08271 Abstracts Collection
Topological and Game-Theoretic Aspects of Infinite Computations
— Dagstuhl Seminar —

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Abstract. From June 29, 2008, to July 4, 2008, the Dagstuhl Seminar 08271 "Topological and Game-Theoretic Aspects of Infinite Computations" was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, many participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Automata theory, computability in analysis, dataflow computation, hierarchies, infinite computations, infinite games, reactive systems, specification and verification, topological complexity, Wadge reducibility

08271 Executive Summary – Topological and Game-Theoretic Aspects of Infinite Computations

The theory of the infinite behaviour of continuously operating computing devices is of primary importance for several branches of theoretical and practical computer science. In particular, it is fundamental for the verification and synthesis of reactive systems like microprocessors or operating systems, for the understanding of dataflow computation, and for the development of adequate

mathematical foundations for exact real computation. The seminar brought together researchers from many different disciplines who are working on theoretical or practical aspects of infinite computations. In this summary we describe the topics, the goals, and the contributions of the seminar.

Joint work of: Hertling, Peter; Selivanov, Victor; Thomas, Wolfgang; Wadge, Bill; Wagner, Klaus

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1649

Effective Wadge Reductions and Randomness

Veronica Becher (University of Buenos Aires, AR)

We obtain a large class of significant examples of n-random reals. Any such real is defined as the probability that a universal monotone Turing machine performing possibly infinite computations on infinite inputs produces an output in a given set O. The input space is the Cantor space 2^ω. The output space is the space of equivalent classes of limits of nondecreasing sequences of elements in a given a countable set endowed with a computable order. For instance, $2^{\leq\omega}, \mathbb{R}, \mathbb{Q}, P(\mathbb{N}), C[0,1]$.

We give a general theorem that, roughly, it says "the harder the set O, the more random the associated probability". The proof uses effective Wadge reductions from Σ^0_n (or Π^0_n) classes in the Cantor space. In particular, we develop methods to transfer Σ^0_n or Π^0_n many-one completeness results of index sets to n-randomness of associated probabilities.

Keywords: Effective Wadge reductions, infinite computations, random reals

Joint work of: Becher, Veronica; Grigorieff, Serge

Fixpoints and Games

Julian Bradfield (University of Edinburgh, GB)

We review the connexions between inductive definitions, fixpoint logics, and games, and in particular the equation via the game quantifier between the fixpoint hierarchy and the difference hierarchy over Σ^0_2. We end with a connexion to the Wadge hierarchy.

Keywords: Games, inductive definitions, fixpoints, descriptive set theory

The Effective Borel Hierarchy in Computable Analysis

Vasco Brattka (University of Cape Town, ZA)

We introduce some basic ideas of the representation based approach to computable analysis and we demonstrate how these tools can be applied in effective descriptive set theory. In particular, we discuss Weihrauch's reducibility for functions and a corresponding notion of completeness. This reducibility is an analogue of Wadge reducibility for functions. We discuss several theorems from functional analysis and their Borel complexity.

Keywords: Computable analysis, effective descriptive set theory, Weihrauch reducibility.

The Algebraic Counterpart of the Wagner Hierarchy

Jeremie Cabessa (Univ. de Lausanne, CH)

The Wagner hierarchy is known so far to be the most refined topological classification of ω-rational languages. Also, the algebraic study of formal languages shows that these ω-rational sets correspond precisely to the languages recognizable by finite pointed ω-semigroups. Within this framework, we provide a construction of the algebraic counterpart of the Wagner hierarchy. We adopt a hierarchical game approach, by translating the Wadge theory from the ω-rational language to the ω-semigroup context.

More precisely, we first define a reduction relation on finite pointed ω-semigroups by means of a Wadge-like infinite two-player game. The collection of these algebraic structures ordered by this reduction is then proven to be isomorphic to the Wagner hierarchy, namely a well-founded and decidable partial ordering of width 2 and height ω^ω. We next describe a decidability procedure of this algebraic hierarchy based on a graph representation of finite pointed ω-semigroups. It follows that the Wagner degree of every ω-rational language can now be computed directly on its syntactic image.

Keywords: ω-automata, ω-rational languages, ω-semigroups, infinite games, hierarchical games, Wadge game, Wadge hierarchy, Wagner hierarchy.

Automata on Linear Orderings

Olivier Carton (LIAFA - Université Paris Diderot and CNRS, FR)

In this talk, we present automata accepting words indexed by linear orderings. These automata are simple and they generalize naturally automata on finite words, infinite words, biinfinite words and transfinite words. Languages accepted by these automata can also be described by suitable rational expressions (Kleene Theorem). These expressions generalize the usual Kleene operations. We also discuss the complementation problem and connexions to logic.

Weak Alternating Automata

Jacques Duparc (Univ. de Lausanne, CH)

In this joint work with F. Murlak, we show that the family of tree languages recognized by weak parity automata is closed under set theoretic operations that correspond to sum, multiplication by ordinals below ω^ω, and exponentiation of base ω_1, once sets are replaced by their Wadge degrees.

As a consequence, the Wadge hierarchy of weakly recognizable tree languages - as well as the one of infinite models of μ-calculus formulas of alternation depth 1 - has the height of at least ε_0 (the least fixed point of the exponentiation of base ω).

Joint work of: Duparc, Jacques; Murlak, Filip

There Exist Some omega-Powers of any Borel Rank

Olivier Finkel (ENS - Lyon, FR)

The operation of taking the omega-power V^ω of a language V is a fundamental operation over finitary languages leading to omega-languages. Since the set X^ω of infinite words over a finite alphabet X can be equipped with the usual Cantor topology, the question of the topological complexity of omega-powers of finitary languages naturally arises and has been posed by Niwinski (1990), Simonnet (1992), and Staiger (1997).

We investigate the topological complexity of omega-powers. We prove the following very surprising result which shows that omega-powers exhibit a great topological complexity: for each non-null countable ordinal ξ, there exist some Σ^0_ξ-complete ω-powers, and some Π^0_ξ-complete ω-powers. This result has some effective versions.

Keywords: Infinite words, omega-languages, omega-powers, Cantor topology, topological complexity, Borel sets, Borel ranks, Wadge hierarchy, Wadge degrees, complete sets

Joint work of: Finkel, Olivier; Lecomte, Dominique

Full Paper:
http://fr.arxiv.org/abs/0706.3523

Topological Complexity of omega-Powers: Extended Abstract

Olivier Finkel (ENS - Lyon, FR)

The operation of taking the omega-power V^ω of a language V is a fundamental operation over finitary languages leading to omega-languages.

Since the set X^ω of infinite words over a finite alphabet X can be equipped with the usual Cantor topology, the question of the topological complexity of omega-powers of finitary languages naturally arises and has been posed by Damian Niwinski (1990), Pierre Simonnet (1992), and Ludwig Staiger (1997).

We investigate the topological complexity of omega-powers. We prove the following very surprising results which show that omega-powers exhibit a great topological complexity: for each non-null countable ordinal ξ, there exist some Σ^0_ξ-complete omega-powers, and some Π^0_ξ-complete omega-powers. On the other hand, the Wadge hierarchy is a great refinement of the Borel hierarchy, determined by Bill Wadge. We show that, for each ordinal ξ greater than or equal to 3, there are uncountably many Wadge degrees of omega-powers of Borel rank $\xi + 1$. Using tools of effective descriptive set theory, we prove some effective versions of the above results.

Keywords: Infinite words, omega-languages, omega-powers, Cantor topology, topological complexity, Borel sets, Borel, ranks, complete sets, Wadge hierarchy, Wadge degrees, effective descriptive set theory, hyperarithmetical hierarchy

Joint work of: Finkel, Olivier; Lecomte, Dominique

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2008/1650

Infinite Game Semantics for Logic Programs with Negation

Chrysida Galanaki (National and Kapodistrian University of Athens, GR)

We present Infinite Games of Perfect Information that capture the well-founded semantics of Logic Programs with negation. In the games, two players, the Believer and the Doubter, compete by trying to prove (respectively disprove) a query. The games use degrees of winning and losing for the two players. In the case of finite logic programs, the game is proved to be determined and corresponds to the Infinite-valued Minimum Model Semantics of negation. In the case of logic programs with a countably infinite number of rules we define a refined game in which the two players in their first moves make a bet in the form of a countable ordinal. Each ordinal can be considered as a kind of clock that imposes a "time limit" to the moves of the corresponding player.

Joint work of: Galanaki, Chrysida; Rondogiannis, Panos; Wadge, William W.

General Logic Programs as Infinite Games

Chrysida Galanaki (National and Kapodistrian University of Athens, GR)

In [vE86] M.H. van Emden introduced a simple game semantics for definite logic programs. Recently [RW05,GRW05], the authors extended this game to apply to logic programs with negation.

Moreover, under the assumption that the programs have a finite number of rules, it was demonstrated in [RW05,GRW05] that the game is equivalent to the well-founded semantics of negation. In this paper we present work-in-progress towards demonstrating that the game of [RW05,GRW05] is equivalent to the well-founded semantics even in the case of programs that have a countably infinite number of rules. We argue however that in this case the proof of correctness has to be more involved. More specifically, in order to demonstrate that the game is correct one has to define a refined game in which each of the two players in his first move makes a bet in the form of a countable ordinal. Each ordinal can be considered as a kind of clock that imposes a "time limit" to the moves of the corresponding player. We argue that this refined game can be used to give the proof of correctness for the countably infinite case.

Keywords: Infinite Games, Negation in Logic Programming, Well-Founded Semantics

Joint work of: Galanaki, Chrysida; Rondogiannis, Panos; Wadge, William W.

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1651

Effective Wadge reductions (Wadge Games with Computable Strategies)

Serge Grigorieff (LIAFA - Université Paris Diderot and CNRS, FR)

We consider computable reductions from the Cantor space 2^ω to Scott domains. Such reductions correspond to winning strategies in Wadge games where player I plays successive digits of an infinite binary sequence whereas player II plays successive approximations of an element of the Scott domain.

Focusing on the Scott domain $\mathfrak{P}(\mathbb{N})$, we transfer some classical many-one hardness of index sets into hardness with respect to computable reductions $2^\omega \to \mathfrak{P}(\mathbb{N})$. Sometimes such a transfer theorem leads to almost everywhere reductions (i.e., the strategy is winning almost for sure, not for sure).

Keywords: Wadge reduction, many-one reduction, index set, Scott domain

Joint work of: Grigorieff, Serge and Becher, Veronica

Model Checking Games for the Quantitative μ-Calculus

Łukasz Kaiser (RWTH Aachen, DE)

We investigate quantitative extensions of modal logic and the modal μ-calculus, and study the question whether the tight connection between logic and games can be lifted from the qualitative logics to their quantitative counterparts.

It turns out that, if the quantitative μ-calculus is defined in an appropriate way respecting the duality properties between the logical operators, then its model checking problem can indeed be characterised by a quantitative variant of parity games. However, these quantitative games have quite different properties than their classical counterparts, in particular they are, in general, not positionally determined.

The correspondence between the logic and the games goes both ways: the value of a formula on a quantitative transition system coincides with the value of the associated quantitative game, and conversely, the values of quantitative parity games are definable in the quantitative μ-calculus.

Full Paper:
http://www.logic.rwth-aachen.de/Publications/kaiser.html

Well-Founded Semantics for Boolean Grammars

Vassilis Kountouriotis (National and Kapodistrian University of Athens, GR)

Boolean grammars are a promising extension of context-free grammars that supports conjunction and negation. In this paper we give a novel semantics for Boolean grammars which applies to all such grammars, independently of their syntax. The key idea of our proposal comes from the area of negation in logic programming, and in particular from the so-called well-founded semantics which is widely accepted in this area to be the "correct" approach to negation. We show that for every Boolean grammar there exists a distinguished (three-valued) language which is a model of the grammar and at the same time the least fixed point of an operator associated with the grammar.

Then, we demonstrate that every Boolean grammar can be transformed into an equivalent (under the new semantics) grammar in normal form. Based on this normal form, we propose an $O(n^3)$ algorithm for parsing that applies to any such normalized Boolean grammar. In summary, the main contribution of this paper is to provide a semantics which applies to all Boolean grammars while at the same time retaining the complexity of parsing associated with this type of grammars.

Keywords: Grammar, boolean, context-free, well-founded, model, semantics

Joint work of: Kountouriotis, Vassilis; Nomikos, Christos; Rondogiannis Panos

On the Semantic Approaches to Boolean Grammars

Vassilis Kountouriotis (National and Kapodistrian University of Athens, GR)

Boolean grammars extend context-free grammars by allowing conjunction and negation in rule bodies. This new formalism appears to be quite expressive and still efficient from a parsing point of view.

Therefore, it seems reasonable to hope that boolean grammars can lead to more expressive tools that can facilitate the compilation process of modern programming languages. One important aspect concerning the theory of boolean grammars is their semantics. More specifically, the existence of negation makes it difficult to define a simple derivation-style semantics (such as for example in the case of context-free grammars). There have already been proposed a number of different semantic approaches in the literature. The purpose of this paper is to present the basic ideas behind each method and identify certain interesting problems that can be the object of further study in this area.

Keywords: Boolean Grammars, Negation in Formal Grammars, Well-Founded Semantics

Joint work of: Kountouriotis, Vassilis; Nomikos, Christos; Rondogiannis, Panos

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1652

Gandy Theorems and Computability over Structures

Oleg V. Kudinov (Sobolev Institute of Mathematics - Novosibirsk, RU)

The central role of Gandy theorem in admissible set theory is well-known, providing many helpful consequences like the construction of universal $\Sigma-$predicates and nice properties of $\Sigma-$definability. It is natural to develop and modify corresponding theory in two different directions - for superstructures $\mathbf{HF}(\mathbf{M})$ in languages without equality and for partially ordered structures under some restrictions on them. Some applications of obtained theorems like bi-interpretations illustrate the power of the considered approach.

Keywords: Least fixed points, Σ-definability, computability theory, bi-interpretations

Recursive and Automatic Graphs with Hamiltonian and Eulerian Paths

Dietrich Kuske (Universität Leipzig, DE)

I indicate proofs of the following results (that refer to the arithmetical and the analytical hierarchy):

(1) the existence of an Eulerian path is complete for - $D_3^0 = \{K \setminus L \mid K, L \in \Pi_3^0\}$ for recursive graphs - Π_3^0 for (planar) locally finite recursive graphs (of degree 4) - Π_2^0 for (planar) automatic graphs (of degree 4)
(2) The existence of a Hamiltonian path is Σ_1^1-complete for planar automatic graphs of bounded degree.

These results sharpen insights by Beigel and Gasarch (as far as (1) for recursive graphs is concerned) and by Hirst and Harel (regarding (2)).

Keywords: Arithmetical and analytical hierarchy, infinite graphs, recursive graphs, automatic structures

Joint work of: Kuske, Dietrich; Lohrey, Markus

Real-World Programming with TransLucid

Blanca Mancilla (Univ. of New South Wales, AU)

Real-world programming is a term used in Cartesian programming to describe various forms of computing in which some of the dimensions may be subject to physical constraints, as occurs in reactive, real-time, pervasive and ubiquitous computing.

This work presented is a first step towards developing a declarative model for describing and programming different kinds of distributed systems.

In real-world programming, the set of equations making up a program is itself context-dependent, i.e., in a different context, the set of equations will possibly be different. A computational thread corresponds to the navigation through a sequence of physical contexts, with appropriate demands for computation and placement of new equations along the way. When multiple computational threads interact, they do so by passing through the same or "neighbouring" physical contexts at the same time. We examine different forms of interaction and how they can be encoded using this vision.

Keywords: Distributed computing, Intensional threads

Declarative Synchronous Multithreaded Programming

Blanca Mancilla (Univ. of New South Wales, AU)

We demonstrate how TransLucid can be used as a reactive system. At each instant, there is a set of active ports, where sets of equations, demands and threads are all registered. Each thread defines a sequence of (state, demand) pairs, and threads may interact through the overall set of equations. The entire system remains fully declarative.

Keywords: Synchronous programming, distributed computing, declarative programming, Cartesian programming, multidimensional programming

Joint work of: Mancilla, Blanca; Plaice, John

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1653

Partial Metric Spaces

Steve G. Matthews (University of Warwick, GB)

Introduced in 1992, a *partial metric space* is a generalisation of the notion of *metric space* defined in 1906 by Maurice Fréchet such that the distance of a point from itself is not necessarily zero.

Motivated by the needs of computer science for non Hausdorff Scott topology, we show that much of the essential structure of metric spaces, such as Banach's contraction mapping theorem, can be generalised to allow for the possibility of non zero self-distances $d(x, x)$.

This talk will introduce the essential motivation, theory, and applications for partial metric spaces, leading to the conclusion that the non Hausdorff nature of topology in computer science is calling upon metric topology to reconsider its foundations.

Wadge Degrees of Deterministic Tree Languages

Filip Murlak (University of Edinburgh, GB)

I will present results obtained in an attempt to generalize the Wagner hierarchy to the case of tree languages.

I will provide a structural description of the Wadge hierarchy restricted to deterministically recognizable tree languages, together with an algorithm calculating the exact position of a given language in the restricted hierarchy. I will also show how to compute the Wadge degree of a given tree language, thus providing an effective description of the way the restricted hierarchy embeds into the full hierarchy.

This is just a part of the work that needs to be done in order to have a tree analogue of the Wagner hierarchy. What is still ahead, is the non-deterministic case. Apart from some partial results on weakly recognizable tree languages, it is a big unknown.

Keywords: Deterministic automata, infinite trees, Wadge hierarchy, Wadge degree, decidability

Topological Arguments for Automata-Theoretic Hierarchies

Damian Niwinski (University of Warsaw, PL)

Automata on infinite objects can be classified along several axes:

- as working on (infinite) words or trees,
- in deterministic, non-deterministic, or alternating mode,
- with a certain accepting condition, and in particular some Rabin–Mostowski index.

These classifications are relevant for the computational complexity of the automata-based verification methods.

Investigation over years have revealed that the automata-theoretic complexity is often, but not always, underlined by the topological complexity. In particular, the strictness of the Rabin–Mostowski index hierarchy for deterministic as

well as for alternating tree automata can be deduced from the non-existence of Wadge's reductions between the witnessing properties. However this argument fails for non-deterministic automata. In a similar vein, all Rabin recognizable tree languages recognized by Buchi automata are analytic, but the converse, somewhat surprisingly, is not true. We also show how the co-analytic inseparability reads in the automata-theoretic context, thus completing some previous work by Arnold and Santocanale.

The talks presents some results obtained jointly with Andre Arnold, Szczepan Hummel, and Henryk Michalewski.

Keywords: Tree automata, index hierarchy, Wadge reducibility

Profinite Topologies and Wadge Reductions: a Work Programme

Jean-Eric Pin (LIAFA - Université Paris Diderot and CNRS, FR)

Profinite topologies are a major tool in the study of regular languages. They can be used in particular to define reductions similar to Wadge's reduction on Polish spaces.

This lecture will present an overview of this topic, including some recent results, and propose a work programme for the future.

Keywords: Profinite topology, Wadge reduction, regular language, logic

Synthesis from Temporal Specifications

Nir Piterman (Imperial College London, GB)

We consider the problem of synthesizing digital designs from their LTL specification. In spite of the theoretical double exponential lower bound for the general case, we show that for many expressive specifications of hardware designs the problem can be solved in time N^3, where N is the size of the state space of the design.

We show the practicality of this method by synthesizing a generalized buffer and an arbiter for ARM's AMBA AHB bus from specifications given in PSL. These are the first industrial examples that have been synthesized automatically from their specifications.

Keywords: Synthesis, ltl

Joint work of: Piterman, Nir; Pnueli, Amir; Sa'ar, Yaniv

Full Paper:
http://dx.doi.org/10.1007/11609773_24

See also: Proc. 7th International Conference on Verification, Model Checking and Abstract Interpretation, volume 3855 of Lecture Notes in Computer Science, pages 364–380. ľ Springer-Verlag

Cartesian Programming: The TransLucid Programming Language

John Plaice (Univ. of New South Wales, AU)

Cartesian programming is a form of declarative programming in which the meaning of a variable depends on a multidimensional context that is encoded as a unary function mapping dimensions to values.

The TransLucid programming language is a low-level language for Cartesian programming, sufficiently rich to be the target language for translating the common programming paradigms into it, while still being fully declarative. The objects manipulated by TransLucid, called hyperdatons, are arbitrary-dimensional infinite arrays, indexed by multidimensional tuples of arbitrary types.

We present the syntax and operational semantics for the complete TransLucid language, including side-effects, limited control of computation, and context-dependent type declarations and variable definitions. We demonstrate that dataflow, functional and imperative programs can all be naturally translated into TransLucid. We conclude with a description of a demand-driven execution mechanism, supporting multiple-thread execution.

Keywords: Declarative programming, Multidimensionality

Cartesian Programming: The TransLucid Programming Language

John Plaice (Univ. of New South Wales, AU)

The TransLucid programming language is a low-level intensional language, designed to be sufficiently rich for it to be the target language for translating the common programming paradigms into it, while still being fully declarative. The objects manipulated by TransLucid, called hyperdatons, are arbitrary-dimensional infinite arrays, indexed by multidimensional tuples of arbitrary types.

We present the syntax, denotational and operational semantics for a simple TransLucid system, consisting of 1) a header detailing how expressions should be parsed, 2) a set of libraries of types, and operations thereon, defined in a host language, 3) a set of TransLucid equations, and 4) a TransLucid demand to be evaluated.

The evaluation of a demand for an (identifier, context) pair is undertaken using eduction, where previously computed pairs are stored in a cache called a

warehouse. The execution ensures that only those dimensions actually encountered during the execution of an expression are taken into account when caching intermediate results.

Keywords: Cartesian programming, Lucid language, declarative programming, multidimensional programming, context-aware programming, semantics.

Joint work of: Plaice, John; Mancilla, Blanca

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1654

The Church Problem for Countable Ordinals

Alexander Rabinovich (Tel Aviv University, IL)

A fundamental theorem of Büchi and Landweber shows that the Church synthesis problem is computable. Büchi and Landweber reduced the Church Problem to problems about ω-games and used the determinacy of such games as one of the main tools to show its computability.

We consider a natural generalization of the Church problem to countable ordinals and investigate games of arbitrary countable length. We prove that determinacy and decidability parts of the Büchi and Landweber theorem hold for all countable ordinals and that its full extension holds for all ordinals $< \omega^\omega$.

Negation in Logic Programming and Infinite-Valued Logics

Panos Rondogiannis (Univ. of Athens, GR)

We present a purely model theoretic semantics for negation-as-failure, based on a novel infinite-valued-logic. We then propose an alternative definition of the semantics of negation, based on infinite games of perfect information. We argue that these two approaches are equivalent and provide a better understanding of well-founded negation.

Keywords: Negation-as-Failure, Infinite-Valued Logics, Infinite Games

Joint work of: Rondogiannis, Panos; Wadge, William

The sequential topology on $\mathbb{N}^{\mathbb{N}^{\mathbb{N}}}$ is not regular

Matthias Schröder (Univ. der Bundeswehr - Neubiberg, DE)

We prove the surprising result that the sequential topology on the Kleene-Kreisel continuous functionals of type k is not regular for $k \geq 2$ and thus not zero-dimensional. We relate this result to an open problem in exact real-number computation.

Keywords: Sequential topologies, Kleene-Kreisel continuous functionals, Computable Analysis

Wadge Reducibility and Infinite Computations

Victor Selivanov (Pedagogical University - Novosibirsk, RU)

We discuss topological properties of infinite computations with the emphasis on the Wadge reducibility and its effective variants. Along with a survey of some basic facts, we discuss the related technique, some fresh, not yet published results and open questions.

Keywords: Baire space, Baire domain, Wadge reducibility

Questions about Descriptive Set Theory, Effective Set Theory, and Automata on Infinite Structures

Pierre Simmonet (Université de Corse, FR)

I Dependence on Models of ZF

In order to start this exposition I will talk about the square of oppositions of John W. Addison. Traditionally, the square AEIO is equipped with formulas as follows:

A Every human being is mortal.
E No human being is mortal.
I Some human being is mortal.
O Some human being is immortal.

We decorate this square with formulas of second order:

A Every Σ^1_2 set is Lebesgue measurable.
E No Σ^1_2 set is Lebesgue measurable.
I Some Σ^1_2 set is Lebesgue measurable.
O Some Σ^1_2 set is not Lebesgue measurable.

The opposition I/E does not pose any problems because the empty set is Lebesgue measurable. In contrast, the opposition A/O depends on the set theoretic model. This was suspected by Lusin who did not believe in the law of excluded middle as far as projective sets are concerned. Gödel showed that in the model of constructible sets there is a Σ^1_2 set that is not Lebesgue measurable. It was Addison who wrote this up. Solovay showed that the relative consistency of ZF + countable choice + the existence of large cardinals implies that all sets of real numbers are Lebesgue measurable. The recent results of Olivier Finkel show that there is a possibility that such a kind of phenomena, well known among Banach people, may appear also in the theory of automata over infinite structures: a statement about automata on infinite structures may depend on the set theoretic model.

II Differentiability and functions defined by synchronous automata on reals in Pisot base
For an example of a Cantor/Scheffer/Lebesgue/Sierpinski function, a strategy definable by an automaton. Is the set of points of differentiability of a function, defined by an automaton on reals in Pisot base, definable in S1S?

III The operation L^ω for rational L and the Wadge hierarchy
(The question was posed in 1988 in the Louveau seminar in the presence of Dominique Lecomte and Gabriel Debs; most probably it was posed earlier by Ludwig Staiger.) Which degrees are obtained when one restricts the Wadge hierarchy to sets L^ω for rational L? In particular, can one find a rational L such that L^ω is Σ^0_2-complete? (Luwdig Staiger)

IV "Jeux de Mistigri" (1984, Louveau and Saint-Raymond)
Keywords: classes of WAGner ${}_AW$ "W A Gauche" and classes of WADge W_A "W A Droite"; the trace theorem; the effective version by Louveau (proved originally using the topology of Gandy and Harrington); having its origin in works of Hurewicz; Borel sets intersected with K_σ.

V Game quantifiers and automata on infinite trees (Louveau seminar, 1988)

VI Is there a version definable in S1S of the dichotomy theorem of Harrington/Kechris/Louveau about the Borel equivalence relations?

Information-Theoretic Properties of Regular ω-Languages

Ludwig Staiger (Martin-Luther-Universität Halle-Wittenberg, DE)

The talk deals with the problem of how much information must be provided on the average in order to specify a particular symbol of an infinite word in a given set of infinite words.

It gives a survey on results showing that estimating the information needed to specify the number n-length prefixes and then averaging by the length n is not necessarily the correct way though it works fine for regular sets which are closed in the Cantor topology.

As a way out we propose to use Hausdorff dimension as a measure of information and we show that for regular sets this measure behaves well with respect to topological density and self-similarity.

Keywords: Infinite words, ω-languages, information, entropy, Hausdorff dimension, self-similarity

Infinite Games in Synthesis and Verification

Wolfgang Thomas (RWTH Aachen, DE)

Starting from Church's Synthesis Problem, posed in 1957 and solved in 1969 by Büchi and Landweber, we survey the essential steps to solve infinite games with MSO-definable winning conditions: transformation to Muller games over finite graphs, their transformation into parity games, and the solution of parity games with positional winning strategies.

In the second part of the talk, we briefly address three kinds of application:

(1) Complementation of automata over infinite trees, (2) Algorithmic synthesis of finite-state controllers (with an emphasis on so-called "request-response games" and the question of time-optimal solutions), and (3) model-checking games, in particular for the modal μ-calculus.

The Complexity of Pure-Strategy Nash Equilibria in Stochastic Multiplayer Games

Michael Ummels (RWTH Aachen, DE)

We study the complexity of pure-strategy Nash equilibria in (turn-based) stochastic multiplayer games with omega-regular winning conditions. Such games are played by multiple players on a directed graph, where every vertex of the graph is either controlled by one of the players or a stochastic vertex. If a play of the game arrives at a stochastic vertex, the next vertex of the play is determined by a fixed probability distribution.

A pure-strategy Nash equilibrium of such a game is a profile of pure strategies (i.e. the players are not allowed to randomise over actions) such that no player can increase her probability of winning by switching to a different strategy. Chatterjee et al. showed that every such game has a pure-strategy Nash equilibrium, and they gave an algorithm for computing one. However, in applications, one might not just look for an arbitrary Nash equilibrium, but one that fulfils certain requirements. For example, one might look for a Nash equilibrium where certain players win the resulting play with a certain probability.

Our main result is that most of these questions are undecidable. In particular, we show that the following problem is undecidable: Given a game G and some distinguished player p, does there exist a Nash equilibrium of G where player p wins with probability 1. The proof for undecidability reveals a connection to the theory of stochastic games with branching-time winning conditions, as studied by Brazdil et al.

Keywords: Stochastic Games, Nash Equilibria

Joint work of: Ummels, Michael; Wojtczak, Dominik

Computing with Streams

Bill Wadge (University of Victoria, CA)

I describe how modest efforts to make conventional programs more amenable to verification inexorably led to a new language (Lucid) in which the program is a system of equations defining infinite streams of data (much as originally suggested by Gilles Kahn).

This not only eases verification, it enables an implementation in terms of a dataflow computation model.

This last advantage may in the end prove to be decisive, given the difficulty that conventional imperative languages have in realizing the parallelism inherent in the emerging multicore architectures.

Keywords: Lucid dataflow streams multicore

See also: Lucid the Dataflow Programming Language, W. Wadge and E. Ashcroft, Academic Press, 1985

Definability in the h-Quasiorder of Labeled Forests

Anton Zhukov (Pedagogical University - Novosibirsk, RU)

This talk presents the joint work with O. Kudinov and V. Selivanov on the definability in the structures of the h-quasiorder, which are connected with the Wadge reducibility and Boolean hierarchy of partitions.

We prove that for any k>2 each element in the quotient structure of finite k-labeled forests with the h-quasiorder is definable in the ordinary first-order language, and, respectively, each element in the structure of (at most) countable k-labeled forests is definable in the language $L_{\omega_1\omega}$, in both cases provided that the minimal non-smallest elements are allowed as parameters. As corollaries, we characterize the automorphism groups of both structures and show that the structure of finite k-forests is atomic. Similar results hold true for the quotient structures of finite (resp. countable) k-labeled trees and finite (resp. countable) k-labeled trees with a fixed label of the root element.

We prove also the first-order definability of the operations of adding a root with a fixed label (in both finite and countable cases).

Keywords: Definability, h-quasiorder, labeled forest, labeled tree

Joint work of: Kudinov, Oleg; Selivanov, Victor; Zhukov, Anton

Revising Type-2 Computation

Martin Ziegler (Universität Paderborn, DE)

We report on several recent works combining revising (or limit) computation on finite strings with the Type-2 Theory of computing on infinite strings and other continuum universes. Can the requirement of the so-called Main Theorem that any computable function must be continuous be lifted? And if so, how discontinous may revisingly computable get?

Keywords: Revising computation, type-2 theory, recursive analysis, hypercomputation, descriptive set theory, turing machines