Diagonal Circuit Identity Testing and Lower Bounds

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Abstract of Dagstuhl Talk

In this talk we give a deterministic polynomial time algorithm for testing whether a *diagonal* depth-3 circuit $C(x_1, \ldots, x_n)$ (i.e. C is a sum of powers of linear functions) is zero. We also prove an exponential lower bound showing that such a circuit will compute determinant or permanent only if there are exponentially many linear functions. Our techniques generalize to the following results:

1. Suppose we are given a depth-3 circuit of the form:

$$C(x_1, \ldots, x_n) := \sum_{i=1}^{k} \ell_{i,1}^{e_{i,1}} \cdots \ell_{i,s}^{e_{i,s}}$$

   where, $\ell_{i,j}$'s are linear functions living in $\mathbb{F}[x_1, \ldots, x_n]$. We can test whether C is zero in deterministic time $\mathit{poly}\,(nk, \mathit{max}\{(1+e_{i,1}) \cdots (1+e_{i,s}) \mid 1 \leqslant i \leqslant k\})$. This immediately gives a deterministic $\mathit{poly}(nk2^d)$ time identity test for general depth-3 circuits of degree d.

2. We prove that if the above circuit $C(x_1, \ldots, x_n)$ with a "small" $s = o\left(\frac{m}{\log m}\right)$ computes the determinant (or permanent) of an $m \times m$ matrix then $k = 2^{\Omega(m)}$.

Our results work for all fields $\mathbb{F}$. (Previous exponential lower bounds for depth-3 only work for nonzero characteristic.)

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