determined by its fractional and feasibility polymorphisms.

We also show that any language without any fractional polymorphisms must be intractable.

Keywords: VCSP, complexity, polymorphism

A Unifying Theory of Structural Tractability for Constraint Satisfaction Problems

David Cohen (RHUL - London, GB)

In this talk (draft paper) we develop the theory of structural decompositions for the CSP. We begin with the very general notion of a guarded decomposition and make several simplifying assumptions to arrive a the definition of an acyclic guarded cover.

We show how many existing decompositions can seen as acyclic guarded covers. We develop a generic algorithm for discovering acyclic guarded covers under the further assumption that they have a join tree satisfying a simple extra condition. We show that many existing decompositions do in fact satisfy this extra condition.

Using this theory we are able to describe a new class of structural decompostion which we call spread cuts. These generalise many existing decomposition methods. We present a class of hypergraphs whose spread cut width is significantly smaller than their hypertree width. The definition of a guarded decomposition and the algorithm for discovering them were motvated by the similar algorithms developed by Gottlob, Scarcello and Leone in their work on hypertrees. The authors also wish to acknowledge that an acyclic guarded decomposition is very similar to a generalised hypertree decomposition as described in the hypertree literature.

Keywords: Structural decomposition, spread cut

Joint work of: Cohen, David; Gyssens, Marc; Jeavons, Peter

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2006/801

Definability of CSPs in Fixed-point logics

Anuj Dawar (Cambridge University, GB)

We consider the question of what constraint satisfaction problems are definable in least-fixed-point logic, its extension with counting and corresponding infinitary logics. We relate this to the classification of CSPs on the basis of universal algebra. We also relate it to conjectures on the characterisation of CSPs that are definable in Datalog.

This reports joint work with Albert Atserias and Andrei Bulatov and some work in progress that is joint with Stephan Kreutzer.

Enumeration Complexity of Query Problems and Quantifier Elimination

Arnaud Durand (Université Paris VII, F)

We consider the problem of query evaluation for fragments of first-order logic. We revisit the complexity of these problems and exhibit a logical and combinatorial approach that permits to obtain tractability results for various classes : in particular for acyclic conjunctive queries and first-order queries on structures of bounded degree and on trees. This method simply tries to eliminate (under reasonable i.e. linear cost) variables of the formula while preserving the result of the query.

Then, we consider query problems as generation problems for which the complexity measure is the delay between two consecutive tuples of the result, and we show some interesting consequences for the complexity of these problems of the above described method.

Keywords: First-order query, complexity

Constraint Satisfaction with Succinctly Specified Relations

Martin Grohe (HU Berlin, D)

The general intractability of the constraint satisfaction problem has motivated the study of the complexity of restricted cases of this problem.

Thus far, the literature has primarily considered the formulation of the CSP where constraint relations are given explicitly.

We initiate the systematic study of CSP complexity with succinctly specified constraint relations.

Keywords: Constraint satisfaction, complexity, succinct representations

Joint work of: Chen, Hubie; Grohe, Martin

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2006/802

A short overview of hinges

Marc Gyssens (Hasselt University, B)

The talk gives a retrospective birds-eye view on hinges and hinge trees and their applications. Originally, hinges ans hinge trees were introduced in the context of databases, more specifically join dependencies (JDs) and hypergraphs. Hinge trees are a tool to decompose a JD into an equivalent set of JDs with fewer components. In addition, (1) hinge trees can be computed in uniform polynomial time, (2) the largest number of edges in a hinge of a hinge tree is an invariant, called the degree of cyclicity (or hinge tree width), and (3) the degree of cyclicity can be computed in polynomial time. A classical characterization of acyclicity can be generalized in that degree of cyclicity at most k is equivalent to k-wise consistency implying global consistency. Here, rephrased in CSP terminology, k-wise consistency means that each constraint tuple can be extended to a partial solution on any k scopes including the scope of the given constraint tuple, and global consistency, also known as minimality, means that every constraint tuple can be extended to a global solution. Later, hinges resurfaced in the context of constraint satisfaction problems (CSPs). It was shown (under moderate conditions on the size of the domains) that, in the context of minimal constraints, a set of scopes is a hinge if and only if every partial solution to that set can always be extended to a global solution. Also, a hybrid algorithm was proposed, where

tree clustering is applied to the nodes of a hinge tree. Finally, the place of hinge trees in the theory of decompositions was discussed in the talks of Scarcello and Cohen.

Keywords: Hinge, hinge tree, join dependency, hypergraph, consistency, constraint satisfaction problem

Complexity of Clausal Constraints Over Chains

Miki Hermann (Ecole Polytechnique - Palaiseau, F)

We investigate the complexity of the satisfiability problem of constraints over finite totally ordered domains. In our context, a clausal constraint is a disjunction of inequalities of the form $x \geq d$ and $x \leq d$.

We classify the complexity of constraints based on clausal patterns.

A pattern abstracts away from variables and contains only information about the domain elements and the type of inequalities occurring in a constraint. Every finite set of patterns gives rise to a (clausal) constraint satisfaction problem in which all constraints in instances must have an allowed pattern. We prove that every such problem is either polynomially decidable or NP-complete, and give a polynomial-time algorithm for recognizing the tractable cases. Some of these tractable cases are new and have not been previously identified in the literature.

Joint work of: Creignou, Nadia; Krokhin, Andrei; Salzer, Gernot; Hermann, Miki

Exponential Lower Bound for tree-like Lovasz-Schrijver proof systems

Dmitry Itsykson (Steklov Inst. - St. Petersburg, RUS)

We prove an exponential lower bound on the size of static Lovasz-Schrijver calculus refutations of Tseitin tautologies.

We use several techniques, namely, translating static LS proof into Positivstellensatz proof of Grigoriev et al., extracting a "good" expander out of a given graph by removing edges and vertices of Alekhnovich et al., and proving linear lower bound on the degree of Positivstellensatz proofs for Tseitin tautologies.

Keywords: Proof complexity

Joint work of: Itsykson, Dmitry; Kojevnikov, Arist

Full Paper:
http://dx.doi.org/10.1007/11786986_29

Random Constraint Optimization: The Case of Max-Cut

Lefteris M. Kirousis (CTI & University of Patras, GR)

We give a deterministic polynomial-time algorithm which for any given average degree d and *asymptotically almost all* random graphs G in $\mathcal{G}(n, m = \lfloor \frac{d}{2} n \rfloor)$ outputs a cut of G whose ratio (in cardinality) with the maximum cut is at least 0.952. We remind the reader that it is known that unless P=NP, for no constant $\epsilon > 0$ is there a MAX-CUT approximation algorithm that for *all inputs* achieves an approximation ratio of $(16/17) + \epsilon$ $(16/17 < 0.94118)$.

Full Paper:
http://www.springerlink.com/content/b16g526026502542/fulltext.pdf

Constraint satisfaction problems and dualities

Andrei Krokhin (University of Durham, GB)

A constraint satisfaction problem for structure B has a duality if the existence of a homomorphism from a given structure A to B is equivalent to the non-existence of a homomorphism to A from a structure belonging to a certain well-behaved class. This talks surveys results concerning finite duality, bounded pathwidth duality, and bounded treewidth duality, where the above mentiooned class of "obstructions has the corresponding property.

We discuss the following aspects for structures with a given duality: definability of the corresponding CSP in logics, sufficient algebraic conditons, and the complexity of the meta-problem (of recognizing structures with the given duality).

Keywords: Constraint satisfaction problems, homomorphism, duality

Constraint satisfaction problems in clausal form: Autarkies, minimal unsatisfiability, and applications to hypergraph inequalities

Oliver Kullmann (University of Wales - Swansea, GB)

Generalised CNFs are considered using such literals, which exclude exactly one possible value from the domain of the variable. First we consider poly-time SAT decision (and fixed-parameter tractability) exploiting matching theory. Then we consider irredundant generalised CNFs, and characterise some extremal minimally unsatisfiable CNFs.

Keywords: Signed CNF, autarkies, minimal unsatisfiable, hypergraph colouring, block designs

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2006/803

Practical SAT algorithms

Oliver Kullmann (University of Wales - Swansea, GB)

An overview on current practical SAT algorithms is given, concentrating on the three dominant paradigms: Local search, conflict-driven DPLL, look-ahead DPLL. Also some background regarding resolution and reductions.

Some new material has been added (not presented in the talk), especially concerning the comparison with local consistency notions for CSP.

Keywords: SAT algorithms, conflict-driven, look-ahead, local search

Non FO CSP's are Logspace hard

Benoit Larose (Champlain Regional College - St. Lambert, CA)

We outline a proof that CSP's that are not first order definable are hard for Logspace under first order reductions.

Constraint satisfaction, structure homomorphisms and fragments of second order logic.

Florent R. Madelaine (University of Durham, GB)

Constraint satisfaction problems (CSPs) can be modelled in terms of existence of homomorphism between structures.

Feder and Vardi have introduced a fragment of Monadic Second Order logic, MMSNP, and proved that the class of problems captured by MMSNP is computationally equivalent to the class of CSPs. However, this computational equivalence involve a change of signature and a highly non-trivial derandomisation due to Kun. In fact, in terms of Descriptive Complexity, the picture is quite different: there are problems in MMSNP that are not CSPs (over the same signature). Moreover, in general, Bodirsky proved that problems captured by MMSNP are in fact (finite union of) well behaved *infinite* Constraint Satisfaction Problems. This yields the following question: *Given a sentence of MMSNP, can we decide whether it captures a finite or an infinite CSP?* This question, when restricted to the first-order fragment of MMSNP is related to the notion of *homomorphism duality* studied in Structural Combinatorics. Another popular concept in this field is that of *restricted homomorphism duality*, which corresponds in our setting to the following question: *Given a sentence of MMSNP, can we decide whether it captures a finite or an infinite CSP, when restricted to a class of input* $\mathcal{K}$*?* We will present an overview of results concerning MMSNP, CSP and duality.

Keywords: Constraint satisfaction problems, monadic second order logic, homomorphism duality

Connectivity of Boolean Satisfiability: Structural and Computational Dichotomies

Elitza Maneva (Univ. California - Berkeley, USA)

Recent work on heuristics and the satisfiability threshold of random constraint satisfaction problems has centered around the structure and connectivity of the solution space. I will review this area, and present new dichotomy theorems that are motivated by it. In particular, we study onnectivity properties of the space of solutions of Boolean CSPs and establish both computational and structural dichotomies in Schaefer's framework.

We establish dichotomy theorems for the complexity of the connectivity and st-connectivity questions for the graph of solutions of Boolean formulas, as well as for the diameter of the connected components of the solution space. Our results assert that the intractable side of the computational dichotomies is PSPACE-complete, while the tractable side - which includes but is not limited to all problems with polynomial time algorithms for satisfiability - is in P for the st-connectivity question, and in coNP for the connectivity question. The diameter of components can be exponential for the PSPACE-complete cases, whereas in all other cases it is linear. Thus, small diameter and tractability of the connectivity problems are remarkably aligned.

The crux of our results is an expressibility theorem showing that a large class of Boolean CSPs can be reduced to each other in a way that preserves the structural properties of the solution space.

Keywords: Boolean constraint satisfaction problems, dichotomy

Joint work of: Gopalan, Parikshit; Kolaitis, Phokion; Maneva, Elitza; Papadimitriou, Christos

Full Paper:
http://arxiv.org/abs/cs.CC/0609072

See also: Proceedings of ICALP 2006, pp 346-357

A New Look at Survey Propagation

Elitza Maneva (Univ. California - Berkeley, USA)

Recently a new class of algorithms for constraint satisfaction problems has emerged. These are known as message-passing algorithms. When applied to random instances of 3-SAT with density below the satisfiability threshold these algorithms have very high success probability even without doing any backtracking. The survey propagation algorithm - introduced by Mezard, Parisi and Zecchina in 2002 - is one such message-passing algorithm which is successful for very high density of clauses, i.e. very close to the conjectured satisfiability threshold. It is

originally based on sophisticated statistical physics approximation methods. We show that it can also be interpreted as a belief propagation algorithm applied to a novel distribution on partial assignments. These partial assignments encode clusters of solutions of the formula.

Keywords: Random satisfiability, 3-SAT, belief propagation, survey propagation, message-passing algorithms

Joint work of: Maneva, Elitza; Mossel, Elchanan; Wainwright, Martin

Full Paper:
http://arxiv.org/abs/cs.CC/0409012

See also: Proceedings of SODA 2005, pp.1089-1098; long version to appear in JACM

On XPath Dialects with Variables

Joachim Niehren (INRIA Futurs - Lille, F)

Variables are fundamental to XPath 2.0, since navigation paths with n free variables can express queries for n-tuples of nodes in trees. We study the expressiveness of hybrid XPath dialects systematically. We show that an XPath dialect with variables is complete for n-ary first-order queries if its variable free fragment is complete for binary first-order queries. Furthermore, we distinguish efficient fragments of hybrid XPath languages – including a fragment of the logical core of XPath2.0 – that capture the class of n-ary first-order queries.

Keywords: Database theory, logic in computer science

Full Paper:
http://www.ps.uni-sb.de/Papers/abstracts/xpath-variables.html

Complexity of Maximum Solution: Max-Ones Generalised to Larger Domains

Gustav Nordh (Linköping University, S)

We study a family of problems, called Maximum Solution, where the objective is to maximise a linear goal function over the feasible integer assignments to a set of variables subject to a set of constraints. This problem is closely related to Integer Linear Programming.

When the domain is Boolean (i.e., restricted to $\{0, 1\}$), the maximum solution problem is identical to the well-studied Max Ones problem, and the approximability is completely understood for all restrictions on the underlying constraint language.

We continue this line of research by considering domains containing more than two elements. We observe that the approximability of the maximum solution problem is completely determined by the polymorphisms of the underlying constraint language. Using this observation we manage to classify the complexity of the maximum solution problem for several large classes of constraint languages.

Joint work of: Jonsson, Peter; Kuivinen, Fredrik; Nordh, Gustav

Islands of Tractability for Uniform CSP, and their Relationship to Database Theory

Francesco Scarcello (University of Calabria, I)

One way of coping with the NP-hardness of the Uniform CSP is to identify significantly large classes C of relational structures that are recognizable in polynomial time, and such that, for each pair (A,B), with A belonging to C, the problem instance CSP(A,B) is solvable in polynomial time. There are different proposals in the literature for identifying such tractable classes, exploiting the structural properties of the (hyper)graphs associated to the CSP instances.

The talk describes the main notions and their relationship to database theory. Successful applications of these structural decomposition methods are also illustrated, as well as recent interesting advances in this field. In particular, the presentation focuses on the notion of hypertree decomposition and on some proposals for extending this method.

Finally, further structural approaches that do not require the explicit computation of a hypergraph decomposition are illustrated and discussed.

Keywords: Uniform CSP, Strctural Decomposition Methods, Hypertree width

Enumerating all Solutions forConstraint Satisfaction Problems

Henning Schnoor (Universität Hannover, D)

We contribute to the study of efficient enumeration algorithms for all solutions of constraint satisfaction problems. The only algorithm known so far, presented by Creignou and Hébrard and generalized by Cohen, reduces the enumeration problem for a constraint language Γ to the decision problem for a slightly enlarged constraint language Γ^+, i.e., it yields an efficient enumeration algorithm for the case where CSP(Γ^+) is tractable. We develop a new class of algorithms, yielding efficient enumeration algorithms for a broad class of constraint languages. For the three-element domain, we achieve a first step towards a dichotomy theorem for the enumeration problem.

Keywords: Complexity, constraint satisfaction, enumeration

Joint work of: Schnoor, Henning; Schnoor, Ilka

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2006/804

Full Paper:
http://www.thi.uni-hannover.de/forschung/publikationen/daten/schsch06.pdf

New Algebraic Tools for Constraint Satisfaction

Ilka Schnoor (Universität Hannover, D)

The Galois correspondence involving polymorphisms and co-clones has received a lot of attention in regard to constraint satisfaction problems.

However, it fails if we are interested in a reduction giving equivalence instead of only satisfiability-equivalence. We show how a similar Galois connection involving weaker closure operators can be applied for these problems. As an example of the usefulness of our construction, we show how to obtain very short proofs of complexity classifications in this context.

Keywords: Constraints, Partial Clones, Galois Correspondence

Joint work of: Schnoor, Henning; Schnoor, Ilka

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2006/805

Fixed-parameter tractable constraint satisfaction

Stefan Szeider (University of Durham, GB)

CSPs can be solved in polynomial time when certain structural parameters like treewidth or hypertree-width are bounded by a constant. However, there is a trade-off between generality and performance: larger bounds on the parameters yield worse time complexities. It is desirable to pay for more generality only by a constant factor in the running time, not by a larger degree of the polynomial. Algorithms with such a uniform polynomial time complexity are known as fixed-parameter algorithms. Parameterized complexity offers tools for developing fixed-parameter algorithms and for gathering strong theoretical evidence that certain problems are inaccessible to fixed-parameter algorithms.

In this talk I will review some recent results on the parameterized complexity of CSP with respect to combinations of the following parameters: the treewidth of primal, dual, and incidence graphs, the hypertree-width of constraint hypergraphs, the domain size, the maximum arity of constraints, and the maximum size of overlaps of constraint scopes. Furthermore, I will compare the parameterized complexity of SAT and Boolean CSP. This is joint work with Marko Samer.

Keywords: Constraint satisfaction, treewidth, fixed-parameter tractability

Systems of equations over finite semigroups and the #CSP dichotomy

Pascal Tesson (Laval University – Quebec, CA)

The #CSP problem consists of counting the number of satisfying of assignements for a given CSP. It is conjectured that for each constraint language B, the #CSP(B) problem is either solvable in P-time or #P-complete.

We study the particular case of the problem which requires counting the number of solutions to a system over a fixed finite semigroup. We show that for each semigroup this problem is indeed in FP or #P-complete and provide the precise classification. We also discuss the light shed by these results on the current status of the #CSP-dichotomy conjecture.

Keywords: Constraint satisfaction problems, #CSP, counting problems, semigroups

Joint work of: Klíma, Ondrej; Larose, Benoît; Tesson, Pascal

See also: Systems of Equations over Finite Semigroups and the #CSP Dichotomy Conjecture, in Proc. MFCS 2006

Advanced Universal Algebra and the CSP

Matt Valeriote (McMaster University, CA)

I will go into some detail on what Tame Congruence Theory is and how it has and can be used to advance work on the Dichotomy Conjecture for constraint languages. I will describe some recents results that provide alternate characterizations of the tractability conjecture in terms of weak near unanimity operations. To conclude, I will discuss the finite width conjecture and an extension of a tractability result due to Victor Dalmau to a more general setting.

Keywords: Universal algebra, tame congruence theory, dichotomy conjecture few subpowers, finite width

Boolean Constraint Satisfaction Problems

Heribert Vollmer (Universität Hannover, D)

We survey complexity classifications (classical and recent) for different computational problems for Boolean constraints. Among the problems addressed are the satisfiability problem for Boolean constraints, the evaluation problem for quantified constraints, both with and without bound on the number of quantifier alternations, the problem to count the number of solutions for quantified

constraints, again with and without bounding quantifier alternations. Particular attention is paid to the question of how much the theory of Boolean clones and Post's lattice can be used to obtain the desired classifications. While for all mentioned problems the classification is obtained making heavy use of the structure of the lattice ("the Galois connection holds a priori"), other problems are pointed out for which we only notice after obtaining the classification by different means that it actually obeys the border of Boolean relational clones ("the Galois connection holds a posteriori") or for which the classifaction cuts across the border of relational clones ("the Galois connection does not hold"). Finally, a further question is addressed in which the lattic of relational clones is only of limited use; this concerns a fine classification of the complexity of the satisfiability problem for Boolean CSPs using very strict reductions such as FO.

Keywords: Boolean constraints, satisfiability, quantified constraints, Post's lattice, Galois connection