On the Design of Simple Multi-unit Online Auctions

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The increased use of online market places (like eBay) by professional traders and small businesses goes along with an increase in demand for online multi-unit auction designs. A seller with many objects for sale might consider it inconvenient to initiate and monitor a single auction for each individual item and thus might favour the use of a multi-unit auction.[2] However, the design of online multi-unit auctions can be substantially more difficult than that of single-unit auctions. In fact, the theoretical as well as empirical literature on multi-unit auctions is much less developed. New difficulties such as market power and computational complexities arise when objects are heterogeneous or bidders demand multiple items. In addition, there is a conflict between simplicity of auction rules and their efficiency (and revenue). If objects for sale are complements, to obtain the optimal performance (at least from a theoretical point of view) the auction design usually requires that bidders specify their preferences on any possible package of the N objects. Thus each bidder has to submit 2^N-1 numbers (as he might value any subset of the items for sale differently) . Especially for a large number of objects such an auction is often infeasible.[3]

Multi-unit auction design is considerably simpler if one can assume that each bidder just demands one object (or, more generally, if objects are substitutes). As we will argue below, under this unit-demand assumption, the standard single-unit auction format used on eBay can be naturally extended to a multi-unit design.

In what follows we will first describe the single-unit auction format used by eBay and then demonstrate how it can be adjusted to allow for the simultaneous sale of many objects. We

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2 As long as bidders just demand one unit each and objects for sale are similar and auctioned in the right order, it should (under certain assumptions) not matter whether items are sold sequentially or simultaneously (see Kittsteiner et al. (2004) and references cited therein).

3 It is not only infeasible because of the huge amount of information that needs to be transmitted but also because of the computational complexity involved in the determination of the allocation of the objects (see de Vries and Vohra (2003)). For a comprehensive overview on the most recent developments in combinatorial auction design refer to Milgrom (2004).

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will discuss some of the drawbacks of this multi-unit design and offer another simple design that can circumvent some of these.

Ebay's single unit auction format essentially works as follows. The seller specifies a minimum bid which is a lower bound for all bids. Each bidder can then submit bids, each of which needs to be a specified increment above the highest standing bid. The highest standing bid is the highest of all previous bids and the minimum bid. After a pre-specified time the auction ends and the bidder with the highest bid wins the object and pays this bid to the seller. To facilitate incremental bidding in a way that does not require bidders to follow the entire bidding process, eBay offers bidders to submit their (maximum) bids to a proxy-agent. During the auction, this proxy-agent increases a bidder's bid by the smallest amount necessary to become the highest bidder, as long as this bid is below or equal to the submitted maximum bid. That is, the proxy-agent bids incrementally on behalf of the bidder up to the maximum bid. Consequently, if one abstracts away a couple of frictions,[4] the optimal strategy for a bidder is to tell the proxy-agent his willingness to pay, as he knows that, as a winner, he will usually not pay this price but a price equal to the second-highest maximum bid (plus at most an increment).[5] Note that this format resembles the so called second-price or Vickrey auction (see Vickrey (1961)): each bidder submits a (sealed) bid. The highest bidder wins and pays a price equal to the second highest bid.

This single-unit format can be modified to accommodate for the sale of many identical objects. Rather than putting one object for sale the seller has to announce how many objects he offers. As in the single-unit auction every bidder submits a (proxy) bid to the proxy-agent. Assume first that each bidder only desires one object and there are N objects offered for sale. Then the auction rules specify that the N highest bidders win one object each and pay a price that is an increment above the N+1'th highest (proxy) bid, i.e., the price is (almost) equal to the highest losing bid. Note that it is important that the price is equal to the highest losing bid and not, e.g. equal to the lowest winning bid. If the latter was the case one of the winners (the bidder with the N′th highest bid) would determine the price. If bidders use the proxy (and do not bid incrementally) this price will be different from the N+1′th highest bid. Thus if the N+1th highest bid was known to a winning bidder during the bidding he would reduce the price by lowering his bid to an amount just above the highest losing bid. But because the highest losing bid is usually not known before the auction is over, it either needs to be

[4] For a discussion of differences resulting from the dynamics and the minimum increment on eBay, see Roth and Ockenfels (2002) and Ockenfels and Roth (2006). We abstract from the issues mentioned there and note that they are equally relevant for the discussion of multi-unit auctions.

[5] This is the optimal strategy as explained and recommended on eBay's German help page.

estimated by bidders if they want to submit the optimal proxy bid or bidders need to bid incrementally to avoid bidding (substantially) above the N+1th bid. In both cases bidding is more difficult and/or less convenient than in the single-unit format discussed above. The nice properties of the single-unit format only translate to the multi-unit format if the highest losing bid defines the price: then, as in the single-unit auction, it is optimal for a bidder to simply submit his willingness to pay to the proxy-agent. Arguments similar to these convinced eBay Germany to change their format away from one where the price is determined by the lowest winning bid to one where it is the highest losing bid (plus an increment), see also Kittsteiner and Ockenfels (2006).

To accommodate for the possibility that some buyers might want to purchase more than one object, online multi-unit auctions allow for bidders to indicate their desired quantity. A bid consists of a pair of two numbers: the amount requested and the willingness to pay (for one item). However, in such a more general auction environment where at least one bidder demands more than one item, neither of the two multi-unit formats discussed above[6] share the desired properties of the second-price auction in the single-object case (discussed above). The reason is that if a bidder demands more than one unit in eBay's multi-unit format, there is a positive probability that his bid is pivotal: he might only win some of the desired items. His bid determines the price (as it is the highest losing bid) but at the same time the bidder also pays this price as he wins a subset of his demanded items. Thus, similar to the case of unit demand where the price is given by the highest winning bid, the bidder will understate his willingness to pay, or (equivalently) reduce demanded quantities, hampering revenue and efficiency.[7]

Furthermore, efficiency cannot be expected in case of complementarities; that is, when the value of a bundle of objects is larger than the sum of values of each object separately. In such cases, a bidder may end up stuck with objects that are worth little because he failed to win all desired objects (exposure problem), and may quit early because of fear of this. As a result, inefficiencies are likely to arise in all auction formats, in which bidders cannot make sure that they purchase the desired number of objects. As already explained, avoiding the latter makes the auction design complicated to implement and difficult to understand for sellers and

[6] The multi-unit auction rules described above (for bidders with unit demand) can easily be adapted to multi-unit bids. A bid of b for q objects is considered as q individual bids of b, each for one object.

[7] Several field studies provide direct evidence of strategic demand reduction in electronic auction markets, such as Grimm et al. (2003), Klemperer (2004)) and Cramton (1995)). This field evidence is supported by laboratory evidence (e.g., Kagel and Levin (2001), Engelmann and Grimm (2004)) and controlled field experiments (List and Lucking-Reiley (2000)).

bidders. EBay resolved this trade-off between simplicity and efficiency/ flexibility in favour of simplicity. Their multi-unit format, from a theoretical point of view, appears adequate if bidders demand only one object each (which is probably the most common situation) and it can easily be understood by bidders who are already familiar with eBay's single-unit auction design. EBay could address the exposure problem (that arises if bidders have multi-unit demand) by allowing bidders, who receive less objects than demanded, to withdraw the bid for all units. This solves the exposure problem but poses some difficult design questions. For instance, what is the price to be paid by the winners when, after the auction, a bid is withdrawn? If the withdrawn bid counts, the winner may rightly ask why he has to pay the higher price even though there is, after the withdrawal, no competition that justifies the higher price. If the withdrawn bid does not count bidders could collude to drive up prices to a pre-emptive level and then finally withdraw the price defining bid such that the objects are sold for a very low price. Maybe because of these problems EBay decided not to allow for bid withdrawal.

A more radical design for an online multi-unit auction could involve, for instance, a declining price: the price decreases from a high initial price and then declines by a predetermined decrement at predetermined times so that a bidder can indicate how many items he is willing to buy at the current price. The auction ends when all objects are sold (or when the auction runs out of time). As bidders who demand a certain amount of objects know at any time whether they will be able to win that bundle, one may expect bids (and revenue) to be higher than in eBay's multi-unit auction, where, due to the exposure problem, bidders might be reluctant to bid. Thus, such an approach is simple and avoids some of the problems of the multi-unit design discussed above. On the negative side, the declining price design represents a substantial departure from most existing single-unit online auction formats, where prices typically increase rather than decrease. Thus it might be less acceptable for bidders, who are reluctant to bid within a unfamiliar auction framework.

We presented some ideas related to the design of multi-unit online auctions. We describe how simple existing single-unit formats can be and have been adjusted by online auction houses to accommodate for the sale of multiple units. Obviously, an optimal design will depend on many other important factors as well. As optimal auction and market design typically has to account for many specific details and conflicting objectives, a comprehensive analysis has to be beyond the scope of this article. A more comprehensive overview, where some of the ideas presented here are further amplified, can be found in Ockenfels et al. (forthcoming).

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