07211 Abstracts Collection
Exact, Approximative, Robust and Certifying Algorithms on Particular Graph Classes
— Dagstuhl Seminar —

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Abstract. From May 20 to May 25, 2007, the Dagstuhl Seminar 07211 "Exact, Approximative, Robust and Certifying Algorithms on Particular Graph Classes" was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Graph theory, approximation algorithms, certifying algorithms, exact algorithms

07211 Summary – Exact, Approximative, Robust and Certifying Algorithms on Particular Graph Classes

The aim of this seminar was to bring together experts working on exact, approximative, robust and certifying algorithms on particular graph classes. Given the fast advances in various areas of graph algorithms on particular graph classes we have witnessed in the past few years, we believe that it was very important to offer researchers in these areas a forum for the exchange of ideas in the relaxed and inspiring workshop atmosphere that Dagstuhl always offers.

There was a strong interaction and healthy exchange of ideas which resulted in successful applications of exact, approximative, robust and certifying graph algorithms; in particular, the seminar dealt with the following topics and their interactions:

- Exact algorithms require that the algorithm provides exactly the result requested. The approach is interesting for NP-hard problems. Two different approaches are exponential-time algorithms and fixed-parameter algorithms. Exponential-time algorithms must solve the problem for all possible inputs exactly. The goal is to obtain an exponential running time being as small as possible as described in the important survey [G. Woeginger, Exact algorithms for NP-hard problems: A survey. In: Combinatorial Optimization - Eureka! You shrink!. M. Juenger, G. Reinelt and G. Rinaldi (eds.). LNCS 2570, Springer, 2003, pp 185-207.]

  Fixed-parameter algorithms are supposed to solve the problem exactly as long as the result is not larger than the given value of the parameter. In many cases fixed-parameter algorithms are tuned for "small parameters". Fixed-parameter algorithms have been studied extensively by Downey and Fellows [Fixed parameter complexity, Springer, 1999], and recent monographs by Niedermeier, and by Flum and Grohe.
- For approximative algorithms, two new concepts shall be discussed, which improve the running time considerably. The first one deals with parameterized complexity, where various parts of the input such as the number n of vertices or the size k of a maximum independent set play the role of a parameter and the running time of the algorithm is optimized with respect to the parameters. This approach is promising for polynomial approximation schemes.

  The second one concerns methods of (non-)linear programming for graph-theoretic problems. There are various optimization problems such as special network-flow problems or determining a maximum independent set in a perfect graph which have polynomial time algorithms but these are far from being really efficient since the algorithms have to solve large linear programming instances. The algorithms become much more efficient, however, if only approximative solutions (with good factors) are required and this is done using methods of (non-)linear programming.
- A robust algorithm for a graph class $\mathcal{C}$ and an algorithmic problem Π is always giving a correct answer: If the input graph G is in the class $\mathcal{C}$ then the problem Π will be correctly solved, and if G is not in $\mathcal{C}$ then either Π will be correctly solved or the algorithm finds out that G is not in $\mathcal{C}$. In both cases, the answer is correct, and the algorithm avoids recognizing $\mathcal{C}$. This can be of big advantage if recognizing $\mathcal{C}$ is NP-complete or even harder. There are various degrees of verification in the case that G is not in $\mathcal{C}$; a witness for this is desirable.
- Certifying recognition algorithms provide a proof respectively certificate for membership and non membership. Certifying algorithms are highly desirable in practice since implementations of correct algorithms may have bugs. Furthermore since the software producing the certificates may have bugs, the certificates have to be authenticated, and this should use a simple and efficient algorithm. A good example is the linear time certifying recognition algorithm for interval graphs [D. Kratsch, R. M. McConnell, K. Mehlhorn,

J. Spinrad, Certifying algorithms for recognizing interval graphs and permutation graphs, SODA 2003: 158-167].

As always, Schloss Dagstuhl and its stuff provided a very convenient and stimulating environment. The organizers wish to thank all those who helped to make the seminar a fruitful research experience.

A. Brandstädt, K. Jansen, D. Kratsch, J.P. Spinrad

Covering and packing planar and delta-hyperbolic graphs with balls

Victor Chepoi (Université de la Méditerrannée - Marseille, F)

In the talk, we will outline the proofs of the following two results:

Theorem 1. *There exists a universal constant c such that any planar graph G of diameter $2R$ can be covered with at most c balls of radius R.*

(This result was conjectured by C. Gavoille, D. Peleg, A. Raspaud, and E. Sopena.)

Theorem 2. *For any δ-hyperbolic graph G, the minimum number of balls of radius $R + \delta$ covering G is less or equal to the maximum number of pairwise disjoint balls of radius R of G. Such a covering and packing can be computed in polynomial time.*

Keywords: Covering, packing, balls, planar graph, delta-hyperbolic space, VC-dimension, primal-dual algorithm

Joint work of: Victor Chepoi, Bertrand Estellon, and Yann Vaxes

Flow Spanners

Feodor F. Dragan (Kent State University, USA)

In this talk, motivated by applications of ordinary (distance) spanners in communication networks and to address such issues as bandwidth constraints on network links, link failures, network survivability, etc., we introduce a new notion of *flow spanner*, where one seeks a spanning subgraph $H = (V, E')$ of a graph $G = (V, E)$ which provides a "good" approximation of the source-sink flows in G. We formulate few variants of this problem and investigate their complexities. A special attention is given to the version where H is required to be a tree.

Keywords: Network design, maximum flow preservation, spanners, spanning trees, approximation

Joint work of: Feodor F. Dragan and C. Yan

Full Paper:
http://www.cs.kent.edu/∼dragan/LATIN06.pdf

See also: Network Flow Spanners, F.F. Dragan and C. Yan, In Proceedings of the 7th Latin American Symposium "LATIN 2006: Theoretical Informatics", Valdivia, Chile, March 20-24, Springer, Lecture Notes in Computer Science 3887, pp. 410-422.

On finding another by parity

Jack Edmonds (University of Waterloo, CA)

A *d-oik,* $C = (V, F)$, short for d-dimensional Eulerian complex, $d \geq 1$, is a finite set V of elements called the vertices of C and a family of $d+1$ element subsets of V, called the rooms of C, such that every d element subset of V is in an even number of the rooms.

A 'wall of a room' means a set obtained by deleting one vertex of the room C and so any wall of a room in an oik is the wall of a positive even number of rooms of the oik.

Example 1. A d-dimensional simplicial pseudo-manifold is a d-oik where every d-element subset of vertices is in exactly zero or two rooms, i.e., in a simplicial pseudo-manifold any wall is the wall of exactly two rooms.

An important special case of simplicial pseudo-manifold is a triangulation of a compact manifold such as a sphere.

Example 2. Let $Ax = b, x \geq 0$, be a tableau as in the simplex method, whose solution-set is bounded and whose basic feasible solutions are all non-zero (non-degenerate).

Let V be the column-set of A. Let the rooms be the subsets S of columns such that $V - S$ is a feasible basis of the tableau. This is an $(n-r-1)$-oik where n is the number of columns of A and r is the rank (the number of rows) of A.

In fact it is a triangulation of an $(n-r-1)$-dimensional sphere – in particular it is combinatorially the boundary of a 'simplicial polytope'.

Example 3. Let the n members of set V be colored with r colors. Let the rooms be the subsets S of V such that $V - S$ contains exactly one vertex of each color. This is an $(n - r - 1)$-oik. In fact it is the oik of Example 2 where each column of A is all zeroes except for one positive entry.

Example 4. An Eulerian graph, that is a graph such that each of its vertices is in an even number of its edges (the rooms) is a 1-oik.

Example 5. For any connected Eulerian graph G with n vertices ($n \geq 3$), we have an $(n-2)$-oik (V, K) where V is the set of edges of G and the rooms are the edge-sets of the spanning trees of G.

Example 6. For any connected bipartite graph G with m edges and n vertices we have an $(m-n)$-oik where V is the edge-set of G, and the rooms are the edge-complements of spanning trees of G.

Example 7 (generalizing 5 and 6). Where M is an Eulerian binary matroid, that is a binary matroid of rank r such that each cocircuit, in fact each cocycle, is even, we have an $(r-1)$-oik, where V is the set of elements of the matroid, and the rooms are the bases of the matroid.

A 'binary matroid' M is given by a 0-1 matrix, A, $\mod 2$. The elements of M are the columns. The bases of M are the linearly independent sets of columns. The cocycles are the supports of the row vectors generated by the rows of A. The cocircuits are the minimal cocycles. Matroid M is Eulerian when each row of A has an even number of ones.

Let $M = [(V, F_i) : i = 1, \ldots, h]$ be an indexed collection of oiks (which we call an 'oik-family') all on the same vertex-set V.

The oiks of M are not necessarily of the same dimension. Of course, all of them may be the same oik.

A 'room-family', $R = [R_i : I = 1, \ldots, h]$, for oik-family M, is where, for each i, R_i is a room of oik i (i.e., a member of F_i).

A 'room-partition' R for M means a room-family whose rooms partition V, i.e., each vertex is in exactly one room of R.

Theorem 1. *Given an oik-family M and a room-partition R for M, there exists another different room-partition for M. In fact, for any oik-family M, there is an even number of room-partitions.*

Proof. Choose a vertex, say w, to be special.

A 'w-skew room-family for oik-family M' means a room-family, $R = [R_i : i = 1, \ldots, h]$, for M such that w is not in any of the rooms R_i, some vertex v is in exactly two of the R_i, and every other vertex is in exactly one of the R_i.

Consider the so-called exchange-graph X, determined by M and w, where the nodes of X are all the room-partitions for M and all the w-skew room-families for M. Two nodes of X are joined by an edge of X when each is obtained from the other by replacing one room by another.

It is easy to see that the odd-degree nodes of X are all the room-partitions for M, and all the even-degree nodes of X are the w-skew room-families for M. Hence there is an even number of room-partitions for M.

'Exchange algorithm': An algorithm for getting from one room-partition for M to another is to walk along a path in X, not repeating any edge of X, from one to another.

Where each oik of the oik-family M is a simplicial pseudo-manifold, X consists of disjoint simple paths, and so the algorithm is uniquely determined by M and w.

Where oik-family M consists of two oiks of the kind in Example 2, the exchange algorithm is the Lemke-Howson algorithm for finding a Nash equilibrium of a 2-person game.

Salvani and von Stengel show that the number of steps in the Lemke-Howson algorithm can grow exponentially relative to the size of the two tableaus of the game.

It is not known whether there is a polytime algorithm for finding a Nash equilibrium of a 2-person game. Deng proved a deep completeness result which is regarded as some evidence that there might not be a polytime algorithm. On the other hand, it is well-known that the simplex method for linear programming can grow exponentially with the size of the tableau and yet polytime algorithms were later found for that.

Suppose each oik of M is given by an explicit list of its rooms, each oik perhaps a simplicial pseudo-manifold, perhaps a 2-dimensional sphere. Is the exchange algorithm always polytime, relative to the number of rooms? In other words, is some component of the exchange graph well-bounded by the number of rooms?

How about the exchange algorithm when each oik of M is a 1-oik? If each oik of M is the same 1-oik then the well-known, non-trivial, non-bipartite matching algorithm can be used to find, if there is one, a first and a second room-partition. On the other hand, in that case, the rooms at each vertex can be paired to get effectively a 1-dimensional manifold where the exchange algorithm is trivial.

How about the exchange algorithm where each oik of M is an Eulerian binary matroid? For an oik-family like that, the well-known, non-trivial, 'matroid partition' algorithm can be used to find, if there is one, a first and a second room-partition.

More background and motivation can be found in my talk, 'Second Hamiltonian Paths and Nash Equilibria', posted at
http://www.math.mcgill.ca/~jbav/2nash.pdf, which is a link from Jacque Verstraete's home page.

Full Paper:
http://www.math.mcgill.ca/~jbav/2nash.pdf

Second Hamiltonian Paths and Nash Equilibria

Jack Edmonds (University of Waterloo, CA)

This is the text-slides of a recent related introductory survey about the complexity of algorithms, about second Hamiltonian paths, and about bimatrix games (2-person Nash equilibria).

Keywords: Graphs, parity, complexity, hamiltonian paths, bimatrix games, Nash equilibria

Full Paper:
http://www.math.mcgill.ca/~jbav/2nash.pdf

An $O(n^3)$-time recognition algorithm for hhds-free graphs

Elaine M. Eschen (West Virginia Univ. - Morgantown, USA)

The class of hhds-free graphs properly generalizes the classes of strongly chordal graphs and distance-hereditary graphs, and forms a restriction of the class of perfectly orderable graphs. The problem of recognizing hhds-free graphs in polynomial time was posed by Brandstädt (Problem session: Dagstuhl Seminar No. 04221, 2004), and Nikolopoulos and Palios (Proceedings of the 31st International Workshop on Graph Theoretic Concepts in Computer Science, WG2005) gave an $O(mn^2)$ algorithm. We present an $O(n^3)$-time algorithm for the problem. Our algorithm demonstrates a relationship between hhds-free graphs and strongly chordal graphs similar to that which is known to exist between hhd-free graphs and chordal graphs.

Keywords: Hhds-free, strongly chordal, hhd-free, chordal, graph

Joint work of: Chính Hoàng, Elaine M. Eschen, and R. Sritharan

Bounding the treewidth in terms of the maximum degree for special graph classes

Serge Gaspers (University of Bergen, N)

This talk is about linear upper bounds of the treewidth in terms of the maximum degree for some special graph classes. For each considered graph class, we determine a constant c such that the treewidth of any graph of this graph class is at most c times the maximum degree of the graph. We prove $c = 4$ for circle graphs, $c = 3$ for 4-chordal graphs and $c = 2$ for weakly chordal graphs. These bounds can be used, for example, to obtain faster exponential time algorithms for solving the minimum dominating set problem on these graph classes.

Keywords: Treewidth bounds, maximum degree, circle graphs, 4-chordal graphs, weakly chordal graphs

Joint work of: Serge Gaspers, Dieter Kratsch, Mathieu Liedloff, and Ioan Todinca

Variations on modular decomposition and interesting subset families

Michel Habib (LIAFA - Université Paris VII, F)

In social networks analysis several authors try to define subsets of vertices having the same behavior inside the networks. The concept of a role (Everett and Borgatti 1991) on the other hand seems promising, however its computation unfortunately is NP-hard (Fiala and Paulusma 2005). On the other hand modular decomposition of graphs seems to be too restrictive to be applied in real social networks analysis.

As a natural consequence, there is need for the search of *relaxed*, but *tractable*, variations of the modular decomposition scheme.

This talk presents some results in this research stream, and we propose to weaken the definition of module in order to further decompose.

In this search, we deal with several important subsets families, namely: laminar, intersecting, crossing, partitive and bipartitive. Some of these families admit a tree representation: i.e. a tree in size of the ground set, which allows to generate the whole family as for example for partitive family. We end with some questions on these tree representation theorems.

Joint work of: Michel Habib, Fabien de Montgolfier, Vincent Limouzy, and Binh Minh Bui Xuan

Interval completion with few edges

Pinar Heggernes (University of Bergen, N)

We present an algorithm with runtime $O(k^{2k}n^3m)$ for the following NP-complete problem: Given an arbitrary graph G on n vertices and m edges, can we obtain an interval graph by adding at most k new edges to G? This resolves the long-standing open question, first posed by Kaplan, Shamir and Tarjan, of whether this problem could be solved in time $f(k) \cdot n^{O(1)}$. The problem has applications in Physical Mapping of DNA and in Profile Minimization for Sparse Matrix Computations. For the first application, our results show tractability for the case of a small number k of false negative errors, and for the second, a small number k of zero elements in the envelope.

Our algorithm performs bounded search among possible ways of adding edges to a graph to obtain an interval graph, and combines this with a greedy algorithm when graphs of a certain structure are reached by the search. The presented result is surprising, as it was not believed that a bounded search tree algorithm would suffice to answer the open question affirmatively.

Keywords: Interval graphs, physical mapping, profile minimization, edge completion, FPT algorithm, branching

Joint work of: Pinar Heggernes, Jan Arne Telle, Christophe Paul, and Yngve Villanger

See also: To be presented at STOC 2007 - The 39th ACM Symposium on Theory of Computing

k-Colorability of P_5-free Graphs

Chinh T. Hoàng (Wilfrid Laurier University, CA)

A polynomial time algorithm that determines for a fixed integer k whether or not a P_5-free graph can be k-colored is presented in this paper. If such a coloring exists, the algorithm will produce a valid k-coloring.

Keywords: P_5-free, graph coloring, dominating clique

Joint work of: Chinh T. Hoàng, Marcin Kaminski, Vadim Lozin, J. Sawada, and X. Shu

Certifying algorithms and forbidden induced subgraphs

Dieter Kratsch (Université Paul Verlaine - Metz, F)

Certifying algorithms provide with each answer a certificate that is used to authenticate the correctness of the answer. The authentication algorithm is a separate algorithm that takes as input, the input of the original algorithm, its output and the certificate and checks (independent of the algorithms) the validity of certificate and algorithm's output.

Forbidden induced subgraphs are desirable certificates for certifying algorithms to recognize graph classes. They can be authenticated in time $O(1)$ by a simple algorithm.

We provide certifying recognition algorithms for split graphs, threshold graphs, bipartite chain graphs, cobipartite chain graphs and trivially perfect graphs.

All recognition algorithms run in time $O(n + m)$, the certificate for membership can be authenticated in time $O(n + m)$ and the certificate for nonmembership is a forbidden induced subgraph and can be authenticated in time $O(1)$.

Keywords: Algorithms, graph classes, certifying algorithm, certificate, authentication

Joint work of: Pinar Heggernes and Dieter Kratsch

Linear-time certifying recognition for partitioned probe cographs

Van Bang Le (Universität Rostock, D)

Cographs are those graphs without induced path on four vertices. A graph $G = (V, E)$ with a partition $V = P \cup N$ where N is an independent set is a partitioned probe cograph if one can add new edges between certain vertices in N in such a way that the graph obtained is a cograph. We characterize partitioned probe cographs in terms of five forbidden induced subgraphs. Using this characterization, we give a linear-time recognition algorithm for partitioned probe cographs. Our algorithm produces a certificate for membership that can be checked in linear time and a certificate for non-membership that can be checked in sublinear time.

Keywords: Cograph, probe cograph, certifying graph algorithm

Joint work of: Van Bang Le and H.N. de Ridder

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2007/1270

Exponential time algorithms for the minimum dominating set problem on some graph classes

Mathieu Liedloff (Université Paul Verlaine - Metz, F)

The Minimum Dominating Set problem remains NP-hard when restricted to any of the following graph classes: c-dense graphs, chordal graphs, 4-chordal graphs, weakly chordal graphs and circle graphs.

Developing and using a general approach, for each of these graph classes we present an exponential time algorithm solving the Minimum Dominating Set problem faster than the best known algorithm for general graphs.

Keywords: Exponential time algorithms, minimum dominating set, graph classes

Joint work of: Serge Gaspers, Dieter Kratsch, Mathieu Liedloff, and Ioan Todinca

Edge intersection graphs of single bend paths on a grid

Marina Lipshteyn (Haifa University, IL)

We combine the known notion of the edge intersection graphs of paths in a tree with a VLSI grid layout model to introduce the edge intersection graphs of paths on a grid.

Let $\mathcal{P}$ be a collection of nontrivial simple paths on a grid $\mathcal{G}$. We define the edge intersection graph $EPG(\mathcal{P})$ of $\mathcal{P}$ to have vertices which correspond to the members of $\mathcal{P}$, such that two vertices are adjacent in $EPG(\mathcal{P})$ if the corresponding paths in $\mathcal{P}$ share an edge in $\mathcal{G}$. An undirected graph G is called *an edge intersection graph of paths on a grid* (EPG) if $G = EPG(\mathcal{P})$ for some $\mathcal{P}$ and $\mathcal{G}$, and $(\mathcal{P}, \mathcal{G})$ is an EPG representation of G. We prove that every graph is an EPG graph.

A turn of a path at a grid point is called a *bend*. We consider here EPG representations in which every path has at most a single bend, called B_1-EPG representations and the corresponding graphs are called B_1-EPG graphs. We prove that any tree is a B_1-EPG graph. Moreover, we give a structural property that enables to generate non B_1-EPG graphs.

Furthermore, we characterize the representation of cliques and chordless 4-cycles in B_1-EPG graphs. We also prove that single bend paths on a grid have Strong Helly number 3.

Keywords: Intersection graphs, paths on a grid

Preconditions, Postconditions, and Certifying Algorithms

Ross McConnell (Colorado State University, USA)

A certifying algorithm is one that produces a certificate with each output that proves that it has not been compromised by an implementation bug.

An example is an algorithm for recognizing interval graphs that either returns an interval model, proving that the given graph is an interval graph, or that points out an instance of one of the well-known forbidden subgraphs for the class. This sidesteps the problem of proving that the implementation is bug-free; it shows only that no bug has compromised the output for the given instance.

We survey how the undergraduate curriculum could be modified to incorporate certifying algorithms as a design philosophy. We also touch on a novel type of algorithm that produces a certificate that one of the following has occurred:

1. A correct output was given;
2. A precondition on the input was violated.

It is not necessary that the certificate provide a simple or efficient means of checking which of these two events occurred; in any case the certificate proves that the algorithm has satisfied the requirement that its precondition implies its postcondition. The advantage of these is that they are often easier to come up with, and they can be chained together as steps of certifying algorithms that have no preconditions.

Keywords: Certifying algorithms, software reliability, undergraduate curriculum, graph theory

Joint work of: Ross McConnell and Kurt Mehlhorn

See also: D. Kratsch, R.M. McConnell, K. Mehlhorn, J.P. Spinrad, Certifying algorithms for recognizing interval graphs and permutation graphs, Siam J. on Computing, 36(2) 2006, 326-353.

Linear Problem Kernels for NP-Hard Problems on Planar Graphs

Rolf Niedermeier (Universität Jena, D)

We develop a generic framework for deriving linear-size problem kernels for NP-hard problems on planar graphs. We demonstrate the usefulness of our framework in several concrete case studies, giving new kernelization results for Connected Vertex Cover, Minimum Edge Dominating Set, Maximum Triangle Packing, and Efficient Dominating Set on planar graphs. On the route to these results, we present effective, problem-specific data reduction rules that are useful in any approach attacking the computational intractability of these problems.

Keywords: Fixed-parameter tractability, reduction to a problem kernel, preprocessing, data reduction

Joint work of: Jiong Guo and Rolf Niedermeier

Network Reliability – Network Sharing, Parts of Desired Sizes in Graphs

Dieter B. Rautenbach (TU Illmenau, D)

Motivated by problems arising in relation to network reliability and network sharing we discuss conditions, results and conjectures about the existence of disjoint connected induced subgraphs of specified minimum orders within a given graph.

Keywords: Network Reliability, Network Sharing, connectivity

Joint work of: Dieter B. Rautenbach and L. Volkmann

A new upper bound for the chromatic number of a graph

Ingo Schiermeyer (TU Bergakademie Freiberg, D)

Let G be a graph of order n with clique number $\omega(G)$, chromatic number $\chi(G)$ and independence number $\alpha(G)$. We show that $\chi(G) \leq \frac{n+\omega+1-\alpha}{2}$. Moreover, $\chi(G) \leq \frac{n+\omega-\alpha}{2}$, if either $\omega+\alpha = n+1$ and G is not a split graph or $\alpha+\omega = n-1$ and G contains no induced $K_{\omega+3} - C_5$.

Keywords: Vertex colouring, chromatic number
See also: Discussiones Mathematicae Graph Theory 27(1) (2007) 137-142

Chordal bipartite completion of colored graphs

R. Sritharan (University of Dayton, USA)

Golumbic, Kaplan, and Shamir, in their paper on graph sandwich problems published in 1995, left the status of the sandwich problems for strongly chordal graphs and chordal bipartite graphs open. It was recently shown by de Figueiredo, Faria, Klein, and Sritharan that the sandwich problem for strongly chordal graphs is NP-complete. We show that given graph G with a proper vertex coloring c, determining whether there is a supergraph of G that is chordal bipartite and also is properly colored by c is NP-complete. This implies that the sandwich problem for chordal bipartite graphs is also NP-complete.

Keywords: Vertex coloring, chordal bipartite graphs

Complexity Aspects of Convexity in Graphs

Jayme L. Szwarcfiter (Federal University - Rio de Janeiro, BR)

Let G be a graph. If $u, v \in V(G)$, a $u-v$ *geodesic* of G is a shortest path linking u and v.

The *closed interval* $I[u, v]$ consists of all vertices lying in some $u-v$ geodesic of G. For $S \subseteq V(G)$, the set $I[S]$ is the union of all sets $I[u, v]$ for $u, v \in S$. We say that S is a *convex set* if $I[S] = S$.

A set S is a *geodetic* if $I[S] = V(G)$. The cardinality of a minimum geodetic set of G is the *geodetic number* of G, while the cardinality of a maximum convex set of G, properly contained in $V(G)$, is the *convexity number* of G.

The *convex hull* of S, denoted $I_h[S]$, is the smallest convex set containing S. A set S is a *hull set* of G if $I_h[S] = V(G)$. The cardinality of a minimum hull set of G is the *hull number* of G.

In this talk, we discuss different aspects related to the computation of the above parameters. For each of them, we describe bounds, hardness results and some polynomial-time solvable cases. Finally, we also discuss the problem of partitioning the vertices of a graph into a desired number of convex subsets.

Keywords: Algorithms, convexity, geodesics, graphs, NP-hardness, partitions

Minimal interval completions and pathwidth of circular-arc graphs

Ioan Todinca (Université d'Orleans, F)

An interval completion of an arbitrary graph $G = (V, E)$ is an interval supergraph $H = (V, F)$ (i.e. $E \subseteq F$).

The pathwidth of G is the minimum cliquesize of H, minus one, over all interval completions H of G. Similarily, the profile of G is the minimum number of edges of an interval completion of G. Both pathwidth and profile are NP-hard to compute. Note that optimal completions for pathwidth and profile can be found among the *minimal interval completion* of the graph, i.e. interval completions H such that no proper subgraph of H is an interval completion of G.

We introduce in this talk the notion of *folding* of an interval graph. Based on this tool, we give polynomial time algorithms for (1) extracting a minimal interval completion from an arbitrary one (2) computing the pathwidth of circular-arc graphs.

On (k, l)-Leaf Powers

Peter Wagner (Universität Rostock, D)

Motivated by questions in biology, where the evolutionary history of a set of species is studied, the notion of k-leaf power was introduced by Nishimura, Ragde and Thilikos.

Based on this we introduce the following notion: A graph $G = (V, E)$ is a (k, l)-leaf power if there is a tree T with leaf set V, such that, for any two vertices x and y of G, we have that $xy \in E$ implies that the distance $d(x, y)$ between the leaves x and y in T is at most k and $xy \notin E$ implies that $d(x, y)$ is at least l.

We study the various classes of (k, l)-leaf powers and identify some that occur for infinitely many pairs (k, l), and we discuss the special case of $(6, 8)$-leaf powers, where we give two characterisations, one of them in terms of forbidden subgraphs, which represent some separator properties in chordal graphs.

Keywords: (k,l)-leaf powers; leaf powers; leaf roots; strictly chordal graphs

Joint work of: Brandstädt, Andreas; Wagner, Peter (speaker)

On Graph Sandwich Problems

Celina de Figueiredo (University of Rio de Janeiro, BR)

Golumbic, Kaplan, and Shamir, in their paper on graph sandwich problems published in 1995 (J. Algorithms 19(3), pp.449-473), left the status of the sandwich problems for strongly chordal graphs and chordal bipartite graphs open.

We prove that the sandwich problem for strongly chordal graphs is NP-complete.

We also give some comments on the computational complexity of the sandwich problem for chordal bipartite graphs.