07281 Abstracts Collection
Structure Theory and FPT Algorithmics for Graphs, Digraphs and Hypergraphs
— Dagstuhl Seminar —

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Abstract. From 8th to 13th July 2007, the Dagstuhl Seminar 07281 "Structure Theory and FPT Algorithmics for Graphs, Digraphs and Hypergraphs" was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Parameterized complexity, fixed-parameter tractability, graph structure theory

07281 Executive Summary – Structure Theory and FPT Algorithmics for Graphs, Digraphs and Hypergraphs

Fixed-parameter algorithmics (FPA) is a relatively new approach for dealing with NP-hard computational problems. In the framework of FPA we try to introduce a parameter k such that the problem in hand can be solved in time $O(f(k)n^c)$, where $f(k)$ is an arbitrary computable function, n is the size of the problem and c is a constant not dependent of n or k. When a parameterized problem P admits an algorithm of running time $O(f(k)n^c)$, P is called fixed-parameter tractable (FPT). The ultimate goal is to obtain such $f(k)$ and c that for small or even moderate values of k the problem under consideration can be completely solved in a reasonable amount of time.

Many practical problems can now be tackled using FPA. The aim of the seminar, held from July 9, 2007 to July 14, 2008 was to bring together specialists of fixed-parameter tractability with researchers who could provide new theoretical tools for FPA and with practical computing practitioners who could benefit from FPA in their own application domains.

The possibility of deep and algorithmically useful combinatorial structure theory seems to be closely allied with FPT—in various combinatorial settings these two different aspects, the one mathematical and the other algorithmic, seem to go together. The parameterized problem Graph Minor Testing is FPT, and exposes in its allied structure theory, with such fundamental structural parameters as treewidth, the kinds of connections between parameterized structure theory and FPA that the workshop explored, encouraged and developed.

Beyond treewidth, which turned out to be a surprisingly universal structural parameter, there is a collection of newer related notions which are currently of intense research interest: cliquewidth of graphs, hypertreewidth of hypergraphs and various parameters measuring near-acyclicity of hypergraphs. The latter are of relevance to the natural input distributions in database and constraint satisfaction problems, and it is a major concern of the workshop to motivate and explore to what extent the successful structure theory and FPA of treewidth, etc. of graphs can be lifted to the setting of hypergraphs.

Although graphs have proven to be a hugely flexible computational modeling tool, and the structure theory and allied FPA of graphs has developed strongly, very little can yet be said for digraphs, even though in the grand scheme of things, digraphs are the more important modeling tool: the entire picture for digraphs in terms of structure theory and FPA has lagged far behind graphs. Some of the most important open problems in concrete FPA involve digraphs (e.g., the Directed Feedback Vertex Set problem that has a vast range of potential important applications, and was widely conjectured to be FPT).

During the 5 days of the conference, 23 talks were given by the participants. Two of these talks were 50-minute surveys given by founders of the field: Mike Fellows started the workshop by reviewing the latest technical and methodological developments and Mike Langston reported on recent algorithmic applications in computational biology.

As a highlight of the seminar, Jianer Chen and Igor Razgon presented their very recent work on the Directed Feedback Vertex Set problem.

The complexity status of this very important problem was open for 15 years or so, until two independent groups of researchers proved its fixed-parameter tractability earlier this year. The technical reports of both groups are available in this volume. The solution of the problem required a clever mix of old and new ideas. In recent years the field witnessed a more systematic identification, study, and dissemination of algorithmic ideas, leading to significant new results. There is no doubt that this progress was helped enormously by meetings such as the previous Dagstuhl seminars.

The talks left plenty of time for discussion in the afternoon. An open problem session was held on Monday. Problems raised there were discussed by different groups throughout the seminar.

Joint work of: Erik D. Demaine; Gregory Gutin; Dániel Marx; Stege, Ulrike

Keywords: Parameterized complexity, fixed-parameter tractability, graph structure theory

07281 Open Problems – Structure Theory and FPT Algorithmics for Graphs, Digraphs and Hypergraphs

The following is a list of the problems presented on Monday, July 9, 2007 at the open-problem session of the Seminar on Structure Theory and FPT Algorithmics for Graphs, Digraphs and Hypergraphs, held at Schloss Dagstuhl in Wadern, Germany.

Joint work of: Erik D. Demaine; Gregory Gutin; Dániel Marx; Stege, Ulrike

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/1254

Using Small Dominating Structures in Exact Algorithms for Non-FPT Problems

Faisal Abu-Khzam (Lebanese American University, LB)

The efficiency of a fixed-parameter algorithm depends on the effective use of a problem's input parameter. In most of these algorithms the size of a target solution is the main input parameter. We consider techniques that employ auxiliary parameters in designing exact algorithms for non-FPT problems. In particular, we discuss the utility of a method that consists of confining the exhaustive (super-polynomial time) search of a solution to potentially small subgraphs that we call dominating structures.

As an exemplar, we consider the Maximum Common Subgraph problem (MCS). We use MCS to model the problem of finding similarities between protein interaction networks of different species. We present an MCS algorithm based on the dominating structures method, and discuss the practicality of our method on pairs of protein interaction networks.

Keywords: Maximum Common Subgraph, protein interaction networks, auxiliary parameters

Computing excluded minors

Isolde Adler (HU Berlin, D)

Robertson's and Seymour's proof of Wagner's conjecture implies that every minor closed class of graphs can be characterised by a finite set of excluded minors, a finite so-called "obstruction set".

The theorem is highly non-constructive. For example, the class of all knot free graphs is closed under minors and hence it can be characterised by a finite obstruction set. But we do not know such an obstruction set, and we do not even know whether such a set is computable.

While in general such finite obstruction sets may not be computable, we show that for some classes they are, such as unions of minor closed classes, and for every k the class of graphs of tree-width at most k.

Joint work of: Grohe, Martin; Kreutzer, Stephan; Adler, Isolde

On Minimum Multiway Cut Problem and Its Application to DFVS

Jianer Chen (Texas A&M University, USA)

The (parameterized) Multiway-Cut problem is for a given graph to find a separator of size bounded by k whose removal separates a collection of terminal sets in the graph. In this talk, we develop new techniques that give an algorithm for this problem, which significantly improves the previous algorithm for the problem.

More importantly, we show how the techniques are used to solve a variation of the Multiway-Cut problem, the Ordered Multiway-Cut problem. This result, combined with the results to be presented by Razgon at the same workshop, will lead to a fixed parameter tractable algorithm for the problem of Feedback Vertex Set in directed graphs, thus resolving an outstanding open problem in the area of parameterized computation and complexity.

The second part of this talk is a joint work with Liu, Lu, Razgon, and O'Sullivan.

Keywords: Multiway cut, feedback vertex set, graph separator

Joint work of: Chen, Jianer; Liu, Yang; Lu, Songjian

Subexponential Time and Fixed-Parameter Tractability

Yijia Chen (Shanghai Jiao Tong Univ., PRC)

In the previous work [CG06] we proved that the so-called miniaturization mapping $\mathcal{M}$ faithfully translates subexponential complexity into (unbounded) parameterized complexity, but it was left open what the preimage of the W-hierarchy under $\mathcal{M}$ is. We answer this question and show that for various (classes of) problems their preimages coincide with natural reparameterizations which take into account the amount of nondeterminism needed to solve them. Our results have some interesting consequences, e.g.,

(1) p-CLIQUE is fixed-parameter tractable if and only if there is an algorithm deciding whether a graph $G = (V, E)$ has a clique of size k in time $2^{o^{eff}(k \cdot \log |V|)} \cdot |V|^{O(1)}$.

(2) If p-CLIQUE is fixed-parameter tractable, then there is an algorithm deciding whether a graph $G = (V, E)$ has a clique of size k in time $|V|^{o^{eff}(k)}$.

Joint work of: Chen, Yijia; Flum, Jörg; Grohe, Martin

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/1233

Fixed-Parameter Algorithms for Consecutive Ones Submatrix Problems

Michael Dom (Universität Jena, D)

We develop an algorithmically useful refinement of a forbidden submatrix characterization of 0/1-matrices fulfilling the Consecutive Ones Property (C1P).

This characterization finds applications in new polynomial-time approximation algorithms and fixed-parameter tractability results for the NP-hard problem to find a maximum-size submatrix of a 0/1-matrix such that the submatrix has the C1P.

Moreover, we achieve several problem kernelizations based on polynomial-time data reduction rules.

Keywords: Circular ones property, fixed-parameter tractability, problem kernel, kernelization, exact algorithms

Joint work of: Dom, Michael; Guo, Jiong; Niedermeier, Rolf

Resolving Conflicts using Hypergraphs with Constraints

Patricia Evans (University of New Brunswick, CA)

Resolving conflicts between individuals where the conflicts can have different sources requires the removal of either one of the individuals, or the source of the conflict.

This scenario can be modeled using a hypergraph or Hitting Set instance, with constraints added as parameters to limit the number of individuals and sources that can be removed.

Prior work on kernelization for related graph problems can be combined to produce a good kernelization for this problem.

New Ideas For Getting Problem Kernels

Henning Fernau (Universität Trier, D)

This tentative talk will summarize some ideas that we recently got for obtaining problems kernels when we actually worked on a couple of quite different concrete algorithmic problems:

- How to use local search to obtain kernelizations; in actual fact, these kernel algorithms are rather Turing reductions opposed to the (usual) many-one kernel reductions.
- How to use approximation algorithms to obtain kernelizations; this idea is particularly useful for natural parameterizations of maximization problems.
- How to employ probabilistic arguments to obtain kernels; in this context, we discuss the (somewhat philosophical) question whether or not we really want to see a solution to a given problem instance, or whether we are content with a yes or no answer.

Keywords: Problem kernels, parameterized algorithms

Joint work of: Fernau, Henning; Raible, Daniel

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/1235

Approximating the Crossing Number for Graphs close to "Planarity"

Petr Hlineny (Masaryk University, CZ)

CrossingNumber, that is finding the least number of edge crossings in a drawing of a graph in the plane, is a quite hard algorithmic problem on graphs; NP-complete in general. (BTW, its status for bounded-tree-width graphs is widely unknown... [Seese]) Although there are new FPT algorithms for this problem [Grohe], [Kawarabayashi + Reed], we do not think these to be the right "parametrization" of the problem. Indeed, it is still unknown how to compute the exact crossing number efficiently for almost planar graphs (planar plus one edge)!

Concerning approximation in general, the curent best is a $\log^3 |V(G)|$-factor approximation algorithm for $cr(G) + |V(G)|$ by [Even, Guha and Schieber], assuming bounded degrees. We have shown recently that crossing number can be efficiently approximated up to a constant factor for graphs of bounded degrees that are:

(a) almost planar, i.e. deleting one edge leaves them planar;
(b) embeddable in the projective plane (also with Gitler and Leanos);
(c) or embeddable in the torus, assuming sufficiently "dense" embedding.

The algorithms themselves are quite straightforward and very natural, but interestingly one needs nontrivial tools of structural and topological graph theory to prove the approximation.

A natural next step of our work is to extend these approximations to graphs embedded on other surfaces, but this is not straighforward, unfortunately.

In addition to these results we would like to raise the following questions:

How much "nonplanarity" of a graph one can allow while still being able to compute or approximate its crossing number efficiently?

Is it true that computing the crossing number of an apex graph is already NP-complete?

Keywords: Crossing number, almost-planar, topological graph

Joint work of: Hlineny, Petr; Salazar, Gelasio

See also: Hlinìný, Petr - Salazar, Gelasio. On the Crossing Number of Almost Planar Graphs. Lecture Notes in Computer Science, Berlin : Springer Verlag, 2007, 4372, pp. 162-173.

Experiments with Parameterized Approaches to Hard Graph Problems

Falk Hüffner (Universität Jena, D)

We present algorithmic improvements and experimental results for two problems from bioinformatics. The first concerns the discovery of signaling pathways in protein interaction networks, which is solved with the color-coding technique. We show several speed-ups of color-coding. The second concerns the detection of "monotone subsystems" in gene regulatory networks. We solve this problem by a combination of a data reduction scheme and iterative compression.

Locally Excluding a Minor

Stephan Kreutzer (HU Berlin, D)

We introduce a new measure for the structural complexity of graph classes that we call "locally excluded minors". In short, a class of graphs locally excludes a minor if every neighbourhood of a vertex excludes a fixed minor, where the minor only depends on the radius of the neighbourhood.

Graph classes locally excluding a minor generalise both the concept of excluded minor classes and graph classes with bounded local tree-width. We show that covering problems such as the dominating and independent set problems can be solved by FPT algorithms on any class of graphs locally excluding minors. The results follow from a general result that first-order model-checking is fixed-parameter tractable on any class of graphs locally excluding a minor. This strictly generalises analogous results by Flum and Grohe on excluded minor classes and Frick and Grohe on classes with bounded local tree-width. The main part of the proof is in showing that certain decompositions of graphs excluding a fixed minor which are guaranteed by Robertson & Seymour's structure theory can be computed by an FPT algorithm taking the excluded minor as parameter. As a consequence of this we obtain fixed-parameter algorithms for problems such as dominating or independent set on graph classes excluding a minor, where now the parameter is the size of the dominating set and the excluded minor. Up to now, the question whether these problems were fixed-parameter tractable with this parametrisations was open.

We also study graph classes with excluded minors, where the minor may grow slowly with the size of the graphs and we show that again, first-order model-checking is fixed-parameter tractable on any such class of graphs.

Keywords: Graph Minors, FPT algorithms
Joint work of: Dawar, Anuj; Grohe, Martin; Kreutzer, Stephan

FPT and Computational Biology: A Medley of Recent Algorithmic Applications

Michael A. Langston (University of Tennessee, USA)

I will report on some of the latest applications borne of FPT technology.

Samples will be drawn from diverse topics such as gene expression, linkage disequilibrium, ontological discovery and data integration.

Parameterized Proof Complexity

Barnaby Martin (University of Durham, GB)

We propose a proof-theoretic approach for gaining evidence that certain parameterized problems are not fixed-parameter tractable. We consider proofs that witness that a given propositional formula cannot be satisfied by a truth assignment that sets at most k variables to true, considering k as the parameter (we call such a formula a parameterized contradiction).

One could separate the parameterized complexity classes FPT and W[2] by showing that there is no proof system (for CNF formulas) that admits proofs of size $f(k)n^{O(1)}$ where f is a computable function and n represents the size of the propositional formula. By way of a first step, we introduce the system of parameterized tree-like resolution, and show that this system does not admit such proofs. We obtain this result as a corollary to a meta-theorem, the main result of this paper. The meta-theorem extends Riis's Complexity Gap Theorem for tree-like resolution. Riis's result establishes a dichotomy between polynomial and exponential size tree-like resolution proofs for propositional formulas that uniformly encode a first-order principle over a universe of size n:

(1) either there are tree-like resolution proofs of size polynomial in n, or
(2) the proofs have size at least $2^{\epsilon n}$ for some constant ϵ.

The second case prevails exactly when the first-order principle has no finite but some infinite model.

We show that the parameterized setting allows a refined classification, splitting the second case into two subcases:

(2a) there are parameterized tree-like resolution proofs of size at most $\beta^k.n^\alpha$ for some constants α, β; or
(2b) every parameterized tree-like resolution proof has size at least n^{k^γ} for some constant $0 < \gamma \leq 1$.

The latter case prevails exactly if for every infinite model, a certain associated hypergraph has no finite dominating set. We provide examples of first-order principles for all three cases.

Keywords: Proof Complexity, Parameterized Complexity

Joint work of: Dantchev, Stefan; Martin, Barnaby; Szeider, Stefan

Finding branch-decompositions and rank-decompositions

Sang-il Oum (University of Waterloo, CA)

We will discuss how to obtain a rank-decomposition of width at most k for graphs of rank-width at most k in FPT time.

The previous algorithms are either (i) approximate (can only find a rank-decomposition of width at most f(k) in FPT time), OR (ii) decision-only (can test rank-width<=k, but no rank-decomposition is provided.) OR (iii) not FPT, but P for fixed k.

Keywords: Rank-width, clique-width, branch-width, matroid

Joint work of: Oum, Sang-il; Hlineny, Petr

Fixed-Parameter Algorithms for the Convex Recoloring Problem

Oriana Ponta (Universität Heidelberg, D)

The Convex Recoloring problem measures how strongly a phylogeny deviates from being a perfect phylogeny.

For an input consisting of a vertex colored tree T and an integer k, the problem is to determine whether recoloring at most k vertices can achieve a convex coloring of T. A vertex coloring is convex if for each pair of vertices v and w, both possessing a color γ, all vertices on the path from v to w are colored by γ. The Convex Recoloring problem is motivated by the scenario of introducing a new character to an existing phylogenetic tree and has further applications in the analysis of gene expression data. Convex Recoloring was introduced by Moran and Snir (Convex Recolorings of Phylogenetic Trees: Definitions, Hardness Results and Algorithms. Lecture Notes in Computer Science, 3608, 218-232, 2005), who showed that the problem is NP-hard already for strings. Moran and Snir described dynamic programming-based fixed-parameter tractable algorithms under two generalizations of the problem: the uniform model, in which each vertex has a weight representing the cost of changing its color, and the non-uniform model, where the cost of coloring a vertex v with color γ is an arbitrary non-negative number $cost(v, \gamma)$. Modifying the dynamic programming structure in Moran and Snir's algorithm for the non-uniform weighted model improves their achieved running time to $O(n \cdot n_c \cdot 3^{n_c})$ for the number of vertices n and the number of colors n_c.

Applying the same technique to their algorithm for the uniform model leads to an improved running time of $O(n \cdot n_c^b \cdot 3^{n_c^b})$ for the number of bad colors n_c^b. A bad color is one that does not induce a monochromatic subtree.

This alternative dynamic programming technique has already been used in a draft paper by Bodlaender and Weyer (2005) for a different parameterization of the problem.

Keywords: Convex recoloring, perfect phylogeny, phylogenetic tree

Exact Elimination of Cycles in Graphs

Daniel Raible (Universität Trier, D)

One of the standard basic steps in drawing hierarchical graphs is to invert some arcs of the given graph to make the graph acyclic.

We discuss exact and parameterized algorithms for this problem. In particular we examine a graph class called $(1, n)$-graphs, which contains cubic graphs. For both exact and parameterized algorithms we use a non-standard measure approach for the analysis. The analysis of the parameterized algorithm is of special interest, as it is not an amortized analysis modelled by 'finite states' but rather a 'top-down' amortized analysis. For $(1, n)$-graphs we achieve a running time of $O^*(1.1871^m)$ and $O^*(1.212^k)$, for cubic graphs $O^*(1.1798^m)$ and $O^*(1.201^k)$, respectively. As a by-product the trivial bound of 2^n for Feedback Vertex Set on planar directed graphs is broken.

Keywords: Maximum Acyclic Subgraph, Feedback Arc Set, Amortized Analysis, Exact exponential algorithms

Joint work of: Raible, Daniel; Fernau, Henning

Directed Feedback Vertex Set is Fixed-Parameter Tractable

Igor Razgon (Univ. College Cork, IRL)

We resolve positively a long standing open question regarding the fixed-parameter tractability of the parameterized Directed Feedback Vertex Set problem. In particular, we propose an algorithm which solves this problem in $O(8^k k! * poly(n))$.

Keywords: Directed FVS, Multicut, Directed Acyclic Graph (DAG)

Joint work of: Razgon, Igor; O'Sullivan, Barry

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/1236

Using Fast Set Convolutions to Compute Minimal Dominating Sets

Peter Rossmanith (RWTH Aachen, D)

A recent paper by Björklund, Husfeldt, Kaski, and Koivisto introduced a wonderful new technique to speed up certain kinds of dynamic programming tasks.

This technique can be adapted to compute a minimal dominating set for an undirected graph, if a tree decomposition of width k is given. The running time of the resulting algorithm is $O^*(3^k)$.

Keywords: Dynamic programming, minimum dominanting set, fixed parameter algorithm

Parameterized Algorithms for Directed Maximum Leaf Problems

Saket Saurabh (University of Bergen, N)

We prove that finding a rooted subtree with at least k leaves in a digraph is a fixed parameter tractable problem. A similar result holds for finding rooted spanning trees with many leaves in digraphs from a wide family of graphs that includes all strong and acyclic digraphs. This settles completely an open question of Fellows and solves another one for large class of digraphs in. Our algorithms are based on the following combinatorial result which can be viewed as a generalization of many results for a "spanning tree with many leaves" in the undirected case, and which is interesting on its own: If a strong directed graph of order n with minimum in-degree at least 3 contains a rooted spanning tree, then D contains one with at least $(n/2)1/5 - 1$ leaves.

Keywords: Directed Maximum Leaf Out-Tree, Directed Graphs

Joint work of: Saurabh, Saket; Alon, Noga: Fomin, V. Fedor; Gutin, Gregory; Krivelevich, Michael

Membership of k-Move Games

Allan Scott (University of Victoria, CA)

It has been suggested that AW[*] is the natural home of k-move games, but the list of games proven to be in AW[*] has remained relatively short. In this talk we consider the parameterized membership of several k-move variants of a few specific games, including such well-known games as Chess and Othello.

Keywords: Games, AW[*]

Joint work of: Scott, Allan; Stege, Ulrike

Classes of graphs with a periodic structure

Detlef G. Seese (Universität Karlsruhe, D)

This talk will discuss algorithmic problems for classes of graphs having a periodic structure. An example are grid-structured hierarchical graphs, which are defined by giving a static graph defining the content of a cell of a d-dimensional grid, repeating this static graph in each cell and by connecting the vertices in cells of a local neighborhood corresponding to a finite transit function in a uniform way (introduced in [KMW67] see also [HoeLenWan92]). It is shown that the periodicity can be used to show that all first order (FO) problems and all monotone properties can be solved in constant time $O(1)$ for such classes. The first result improves for this class the linear time computability of FO problems for graphs of bounded degree from [See95], [See96]. As a by product we will show that the FO theory of the related classes of graphs is decidable.

[KMW67] Karp, R.M., Miller, R.E., Winograd, A.: The organization of computations for uniform recurrence equations; Journal of the ACM, 14, 3 (1967) 563-590.
[HoeLenWan92] Hoefting, F., Lengauer, T., Wanke, E.: Processing of hierarchically defined graphs and graph families, In B. Monien und T. Ottmann, (eds.), Data Structures and Efficient Algorithms, LNCS, 594 (1992) 44-69.
[See95] Seese, D.: Linear Time Computable Problems and Logical Descriptions, Proceedings of SEGRAGRA'95, Joint COMPUGRAPH/ SEMAGRAPH Workshop on Graph Rewriting and Computation, Volterra (Pisa, Italy), August 28 - September 1, 1995, A. Corradini, U. Montanari (eds.), in Electronic Notes in Theoretical Computer Science, 2 (1995) 219-232.
[See96] Seese, D.: Linear time computable problems and first-order descriptions; Math. Struct. in Comp. Science (1996) 505-526.

Keywords: Periodic structures, grid structured hierarchical graphs, first order problems, monotone properties, constant time, decidability

Catalan Structures and Dynamic Programming in H-minor free graphs

Dimitrios M. Thilikos (National and Capodistrian University of Athens, GR)

We give an algorithm that, for a fixed graph H and integer k, decides whether an n-vertex H-minor free graph G contains a path of length k in $2^{O(\sqrt{k})} \cdot n^{O(1)}$ steps. Our approach builds on a combination of Demaine-Hajiaghayi's bounds on the size of an excluded grid in such graphs with a novel combinatorial result on certain branch decompositions of H-minor free graphs. This result is used to bound the number of ways vertex disjoint paths can be routed through the separators of such decompositions. The proof is based on several structural theorems from the Graph Minors series of Robertson and Seymour.

With a slight modification, similar combinatorial and algorithmic results can be derived for many other problems. Our approach can be viewed as a general framework for obtaining time $2^{O(\sqrt{k})} \cdot n^{O(1)}$ algorithms on H-minor free graph classes.

Keywords: Branchwidth, catalan numbers, graqph minors, dynamic programming, longest path

Joint work of: Dorn, Frederic; Thilikos, Dimitrios M.; Fomin, Fedor V.

Parameterized Complexity for Graph Linear Arrangement Problems

Anders Yeo (RHUL - London, GB)

A linear arrangement (LA) of a graph G=(V,E) of order n is a bijection $\alpha : V \to \{1, \dots, n\}$. The length (net length) of an edge $uv \in E$ relative to α is defined as $\lambda_\alpha(uv) = |\alpha(u) - \alpha(v)|$ ($\lambda_\alpha(uv) = |\alpha(u) - \alpha(v)| - 1$). The cost (net cost) of an LA α is is the sum of the costs (net costs) relative to α of edges of G. For a LA α of G=(V,E), its *profile* is $prf(G) = \sum_{v \in V} \max\{\alpha(v) - \alpha(u) : u \in N[v], \alpha(u) \leq \alpha(v)\}$.

In his recent Habilitation thesis and at his talk at Dagstuhl in July 2005, H. Fernau considers the following problem: given a graph $G = (V, E)$ and a parameter k, check whether there is an LA of net cost at most k. Fernau asks whether the problem is fixed-parameter tractable (FPT), i.e., whether it can be solved by an algorithm of time complexity $O(f(k)(|V| + |E|)^t)$, where t is a constant independent of k and f is a computable function. We prove that the problem is FPT by deriving an algorithm of complexity $O(|V| + |E| + 5.88^k)$.

M. Serna and D.M. Thilikos (2005) ask whether the following problems are FPT: (i) given a graph $G = (V, E)$ and a parameter k, check whether there is an LA of cost at most $k|V|$; (ii) given a graph $G = (V, E)$ and a parameter k, check whether there is an LA of cost at most $k|E|$; (iii) given a graph $G = (V, E)$ and a parameter k, check whether there is an LA of profile at most $k|V|$. We prove that for any fixed $k \geq 2$ the following problems are NP-complete: check whether $G = (V, E)$ has an LA of cost at most $k|V|$ (cost at most $k|E|$, profile at most $k|V|$). Thus, the Serna-Thilikos problems are not FPT.

Joint work of: Gutin, Gregory; Rafiey, Arash; Szeider, Stefan; Yeo, Anders

Approximating Solution Structure

Iris van Rooij (TU of Eindhoven, NL)

Hen it is hard to compute an optimal solution $y \in \mathit{optsol}(x)$ to an instance x of a problem, one may be willing to settle for an efficient algorithm A that computes an approximate solution $A(x)$. The most popular type of approximation algorithm in Computer Science (and indeed many other applications) computes solutions whose value is within some multiplicative factor of the optimal solution value, *e.g.*, $\max(\frac{val(A(x))}{optval(x)}, \frac{optval(x)}{val(A(x))}) \leq h(|x|)$ for some function $h()$. However, an algorithm might also produce a solution whose structure is "close" to the structure of an optimal solution relative to a specified solution-distance function d, *i.e.*, $d(A(x), y) \leq h(|x|)$ for some $y \in \mathit{optsol}(x)$. Such structure-approximation algorithms have applications within Cognitive Science and other areas. Though there is an extensive literature dating back over 30 years on value-approximation, there is to our knowledge no work on general techniques for assessing the structure-(in)approximability of a given problem.

In this talk, we describe a framework for investigating the polynomial-time and fixed-parameter structure-(in)approximability of combinatorial optimization problems relative to metric solution-distance functions, *e.g.*, Hamming distance. We motivate this framework by (1) describing a particular application within Cognitive Science and (2) showing that value-approximability does not necessarily imply structure-approximability (and vice versa). This framework includes definitions of several types of structure approximation algorithms analogous to those studied in value-approximation, as well as structure-approximation problem classes and a structure-approximability-preserving reducibility. We describe a set of techniques for proving the degree of structure-(in)approximability of a given problem, and summarize all known results derived using these techniques. We also list 11 open questions summarizing particularly promising directions for future research within this framework. (co-presented with Todd Wareham)

Joint work of: Hamilton, Matthew; Müller, Moritz; van Rooij, Iris; Wareham, Todd

Keywords: Approximation Algorithms, Solution Structure

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/1236