Topological and Game-Theoretic Aspects of Infinite Computations

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Abstract. The theory of the infinite behaviour of continuously operating computing devices is of primary importance for several branches of theoretical and practical computer science. In particular, it is fundamental for the verification and synthesis of reactive systems like microprocessors or operating systems, for the understanding of dataflow computation, and for the development of adequate mathematical foundations for exact real computation. The seminar brought together researchers from many different disciplines who are working on theoretical or practical aspects of infinite computations. In this summary we describe the topics, the goals, and the contributions of the seminar.

Key words: Automata theory, computability in analysis, dataflow computation, hierarchies, infinite computations, infinite games, reactive systems, specification and verification, topological complexity, Wadge reducibility.

1 Motivation

The theory of the infinite behaviour of continuously operating computing devices is of primary importance for several branches of theoretical and practical computer science. In particular, it is fundamental for the verification and synthesis of reactive systems like microprocessors or operating systems, for the understanding of dataflow computation, and for the development of adequate mathematical foundations for exact real computation. The theory of infinite computations develops different classifications of the corresponding mathematical objects, for example, languages of infinite words or trees as in automata theory, filters which transform infinite streams in dataflow computation, or functions on infinite words as in computability in analysis.

The study of computability with infinite objects, unlike that with finite objects, is intimately related to topology. It makes essential use of such classification

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tools as hierarchies as in descriptive set theory, reducibilities as in computability theory, and infinite games with perfect information. In the development of this area, ideas and techniques from different fields were employed, e.g. from topology, logic, game theory, Wadge reducibility, and domain theory. This work has produced, in particular, an impressive theory of the specification and verification of reactive systems, which is developing rapidly and is already of practical importance for the computer industry.

Despite the tight connections with several branches of mathematics, researchers from the cited fields often work independently and from time to time rediscover notions and techniques that have been used already in another field. The idea of this seminar was to bring together researchers from different fields, ranging from pure mathematics (descriptive set theory, domain theory, logic, symbolic dynamics) to theoretical and applied computer science (automata theory, computability in analysis, model checking), who work on topological and game-theoretic aspects of infinite computations, using similar notions and tools. The goal of the seminar was to stimulate their work and establish ways for future cooperation and synergy. For example, while the theory of automata over infinite words already has direct applications in the theory of specification and synthesis of reactive systems, it would be very interesting to establish such direct applications also for the theory of infinite games.

2 Organization

The seminar was attended by 35 people from various scientific areas related to the topics of the seminar: by people working in computability, topology, descriptive set theory, automata theory, game theory, and by people working on applications in verification and synthesis of reactive systems and stream computations. In fact, many participants are working in several of these areas and presented results connecting two or more of these areas. There were 5 overview talks of fifty minutes each and 26 contributed talks of thirty minutes each. Besides that, there were many discussions among smaller groups of people.

3 Topics

In the following section we summarize the results presented in the talks. In this section we give a very short description of the topics considered during the seminar.

Infinite computations were the common topic of essentially all talks of the seminar. Infinite computations can be performed by Turing machines or by automata. In both cases, topology is essential for describing what may happen and for classifying problems. Many different hierarchies were used, not only purely topological ones like the Borel hierarchy or the Wadge hierarchy, but also an effective Borel hierarchy and computability-theoretic hierarchies like the arithmetical hierarchy and the analytical hierarchy. Computability-theoretic applications ranged from computable analysis over computability over certain structures to higher type computation and to graph-theoretical problems, and there were also connections to the notion of algorithmic randomness. For the automata-theoretic side, topological and automata-theoretic hierarchies, e.g. the Wagner hierarchy, were important. There were many talks concerned with extending known results in various directions, for example to extend results about regular ω-languages to tree languages. Besides topology, infinite games played a particularly important role and were themselves the topic of investigations. Various applications of automata theory and the theory of infinite games were discussed, for example the specification and synthesis of reactive systems as well as dataflow computation models, applications in Cartesian programming, and applications in the semantics of logic programs and of certain grammars.

4 Results

This section gives an overview of the results presented in the talks of the seminar. In this overview, we follow roughly the keywords listed in the second section.

A main theme of the seminar were topological aspects of infinite computations. Often, the complexity of problems is measured using a topological hierarchy, frequently defined via some reducibility relation. The idea of comparing problems via some reducibility relation is in fact borrowed from computability theory. In his overview talk, Vasco Brattka combined topology and computability theory. He gave an introduction to the representation based approach to computable analysis and showed how an effective analogue (for functions) of Wadge reducibility (for sets) can be used in order to classify problems from analysis, in particular from functional analysis. When a Turing machine is supposed to produce an infinite output string then usually it is not allowed to change already produced output bits. Martin Ziegler analyzed what happens topologically when one drops this condition and allows machines to revise their results. Dietrich Kuske presented new characterisations of the complexity of certain graph theoretic problems in terms of the arithmetical and the analytical hierarchy. Oleg Kudinov presented new results concerning computability theory over certains structures, in particular extensions of the Gandy theorem in admissible set theory. Veronica Becher and Serge Grigorieff considered an effective reducibility relation from the Cantor space to certain ordered sets. On the one hand, they extended known hardness results with respect to many-one reduction to hardness results with respect to this reduction. On the other hand, they showed that there are interesting connections to the notion of algorithmic (n)-randomness.

The results discussed so far were mostly of computability-theoretic nature. Now we come to results that are of topological nature, although they are dealing with objects and questions from either computability theory or automata theory. Matthias Schröder presented a solution to a question in topology which had been open for some time and which was motivated by another yet unsolved problem in higher type computation: he showed that the sequential topology on the Kleene-Kreisel continuous functionals of type k is not regular for $k \geq 2$. Steve Matthews gave an introduction to the motivation, theory and applications in computer science of partial metric spaces. Anton Zhukov considered a topological hierarchy (called h-quasiorder) of k-partitions defined via Wadge reducibility and presented new results concerning definability in this order. In his overview talk, Victor Selivanov discussed this hierarchy and presented many results and open problems concerning this hierarchy and other ones defined via Wadge reducibility and effective versions of Wadge reducibility on various spaces like the Baire space and the Baire domain. Olivier Finkel presented several new results concerning the topological complexity of the ω-power L^ω of a language L, among them the surprising result that for each non-null countable ordinal ξ there exist some Σ^0_ξ-complete ω-powers and some Π^0_ξ-complete ω-powers.

Next, we come to results related to topology and automata. Pierre Simonnet presented a number of open questions about (effective) descriptive set theory and automata on infinite strings, for example concerning the ω-power L^ω of a language L. Ludwig Staiger analyzed information-theoretic properties of regular ω-languages, using Hausdorff dimension as a measure of information. Jean-Eric Pin gave an overview talk on profinite topologies, a tool for studying regular languages. He demonstrated how they can be used to define reducibility relations similar to Wadge reducibility. The Wadge hierarchy has a counterpart over regular ω-languages: the Wagner hierarchy. Jeremie Cabessa constructed an algebraic counterpart to the Wagner hierarchy. Filip Murlak presented several new results obtained in the ongoing attempt to generalize the Wagner hierarchy to tree languages, among them a structural description of the Wadge hierarchy restricted to deterministically recognizable tree languages. Jacques Duparc showed that the family of tree languages recognized by weak parity automata is closed under certain set theoretic operations. Damian Niwinski pointed out that the automata-theoretic complexity of infinite objects (strings or trees) is often, but not always, underlined by the topological complexity. He substantiated this observation by some new results concerning Rabin recognizable tree languages recognized by Büchi automata. Olivier Carton generalized important parts of the theory of automata on finite or infinite strings to automata on linear orderings.

Closely related to the theory of automata on infinite strings is the theory of infinite games. Julian Bradfield reviewed connections between inductive definitions, fixpoints logics, games, and the Wadge hierarchy. Wolfgang Thomas gave an overview talk in which he described how to solve infinite games with MSO-definable conditions. He also described three applications, namely to the complementation problem, to the synthesis of finite-state controllers, and to model-checking games, in particular for the modal μ-calculus. The starting point in this area are the Church Synthesis Problem and its solution by Büchi and Landweber. Alexander Rabinovich generalized this problem to countable ordinals and important parts of the Büchi-Landweber result from games of length ω to games of arbitrary countable length. Łukasz Kaiser looked at quantitative extensions of the modal μ-calculus and analyzed to what extent qualitative statements concerning this calculus and games can be lifted to quantitative statements. Michael Ummels studied the complexity of pure-strategy Nash equilibria in (turn-based) stochastic multiplayer games with omega-regular winning conditions and observed that many natural questions in this context are undecidable. Nir Piterman demonstrated that in many cases of practical importance one can synthesize digital designs from their LTL specifications in reasonable time and gave industrial examples that have been synthesized automatically from their specifications.

Another application of infinite computation can be found in programming. In his overview talk Bill Wadge described how the wish to make programs more amenable to verification led to Lucid, a programming language in which a program is a system of equations defining infinite streams of data. This leads naturally to an implementation in a dataflow computation model. Blanca Mancilla and John Plaice described different aspects of a related language, TransLucid, for Cartesian programming. They described the syntax and operational semantics of TransLucid, demonstrated that many important programming paradigms can be realized in TransLucid, and discussed how it can be used for real-world programming and how computational threads can interact in TransLucid. Chrysida Galanaki and Panos Rondogiannis looked at the semantics of logic programs with negation and suggested a new definition of the semantics of negation using infinite games of perfect information. The idea behind the well-founded semantics of logic programs with negation was applied by Vassilis Kountouriotis in order to define an interesting new semantics for Boolean grammars, an important extension of context-free grammars.