Reference Point Approaches and Objective Ranking

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Abstract

The paper presents a reflection on some of the basic assumptions and philosophy of reference point approaches, stressing their unique concentration on the *sovereignty* of the subjective decision maker. As a new development in reference point approaches also the concept of *objective ranking* is stressed, defined as dependent only on a given set of data, relevant for the decision situation, and independent from any more detailed specification of personal preferences than that given by defining criteria and the partial order in criterion space. Rational objective ranking can be based on reference point approach, because reference levels needed in this approach can be established objectively statistically from the given data set. Examples show that such objective ranking can be very useful in many management situations.

Keywords: multiple criteria optimization and decisions; reference point approaches; objectivity and subjectivity in decision support

1 Introduction

We shall use here the words *subjective* and *objective* not in any derogatory sense, but in their original epistemic sense – *subjective* as resulting from personal cognition or preferences, *objective* as trying to represent outside world without bias and presuppositions. Thus, we can say that all contemporary decision analysis, aiming at supporting the decision maker in using her/his own preferences for selecting best personal decisions, concentrates actually on computerized support of *subjective decisions.* But what means computerized support? It should include at least two aspects:

1) A computerized representation of knowledge (including data, rules, models) about a part of outside reality pertinent for the decision situation, which should be *as objective as possible;*[1]

[1] Full objectivity is obviously, for many reasons, not possible (this is known, e.g., since Heisenberg in 1927 has shown that any act of measurement distorts the measured quantity). However, we can try to be as objective as possible, by try-

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2) A computerized support for combining the *subjective preferences* of an individual decision maker with an objective representation of the pertinent knowledge in selecting the actual decision.

In other words, as formulated by (Emery 1987) when describing the goals of a DSS (decision support system):

"A DSS provides computer-based assistance to a human decision maker. This offers the possibility of combining the best capabilities of both humans and computers. A human has an astonishing ability to recognize relevant patterns among many factors involved in a decision, recall from memory relevant information on the basis of obscure and incomplete associations, and exercise subtle judgments. A computer, for its part, is obviously much faster and more accurate than a human in handling massive quantities of data. The goal of a DSS is to supplement the decision powers of the human with the data manipulation capabilities of the computer."

Here, however, starts the essential dilemma of decision support: *how far is it appropriate to supplement the decision powers of the human?* There are diverse answers to this fundamental question. We could thus distinguish two essential ways of combining the subjective preferences with an objective representation of the pertinent knowledge:

I. Modelling the preferences of the decision maker in the computer and proposing the resulting, in a sense *automated* decision for her/his approval;

II. Consciously foregoing precise modelling of the preferences of the decision maker in order to preserve (as far as possible) the *sovereignty* of her/his decisions.

While there are situations in which the first way is useful (in control engineering, dynamic pricing of tickets, etc.), in a typical decision support it is considered as a too far reaching intervention into the sovereign rights of a human decision maker. In other words, real decision makers – particularly of higher level that might be considered as master experts in decision making, see (Dreyfus and Dreyfus 1986) – resent automated decisions, do not want to reveal their detailed preferences. On the other hand, following the second way in an extreme fashion, insisting on a full preservation of the sovereignty of human decisions would mean not using decision support at all, relying exclusively on the power of intuition of master expert decision makers – which also might be desired in some situations, such as stimulat-

ing to *falsify* every theory or belief, following (Popper 1972). Even if the post-modern sociology of science denies objectivity and ridicules Popperian *falsificationism,* technology includes *falsificationism* in its everyday practice by submitting tools and other technological artefacts to destructive tests in order to determine limits of their applicability. See (Wierzbicki and Nakamori 2006, 2007) and further comments in this chapter.

ing creativity, see (Wierzbicki 1997; Wierzbicki and Nakamori 2006) but not necessarily in all cases.

Thus, there is a need to compromise, to find a middle way between the extremes of the first and the second way. There might be diverse waysof such compromises; one is a *progressive elicitation of preferences during a computer supported decision process;* see (Miettinen et al. 2006). We must warn, however, that such progressive elicitation should not be understood as leading to a full indentification of preferences – otherwise it would turn into full automation of decisions – and that there are diverse approaches to such elicitation, depending, between others, on the assumed degree of preserving the sovereignty of the decision maker.

2 Reference Point Approaches

Reference point approaches have a long history, see (Wierzbicki 1977; 1980, 1999, Wierzbicki et al. 2000); however, we shall limit their description here to their fundamental philosophy, a short description of their basic features and to some contemporary, new developments related to this class of approaches.

Fundamental Assumptions of Reference Point Approaches

During almost 30 years development of reference point approaches, including their diverse applications, several fundamental assumptions of these approaches were clarified, most of them expressing lessons learned from practice of decision making. These are:

1) Separation of preferential and substantive models. This denotes the conviction that in a good decision support system should carefully distinguish between the *subjective* part of knowledge represented in this system, concerning the preferences of the user, thus called a *preferential model* of the decision situation, and the *objective* part, representing in this system some selected knowledge about pertinent aspects of the decision situation –obviously selected never fully objectively, but formulated with objectivity as a goal – called a *substantive model* (sometimes *core model*) of the decision situation. Typically, a substantive model has the following general form:

$$\boldsymbol{y} = \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{a}) \qquad (1)$$

where:

♦ $\boldsymbol{y}$ is a vector of *outcomes* (outputs) y_i, used for measuring the consequences of implementation of decisions;

♦ $\boldsymbol{x}$ is a vector of *decisions* (controls, inputs to the decision making process), which are controlled by the user;

♦ $\boldsymbol{z}$ is a vector of *external impacts* (external inputs, perturbations), which are not controlled by the user;

♦ $\boldsymbol{a}$ is a vector of *model parameters;*

♦ $\boldsymbol{F}(\ )$ is a vector of functions (including such that are conventionally called *objectives* and *constraints*), describing the *relations* between decisions $\boldsymbol{x}$, impacts $\boldsymbol{z}$, parameters $\boldsymbol{a}$, and outcomes $\boldsymbol{y}$.

The compact form of (1) is misleading, since it hides the actual complexity of the underlying knowledge representation: a large model today may have several millions of variables and constraints, even when the number of decision and outcome variables is usually much smaller. Additionally, the substantive model includes *constraint specification* (symbolically denoted by $\boldsymbol{x} \in \boldsymbol{X}_0$) that might have the form of admissible bounds on selected model outcomes, or be just a list of considered decision options $\boldsymbol{x}_k \in \boldsymbol{X}_0$, $k \in \boldsymbol{K} = \{1,\ldots k,\ldots K\}$. While the reference point approach is typically described for the *continuous* case (with a nonempty interior of $\boldsymbol{X}_0$, thus an infinite number of options in this set), we shall concentrate here on the *discrete* case, with a finite number of decision options K, for which case the reference point approach is equally or even particularly suitable.

The actual issue of the separation of preferential and substantive models is that the substantive model should not represent the preferences of the decision maker, except in one aspect: the number of decision outcomes in this model should be large enough for using them in a separate representation of a *preferential structure* $\boldsymbol{P}(\boldsymbol{x}, \boldsymbol{y})$ of the user, needed for selecting a manageable subset of solutions (decisions) that correspond best to user's preferences. The separate representation of preferential structure can have several degrees of specificity, while the reference point approaches assume that this specification should be as general as possible, since a more detailed specification violates the sovereign right of a decision maker to change her/his mind:

♦ The most general specification contains a selection of outcomes y_i that are chosen by the decision maker to measure the quality of decisions, called typically *criteria* (*quality measures, quality indicators*) and denoted here by q_j, $j \in \boldsymbol{J} = \{1,\ldots j,\ldots J\}$. This specification is accompanied by defining a *partial order* in the space of criteria – simply asking the decision maker which criteria should be *maximized* and which *minimized* (while another option, *stabilizing* some criteria around given reference levels, is also possible in reference point approaches, see Wierzbicki et al. 2000). Here we shall consider only the simplest case when all criteria are maximized.

♦ The second level of specificity in reference point approaches is assumed to consist of specification of *reference points* – generally, desired levels of criteria. These reference points might be double, including *aspiration levels,* denoted here by q_j^a (levels of criteria values that the decision maker would like to achieve) and *reservation levels* q_j^r (levels of criteria values that should be achieved according to the decision maker). Specification of reference levels is treated as an alternative to trade-off or weighting coefficient information that leads usually to linear representation of preferences and unbalanced decisions as discussed below, although some reference point approaches (Nakayama 1995) combine reference levels with trade-off information.

♦ The detailed specification of preferences includes full or gradual identification of utility or value functions, see, e.g., (Keeney and Raiffa 1976), (Keeney 1992). This is avoided in reference point approaches that stress learning instead of value identification – according to the reference point philosophy, the decision maker should learn during the interaction with a DSS, hence her/his preferences might change in the decision making process and she/he has full, sovereign right or even necessity to be inconsistent.

2) Nonlinearity of preferences. This denotes the conviction that *human preferences have essentially nonlinear character,* including a preference for balanced solutions, and that any linear approximation of preferences (e.g., by a weighted sum) distorts them, favouring unbalanced solutions.

This is in opposition to the methods taught in most management schools in diverse versions as *the basic approach* to multiple criteria decision making. These methods consist in determining – by diverse approaches, between which the AHP (Saaty 1982) is one of the most developed – *weighting coefficients* w_j for all $j \in J$ and then aggregating the criteria by a weighted sum:

$$q_J = \sum_{j \in J} w_j q_j \tag{2}$$

with the additional requirement on the scaling of weighting coefficients that $\sum_{i \in J} w_i = 1$. Such an aggregation might be sometimes necessary, but it has several limitations. The most serious between them are the following:

- ♦ The weighted sum tends to promote decisions with unbalanced criteria, as illustrated by the *Korhonen paradox* quoted below; in order to accommodate the natural human preference for balanced solutions, a nonlinear aggregation is necessary.
- ♦ The weighted sum is based on a tacit (unstated) assumption that a trade-off analysis is applicable to all criteria: a worsening of the value of one criterion might be compensated by the improvement of the value of

another one. While often encountered in economic applications, this *compensatory character* of criteria is usually not encountered in interdisciplinary applications.

The *Korhonen paradox* is based upon the following example. Suppose we select a partner for life and consider two criteria: *sex-appeal* and *intelligence*.[2] Suppose we have three candidates (options). Candidate 1 has 100 points for sex-appeal, 0 points for intelligence. Candidate 2 has 0 points for sex-appeal, 100 points for intelligence. Candidate 3 has 45 points for sex-appeal and 45 points for intelligence. It is easy to prove that when using a weighted sum for ranking the candidates, candidate 3 will be never ranked first – no matter what weighting coefficients we use. Thus, weighted sum indeed tends to promote decisions with unbalanced criteria; in order to obtain a balanced solution (the first rank for candidate 3), we have either to use additional constraints or a nonlinear aggregation scheme.

Not knowing about the Korhonen paradox but educated in typical management schools, the legislators in Poland introduced a public tender law. This law requires that any institution preparing a tender using public money should publish beforehand all criteria of ranking the offers and all weighting coefficients used to aggregate the criteria. This legal innovation backfired: while the law was intended to make public tenders more transparent and accountable, the practical outcome was opposite because of effects similar to the Korhonen paradox. Organizers of the tenders soon discovered that they are forced either to select the offer that is cheapest and worst in quality or the best in quality but most expensive one. In order to counteract, they either limited the solution space drastically by diverse side constraints (which is difficult but consistent with the spirit of the law) or added additional poorly defined criteria such as the *degree of satisfaction* (which is simple and legal but fully inconsistent with the spirit of the law, since it makes the tender less transparent and opens hidden door for graft).

This recent practical experience shows that we should be very careful when using weighted sum aggregation. In short summary, *a linear weighted sum aggregation is simple mathematically but too simplistic in representing typical human preferences that are often nonlinear; using this simplistic approach resulted in practice in adverse and unforeseen side-effects.*

3) Holistic perception of criteria. The third basic assumption of reference point approaches is that the decision maker selects her/his decision using a holistic assessment of the decision situation; in order to help

[2] This paradox was not officially published, since in the original formulation of it in diverse discussions, Pekka Korhonen used another name for the second criterion: *ability to cook.*

her/him in such holistic evaluation, a DSS should compute and inform the decision maker about *relevant ranges of criteria change.* Such ranges can be defined in diverse ways, while two of them are basic:

- ♦ *Total ranges of criteria* involve the definition of the *lower bound* q_j^{lo} and the *upper bound* q_j^{up}, over all admissible decisions, for all all $j \in J$.
- ♦ *Efficient ranges of criteria* establish also the lower bound and the upper bound, but counted only over Pareto optimal (nondominated, efficient) decisions. The upper bound is called in this case (of all maximized criteria) the *utopia point* or *ideal point* q_j^{uto} and is typically equal to q_j^{up}; the lower bound is called in this case the *nadir point* q_j^{nad}. Generally, $q_j^{nad} \neq q_j^{lo}$ and the nadir point is easy to determine only (see Ehrgott and Tenfelde-Podehl 2000) in the case of bi-criteria problems (for continuous models; for discrete models the determination of a nadir point is somewhat simpler).

No matter which ranges of criteria we use, in reference point approaches we assume that all criteria or quality indicators – and their values q_{jk} for decision options $k \in K$ - are scaled down to a relative scale by the transformation:[3]

$$q_{jk}^{r} = (q_{jk} - q^{lo}_{j})/(q^{up}_{j} - q^{lo}_{j}) * 100\% \qquad (3)$$

We assume that such a transformation is performed and will not later indicate the upper index *r,* stipulating that all further values of quality indicators are measured in a common, relative percentage scale – although in some examples we can modify this assumption, provided all criteria are measured in relative scales.

4) Reference points as tools of holistic learning. While some decision theorists are likely to point out that a decision maker can be mistaken in specifying her/his reference point, another basic assumption of reference point approaches is that those reference (aspiration, reservation) points are treated not as fixed expression of preferences but as a tool of adaptive, holistic learning about the decision situation as described by the substantive model. Thus, even if the convergence of reference point approaches to a solution best preferred by the decision maker can be proved (Wierzbicki 1999), this aspect is never stressed; more important aspects relate to other properties of these approaches. Even if the reference points might be determined in some objective fashion, independently of the preferences of the decision maker, we stress again a diversity of such objective determinations, thus making possible comparisons of resulting ranking lists.

[3] Moreover, it is consistent with measurement theory, see Barzilai (2004) - who points out that all utility and value theory in this respect is not necessarily consistent with measurement theory.

5) Achievement functions as ad hoc approximations of value. Given the partial information about preferences (the partial order in the space of criteria) and their assumed nonlinearity, and the information about the positioning of reference points inside known criteria ranges, the simplest ad hoc approximation of nonlinear value function consistent with this information and promoting balanced solutions can be proposed. Such ad hoc approximation takes the form discussed later, see Eq. (4), (5), of so called *achievement functions.* Achievement functions are determined essentially by max-min terms that favour solutions with balanced deviations from reference points and express the Rawlsian principle of justice (concentrating the attention on worst off members of society or on issues worst provided for, see Rawls 1971); these terms are slightly corrected by regularizing terms, resulting in the efficiency (Pareto optimality) of solutions that maximize achievement functions.

6) Sovereignty of the decision maker. It can be shown (Wierzbicki 1986) that achievement functions have the property of *full controllability*, This means that any Pareto optimal solution[4] can be selected by the decision maker by modifying reference points and maximizing the achievement function; this provides for the full sovereignty of the decision maker. Thus, a DSS based on reference point approach behaves analogous to a perfect analytic section staff in a business organization (Wierzbicki 1983). The CEO (boss) can outline his preferences to her/his staff and specify the reference points. The perfect staff is not afraid to tell the boss that her/his aspirations or even reservations are not attainable, if this is the case; but the staff computes in this case also the Pareto optimal solution that comes closest to the aspirations or reservations. If, however, even the aspirations of the boss are attainable and not Pareto optimal (a better decision might be found), the perfect staff is not afraid (nor too lazy) to present to the boss just the decision that results in the aspirations; the staff presents also a Pareto optimal decision[5] corresponding to a uniform improvement of all

[4] In fact, when using the regularized achievement function (5) with $\varepsilon > 0$, only any ε-properly Pareto optimal solution, i.e., any Pareto optimal solution with trade-off coefficients bounded by $1 + 1/\varepsilon$; for sufficiently small ε, this means almost any properly Pareto optimal solution, see (Wierzbicki 1986), (Wierzbicki et al. 2000). However, for achievement functions with $\varepsilon = 0$, supplemented by nucleolear optimization (Ogryczak 2006) which is easy for discrete problems, full controllability of any Pareto optimal solution is achieved.

[5] This distinguishes the reference point approaches from a similar set of *goal programming* approaches that minimize a distance form the reference point (goal). Since a distance is not a monotone function (it changes from decreasing to increasing when passing zero), special tricks are needed to secure Pareto op-

criteria over the aspirations. In a special case, when the boss is lucky (or experienced and intuitive) enough to specify aspirations or reservations that are Pareto optimal, the perfect staff responds with the decision that results precisely in attaining these aspirations (reservations) – and does not argue with the boss that another decision is better, even if such a decision might result from a trade-off analysis performed by the staff. Only a computerized DSS, not a human staff, can behave in such perfect fashion.

7) **Final aims: intuition support versus rational objectivity.** To summarize the fundamental assumptions and philosophy of reference point approaches, the basic aim of such approaches when supporting an individual, subjective decision maker is to enhance her/his power of intuition (see Wierzbicki 1997) by enabling holistic learning about the decision situation as modelled by the substantive model; the same applies, actually, when using reference point approaches for supporting negotiations and group decision making, see (Makowski 2005). Another basic aim, discussed in one of next sections, might be the support of objective rankings in situations needing rational objectivity.

Basic features of reference point approaches

The large disparity between the opposite ends of the spectrum of preference elicitation – full value or utility identification versus a weighted sum approach – indicates the need a middle-ground approach, simple enough and easily adaptable but not too simplistic. We mentioned above that the reference point approach requires the specification of reference (aspiration and reservation) levels for each criterion. After this specification, the approach uses a relatively simple but nonlinear aggregation of criteria by an achievement function that can be interpreted as an ad hoc and adaptable approximation of the value function of the decision maker based on the information contained in the estimates of the ranges of criteria and in the positioning of aspiration and reservation levels inside these ranges.

In order to formulate an achievement function, we first count achievements for each individual criterion by transforming it (piece-wise linearly) e.g. in the case of maximized criteria as shown in Eq. (4). The coefficients α and β in this formula are typically selected to assure the concavity of this function, see (Wierzbicki and al. 2000); but the concavity is needed only for problems with a continuous (nonempty interior) set of options, for an easy transformation to a linear programming problem. In a ranking prob-

timality in goal programming, while maximizing an achievement function (that must be monotone by definition) in reference point approaches results always in Pareto optimality, without the necessity of special tricks lem with a discrete and finite set of options, we can choose these coefficients to have a reasonable interpretation of the values of the partial achievement function.

$$\sigma_j(q_j, q^a_j, q^r_j) = \begin{cases} 1 + \alpha\,(q_j - q^a_j)/(q^{up}_j - q^a_j), & \text{if } q^a_j \le q_j \le q^{up}_j\,, \\ (q_j - q^r_j)/(q^a_j - q^r_j), & \text{if } q^r_j \le q_j < q^a_j\,, \\ \beta\,(q_j - q^r_j)/(q^r_j - q^{lo}_j), & \text{if } q^{lo}_j \le q_j < q^r_j\,. \end{cases} \qquad (4)$$

The value $\sigma_j(q_{jk}, q^a_j, q^r_j)$ of this achievement function for a given decision option $k \in K$ signifies the satisfaction level with the quality indicator or criterion j for this option. If we assign the values of satisfaction from -1 to 0 for $q^{lo}_j \le q_j < q^r_j$, values from 0 to 1 for $q^r_j \le q_j < q^a_j$, values from 1 to 2 for $q^a_j \le q_j \le q^{up}_j$, then we can just set $\alpha = \beta = 1$.

After this transformation of all criteria values, we might use then the following form of the overall achievement function:

$$\sigma(\boldsymbol{q}, \boldsymbol{q}^a, \boldsymbol{q}^r, \varepsilon) = \min_{j \in J} \sigma_j(q_j, q^a_j, q^r_j) + \varepsilon \sum_{j \in \boldsymbol{J}} \sigma_j(q_j, q^a_j, q^r_j) \qquad (5)$$

where $\boldsymbol{q} = (q_1,...q_j,...q_J)$ is the vector of criteria and $\boldsymbol{q}^a = (q_1{}^a,...q_j{}^a,...q_J{}^a)$, $\boldsymbol{q}^r = (q_1{}^r,...q_j{}^r,...q_J{}^r)$ correspondingly the vectors of aspiration and reservation levels, while $\varepsilon > 0$ is a small regularizing coefficient. The achievement values $\sigma_k = \sigma(\boldsymbol{q}_k, \boldsymbol{q}^a, \boldsymbol{q}^r, \varepsilon)$ for all $k \in K$ can be used either to optimize them or to order the options in an *overall ranking list*, starting with the highest achievement value.

The formulae (4), (5) do not specify the only form of an achievement function; there are many possible forms of such functions as shown in (Wierzbicki et al. 2000). All of them, however, have an important property of *partial order approximation:* their level sets approximate closely the positive cone defining the partial order (Wierzbicki 1986).

Another property of the formulae (4), (5) is the dependence of the slopes of achievement functions on the situation of aspiration and reservation levels q^a_j, q^r_j in the range of objective values $[q^{lo}_j, q^{up}_j]$, which might be interpreted as a dependence of implied weighting coefficients on the currently specified reference points or a way of progressive elicitation of preferences. This property is shared with other more advanced forms of achievement functions, see (Wierzbicki et al. 2000) and (Luque et al. 2007).

As indicated above, the achievement function has also a very important theoretical property of *controllability*, not possessed by utility functions nor by weighted sums: for sufficiently small values of ε, given any point $\boldsymbol{q}^*$ in the (proper) Pareto set of criteria values related to nondominated options (such as the third option in the Korhonen paradox), we can always choose such reference levels - in fact, it suffices to set aspiration levels equal to the components of ***q**** - that the maximum of the achievement function (5) is attained precisely at this point. Conversely, if $\varepsilon > 0$, all maxima of achievement function correspond to Pareto optimal (nondominated) options, similarly as in the case of utility functions and weighted sums.

As noted above, precisely this controllability property results in a fully sovereign control of the decision support system by the user. Alternatively, as shown by (Ogryczak 2006), we can assume $\varepsilon = 0$ and use *nucleolar minimax* approach. In this approach we consider first the minimal, worst individual criterion achievement σ_k computed as in (4), (5) with $\varepsilon = 0$; if, however, two options k_1 and k_2 (or more of them) have the same achievement value, we order them according to the second worst individual criterion achievement, and so on. We can use this approach also for ranking of the options according to the achievement values , or also for classification. In the latter case, we must first split the interval of achievement values, say, [-1; 2], into a given number of subintervals, preferably of equal length $\Delta\sigma$, say, $\Delta\sigma = 0.3$ with 10 subintervals. Then we classify the options as approximately equivalent (belonging to the same class) if their worst individual criterion achievements σ_k computed as in (4), (5) belong to the same subinterval. If we use $\varepsilon = 0$ and *nucleolar minimax* approach, we must check additionally if the second worst (or even third worst) individual criterion achievements for options belonging to one class differ not more than $\Delta\sigma$; if they differ more, they remain in the same class but subdivided into two new subclasses, etc.

3. The issue of objective ranking

We switch now to a different issue of *objective ranking;* we assume here that X_0 is a set of discrete alternatives and the problem is how to rank these alternatives using multiple criteria and their aggregation.

This is a classical problem of multi-attribute decision analysis; however, all classical approaches – whether of (Keeney and Raiffa 1976), or of (Saaty 1982), or of (Keeney 1992) – concentrate on *subjective ranking.* By this we do not mean *intuitive subjective ranking,* which can be done by any experienced decision maker based on her/his intuition,[6] but *rational*

[6] One of the authors of this chapter bought and used in his life well over 10 cars and never applied any decision support for this task, trusting more in his experience and the power of intuition. When he tried to convince the minister of science in Poland that they should use a decision support system for ranking of scientific research grants, the minister (himself also a professor) answered "I was elected to this high political function because my intuition is good enough to decide without any computerized decision support". See also (Wierzbicki

subjective ranking, based on the data relevant for the decision situation – however, using an approximation of personal preferences in aggregating multiple criteria.

And therein is the catch: in many practical situations, if the decision maker wants to have a computerized decision support and rational ranking, *she/he does not want to use personal preferences, prefers to have some objective ranking.* This is often because the decision is not only a personal one, but affects many people – and it is usually very difficult to achieve an *intersubjective rational ranking,* accounting for personal preferences of all people involved. This obvious fact is best illustrated by the following example.

Suppose an international corporation consists of six divisions *A,…, F.* Suppose, for simplicity, that we are considering only these six units, without additionally specifying problems related to these units (which will be subject of the next example). Suppose these units are characterized by diverse data items, such as name, location, number of employees etc. However, suppose that the CEO of this corporation is really interested in the following *attributes* classified as *criteria:*

- *profit* (in %),
- *market share* (*m.share,* in % of supplying a specific part of market, e.g. global market for specific type of memory chips),
- *internal collaboration* (*i.trade,* in % of revenue coming from supplying other divisions of the corporation), and
- *local social image* (*l.s.i.,* meaning public relations and the perception of this division – e.g., of its friendliness to local environment - in the society where it is located, evaluated on a scale 0-100 points);

All these criteria are maximized, improve when increased. An example of decision table of this type is shown in Table 1, while Pareto optimal (nondominated) divisions are distinguished by mark *.

The CEO obviously could propose an intuitive, subjective ranking of these divisions – and this ranking might be even better than a rational one resulting from the table above, if the CEO knows all these divisions in minute detail. However, when preparing a discussion with his stockholders, he might prefer to ask consultants for an *objective ranking.*

1997), (Wierzbicki and Nakamori 2006) on the rational explanation of the power of intuition. However, there are situations when we need a rational analysis of possible decisions; between such situations, many require *objective ranking,* that is, ranking independent from personal preferences.

Division	c_1: *name*	c_2: *location*	c_3: *employ-s*	q_1: *profit*	q_2: *m.share*	q_3: *i.trade*	q_4: *l.s.i.*
A	*Alpha*	*USA*	2500	11 %	8 %	10 %	40
*B**	*Beta*	*Brasilia*	7500	23 %	40 %	34 %	60
*C**	*Gamma*	*China*	4500	16 %	50 %	45 %	70
*D**	*Delta*	*Dubai*	500	35 %	20 %	20 %	44
*E**	*Epsilon*	*Europe*	3500	18 %	30 %	20 %	80
F	*Fi*	*France*	1200	12 %	8 %	9 %	30

Table 1. An example of a multicriteria decision table[7]

Here we must add some philosophical comments on *subjectivity* and *objectivity*. The destruction of the *industrial era episteme* (see Wierzbicki 2005, Wierzbicki and Nakamori 2007) – sometimes called not quite precisely *positivism* or *scientism* - started early, e.g., since Werner Heisenberg (Heisenberg 1927) has shown that not only a measurement depends on a theory and on instruments, but also the very fact of measurement distorts the measured variable. This was followed by diverse philosophical debates, summarized, e.g., by Van Orman Quine (Quine 1953) who has shown that the logical empiricism (neo-positivism) is logically inconsistent itself, that all human knowledge *"is a man-made fabric that impinges on existence only along the edges"*. This means that there is no absolute objectivity; however, this was quite differently interpreted by hard sciences and by technology, which nevertheless tried to remain as objective as possible, and by social sciences which, in some cases, went much further to maintain that all knowledge is *subjective* – results from a discourse, is constructed, negotiated, relativist, depends on power and money, see, e.g., (Latour 1990). This has led to a general divergence of the *episteme* – the way of constructing knowledge - of the three different cultural spheres (of hard and natural sciences, of technology, and of social sciences and humanities), see (Wierzbicki and Nakamori 2007).

Thus, *full objectivity* is obviously – after Heisenberg and Quine – not attainable, but in many situations we must try to be *as much objective as possible*. This concerns not only technology that cannot advance without trying to be objective and, in fact, pursues Popperian *falsificationism* (see Popper 1972) in everyday practice when submitting technological artifacts to destructive tests in order to increase their reliability – while postmodern social sciences ridicule falsificationism as an utopian description how science develops. However, objectivity is needed also – as we show here – in management.

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[7] The data in this Table are consciously distorted, any similarity to an actual corporation is not intended.

For an individual decision maker, this might mean that she/he needs some independent reasons for ranking, such as a dean cannot rank the laboratories in her/his school fully subjectively, must have some reasonable, objective grounds that can be explained to entire faculty, see the next example. For a ranking that expresses the preferences of a group, diverse methods of aggregating group preferences might be considered; but they must be accepted as fair – thus objective in the sense of intersubjective fairness - by the group.

However, it is not obvious how to define the grounds of an objective ranking and how such an objective ranking might be achieved. In multiple criteria optimization, one of similar issues was to propose *compromise solutions,* see, e.g., Zeleny (1974); however, such solutions depend too strongly on the assumed metric of the distance from the utopia or ideal point. *We propose here to define objective ranking as dependent only on a given set of data, agreed upon to be relevant for the decision situation, and independent of any more detailed specification of personal preferences than that given by defining criteria and the partial order in criterion space.*

Given set of data might be such as in Table 1, or a bigger set of data including much more detailed characteristics of the divisions *A,…, F;* generally, any selected data information system, see (Pawlak 1991). The specification of criteria and their partial order (whether to minimize, or maximize them) can be also easily be agreed upon, be objective in the sense of intersubjective fairness.

It is also not obvious how an objective ranking might be achieved. This is because *almost all the tradition of aggregation of multiple criteria concentrated on rational subjective aggregation of preferences and thus ranking,* see (von Neumann and Morgenstern 1944), usually addressed as either a full identification and aggregation of value and utility functions, see (Keeney and Raiffa 1976, Keeney 1992), or as identification of weighting coefficients under the assumption of aggregation by a weighted sum, e.g., as in the AHP method, see (Saaty 1982). While we could try, in the sense of intersubjective fairness, identify group utility functions or group weighting coefficients, both these concepts are too abstract to be reasonably debated by an average group (imagine a stockholder meeting trying to define their aggregate utility function under uncertainty). Thus, neither of these approaches is easily adaptable for rational objective ranking.

The approach that can be easily adapted for rational objective ranking is *reference point approach* as described above - *because reference levels* needed in this approach *can be* either defined subjectively by the decision maker, or *established objectively statistically from the given data set.* We can use this approach also not for objective ranking, but for *objective classification,* using methods as indicated above with objectively defined reference points.

We show below how to apply this approach for the simple example given in Table 1. Recall that we denote by q_{jk} the value of a criterion or quality indicator q_j for the decision option $k \in K$. The achievement values $\sigma_k = \sigma(\boldsymbol{q}_k, \boldsymbol{q}^a, \boldsymbol{q}^r, \varepsilon)$ for all $k \in K$ can be used to order the options in an *overall ranking list*, starting with the highest achievement value. Now, the question is: *how to define aspiration and reservation levels in order to obtain rational objective ranking?* Several ways were listed in (Granat et al. 2006): *neutral, statistical, voting;* we shall concentrate here on statistical determination.

A statistical determination of reference levels concerns values $q^m{}_j$ that would be used as basic reference levels, an upward modification of these values to obtain aspiration levels $q^a{}_j$, and a downward modification of these values to obtain reservation levels $q^r{}_j$; these might be defined as follows:

$$q^m{}_j = \sum_{k \in K} q_{jk} / |K|;\ q^r{}_j = 0.5\,(q_j^{lo} + q^m{}_j);\qquad (6)$$
$$q^a{}_j = 0.5(q_j^{lo} + q^m{}_j)\ \forall j \in J$$

where $|K|$ denotes the number of alternative options, thus $q^m{}_j$ are just average values of criteria in all set of alternative options, aspiration and reservation levels – just averages of these averages and the lower and upper bounds, respectively. However, there are no essential reasons why we should limit the averaging to the set of alternative options ranked; we could use as well a larger set of data in order to define more adequate (say, historically meaningful) averages, or a smaller set – e.g., only the Pareto optimal options denoted by * in Table 1 – in order to define, say, more demanding averages and aspirations. If the data set is much larger than in this illustrative example, we can use, e.g., evolutionary multiple objective optimization (see Deb 2001, Wierzbicki and Szczepański 2003) for an approximation of the Pareto set.

For the data from Table 1, we can thus present two variants of objective ranking: A – based on averages of data from this table; B – based on averages from Pareto optimal options – see Table 2. Note that the more demanding ranking B displays a *rank reversal:* the divisions *C* and *E,* occupying positions 2 and 3 in ranking A, exchange their places in ranking B. This is, however, a natural phenomenon: *average aspirations favour standard though good solutions, truly interesting solutions result from demanding aspirations.* Note also that the rank reversal disappears if, instead of ranking, we classify the divisions into three classes: I: very good, II: good, III: wanting – both divisions *C* and *E* remain in the class II.

Criterion	q_1	Q_2	Q_3	q_4			
Upper bound	35 %	50 %	45 %	80			
Lower bound	11 %	8 %	9 %	30			
Reference A (average)	19.17%	26 %	23 %	54			
Aspiration A:	27.08%	38 %	34 %	67			
Reservation A	15.08%	17 %	16 %	42			
Reference B (average Pareto)	23.0%	35.0%	29.75%	63.5			
Aspiration B	29.0%	42.5%	37.37%	71.7			
Reservation B	17.0%	21.5%	19.37%	46.7			
Ranking A: Division	σ_1	σ_2	σ_3	σ_4	σ	*Rank*	*Class*
A	-1.00	-1.00	-0.857	-0.167	-1.304	5	III
B	0.660	1.23	1.00	0.760	+1.02	1	I
C	0.077	2.00	2.00	1.231	0.608	2	II
D	2.00	0.143	0.222	0.087	0.332	4	II
E	0.243	0.619	0.222	2.00	0.530	3	II
F	-0.755	-1.00	-1.00	-1.00	-1.375	6	III
Ranking B: Division	σ_1	σ_2	σ_3	σ_4	σ	*Rank*	*Class*
A	-1.00	-1.00	-0.904	-0.363	-1.326	5	III
B	0.500	0.881	0.813	0.532	0.773	1	I
C	-0.167	2.00	2.00	0.932	0.309	3	II
D	2.00	-0.111	0.035	-0.162	0.014	4	II
E	0.083	0.405	0.035	2.00	0.335	2	II
F	-0.830	-1.00	-1.00	-1.00	-1.383	6	III

Table 2 An example of objective ranking and classification for the data from Table 1 (ε =0.1; A: average reference levels, B: Pareto average reference levels)

In some management applications the worst ranked options are the most interesting, because they indicate the need of a corrective action. Objective ranking was originally motivated by an actual application[8] when evaluating scientific creativity conditions in a Japanese research university, JAIST, see (Tian et al. 2006). The evaluation was based on survey results. The survey included 48 questions with diverse answers and over 140 respondents with diverse characteristics: school attachment (JAIST consists of

[8] Actually, it is misleading to call it an application; a real life problem was first solved innovatively, which motivated later the development of theory. This often happens in technology development: technology is not necessarily and not only an application of basic natural science, it often precedes theoretical developments – such as invention of a wheel preceded the concept of a circle.

three schools), nationality (Japanese or foreign – the latter constitute over 10 % of young researchers at JAIST), research position (master students, doctoral students, research associates etc.). In total, the data base was not very large, but large enough to create computational problems.

The questions were of three types. The first type was *assessment questions,* assessing the situation between students and at the university; the *most critical* questions of this type might be selected as those that correspond to *worst* responses. The second type was *importance questions,* assessing importance of a given subject; the *most important* questions might be considered as those that correspond to *best* responses. For those two type of questions, responders were required to tick appropriate responses in the scale *vg (very good), g (good), a (average), b (bad), vb (very bad)* – sometimes in an inverted scale if the questions were negatively formulated. The third type was *controlling questions,* testing the answers to the first two types by indirect questioning revealing responder attitudes or asking for a detailed explanation.

Answers to all questions of first two types were evaluated on a common scale, as a percentage distribution (histogram) of answers *vg* – *g* – *a* – *b* – *vb.* It is good if there are many answers specifying positive evaluations *very good* and *good,* and if there are only few answers specifying negative evaluations *bad* and *very bad.* The interpretation of the evaluation *average* was *almost bad;* if we want most answers to be *very good* and *good,* we admit only a few answers to be *average.* Therefore, in this case $J = G \cup B$, $G = \{vg, g\}$, $B = \{a, b, vb\}$; the statistical distributions (percentage histograms) of answers were interpreted in the sense of multiple criteria optimization, with $j \in G = \{vg, g\}$ as positive outcomes (quality indicators that should be maximized) and $j \in B = \{a, b, vb\}$ as negative outcomes (quality indicators to be minimized).

A reference point approach was proposed for this particular case of ranking probability distributions; other approaches are usually more complicated (see, e.g., Ogryczak and Ruszczyński 2001). However, when the dean of the School of Knowledge Science in JAIST, himself a well known specialist in multiple criteria decision support, was asked to define his preferences or preferred aspiration levels, the reality of the managerial situation overcome his theoretical background: he responded *in this case, I want the ranking to be as objective as possible – I must discuss the results with the deans of other schools and with all professors.* This was the origin of reflection on objective versus subjective rational ranking.

Thus, a statistical average of the percentages of answers in the entire data set was taken as the *reference distribution* or *profile.* Since it was realized that such a reference profile might result in good but standard answers, some artificial reference distributions were also constructed as more demanding than the average one; averages over Pareto optimal options were not computed because of the complexity of the data set.

The detailed results of the survey were very interesting theoretically, but also very useful for university management, see (Tian et al. 2006). It was found that seven questions of the first (assessment) type ranked as worst practically did not depend on the variants of ranking, on the schools or on the characteristics of respondents; thus, the objective ranking gave robust results as to the problems that required most urgent intervention by the university management. The best ranked questions of the second (importance) type were more changeable, only three of them consistently were ranked among the best ones in diverse ranking profiles. Moreover, a rank reversal phenomenon was observed: if the average reference distribution was used, best ranked were questions of rather obvious type, more interesting results were obtained when using more demanding reference profile.

4. Conclusions

We presented here some of the basic assumptions and philosophy of reference point approaches, stressing their unique concentration on the *sovereignty* of the subjective decision maker. Between new developments in reference point approaches we stress, however, also *objective ranking defined as dependent only on a given set of data,* relevant for the decision situation, and *independent of any more detailed specification of personal preferences than that given by defining criteria and the partial order in criterion space.* Rational objective ranking can be based on reference point approach, because *reference levels* needed in this approach *can be established objectively statistically from the given data set.* Examples show that such *objective ranking can be very useful in many management situations.*

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