08492 Abstracts Collection

Structured Decompositions and Efficient Algorithms

— Dagstuhl Seminar —

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Abstract. From 30.11. to 05.12.2008, the Dagstuhl Seminar 08492 "Structured Decompositions and Efficient Algorithms " was held in Schloss Dagstuhl – Leibniz Center for Informatics. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Sparse signal representation, optimal signal reconstruction, approximation, compression

08492 Executive Summary – Structured Decompositions and Efficient Algorithms

New emerging technologies such as high-precision sensors or new MRI machines drive us towards a challenging quest for new, more effective, and more daring mathematical models and algorithms. Therefore, in the last few years researchers have started to investigate different methods to efficiently represent or extract relevant information from complex, high dimensional and/or multimodal data. Efficiently in this context means a representation that is linked to the features or characteristics of interest, thereby typically providing a sparse expansion of such. Besides the construction of new and advanced ansatz systems the central question is how to design algorithms that are able to treat complex and high dimensional data and that efficiently perform a suitable approximation of the signal. One of the main challenges is to design new sparse approximation algorithms that would ideally combine, with an adjustable tradeoff, two properties: a provably good 'quality' of the resulting decomposition under mild assumptions on the analyzed sparse signal, and numerically efficient design.

Keywords: Sparse signal representation, optimal signal reconstruction, approximation, compression

Joint work of: Dahlke, Stephan ; Daubechies, Ingrid ; Elad, Michael ; Kutyniok, Gitta ; Teschke, Gerd

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2009/1885

A prediction scheme for the adaptive approximation of nonlinear functions of wavelet expansions

Kai Bittner (Qimonda AG - Neubiberg, DE)

A core ingredient of adaptive wavelet methods for nonlinear operator equations is the adaptive evaluation of nonlinear functions. We present an efficient adaptive method for approximately evaluating nonlinear functions of wavelet expansions using semi-orthogonal spline wavelets. Solving the two tasks of predicting an index set for the approximation in terms of wavelets, as well as the approximative computation of the corresponding wavelet coefficients, we are able to achieve the desired accuracy. The computational complexity of the proposed method has the same asymptotic behavior as the best n-term tree approximation.

The continuous shearlet transform in arbitrary space dimensions

Stephan Dahlke (Universität Marburg, DE)

We shall be concerned with the generalization of the continuous shearlet transform to higher dimensions. Similar to the two-dimensional case, our approach is based on translations, anisotropic dilations and specific shear matrices. We show that the associated integral transform again originates from a square-integrable representation of a specific group, the full n-variate shearlet group. Moreover, we verify that by applying the coorbit theory, canonical scales of smoothness spaces and associated Banach frames can be derived. We also indicate how our transform can be used to characterize singularities in signals.

This is joint work with G. Steidl and G. Teschke

Keywords: Shearlets, coorbit space theory, square-integrable group representations, Banach frames, detection of singularities

See also: S. Dahlke, G. Steidl, and G. Teschke, The continuous shearlet transform in arbitrary space dimensions, Preprint Nr. 2008-7, Philipps-University

The Continuous Shearlet Transform in Arbitrary Space Dimensions

Stephan Dahlke (Universität Marburg, DE)

This note is concerned with the generalization of the continuous shearlet transform to higher dimensions. Similar to the two-dimensional case, our approach is based on translations, anisotropic dilations and specific shear matrices. We show that the associated integral transform again originates from a square-integrable representation of a specific group, the full n-variate shearlet group. Moreover, we verify that by applying the coorbit theory, canonical scales of smoothness spaces and associated Banach frames can be derived. We also indicate how our transform can be used to characterize singularities in signals.

Joint work of: Dahlke, Stephan; Steidl, Gabriele; Teschke, Gerd

Adaptivity, and Stabilization, and Sparsity Recovering Schemes for Transport Problems

Wolfgang Dahmen (RWTH Aachen, DE)

Adaptive solution concepts are meanwhile fairly well understood for well conditioned variational problems when in addition a sparsifying Riesz basis for the corresponding energy space is available.

This is typically no longer the case for important transport dominated problem classes.

This talk outlines ongoing work on restoring stability and ultimately k-term approximation performance in the context of convection-diffusion problems by using sparsity recovering schemes of the type arising in compressed sensing.

Keywords: Convection dominated problems, adaptivity, stabilization, ℓ_1-regularization

Joint work of: Dahmen, Wolfgang; Welper, Gerrit

Representation of Operators in the Time-Frequency Domain and Generalzed Gabor Multipliers

Monika Dörfler (Universität Wien, AT)

Starting from a general operator representation in the time-frequency domain, this paper addresses the problem of approximating linear operators by operators that are diagonal or band-diagonal with respect to Gabor frames.

A characterization of operators that can be realized as Gabor multipliers is given and necessary conditions for the existence of (Hilbert-Schmidt) optimal Gabor multiplier approximations are discussed and an efficient method for the calculation of an operator's best approximation by a Gabor multiplier is derived. The spreading function of Gabor multipliers yields new error estimates for these approximations. Generalizations (multiple Gabor multipliers) are introduced for better approximation of overspread operators. The Riesz property of the projection operators involved in generalized Gabor multipliers is characterized, and a method for obtaining an operator's best approximation by a multiple Gabor multiplier is suggested. Finally, it is shown that in certain situations, generalized Gabor multipliers reduce to a finite sum of regular Gabor multipliers with adapted windows.

Keywords: Operator approximation, generalized Gabor multipliers, spreading function, twisted convolution

Joint work of: Dörfler, Monika; Torrésani, Bruno

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2009/1880

Arbitrary Shrinkage Rules for Approximation Schemes with Sparsity Constraints

Martin Ehler (National Institutes of Health - Bethesda, US)

Finding a sparse representation of a possibly noisy signal is a common problem in signal representation and processing. It can be modeled as a variational minimization with ℓ_τ-sparsity constraints for $\tau < 1$. Applications whose computation time is crucial require fast algorithms for this minimization. However, there are no fast methods for finding the exact minimizer, and to circumvent this limitation, we consider minimization up to a constant factor. We verify that arbitrary shrinkage rules provide closed formulas for such minimizers, and we introduce a new shrinkage strategy, which is adapted to $\tau < 1$.

Keywords: Frames, shrinkage, variational problems, sparse approximation

Joint work of: Ehler, Martin; Geisel, Simone

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2009/1881

A Weighted Average of Sparse Representations is Better than the Sparsest One Alone

Michael Elad (Technion - Haifa, IL)

Cleaning of noise from signals is a classical and long-studied problem in signal processing.

Algorithms for this task necessarily rely on an a-priori knowledge about the signal characteristics, along with information about the noise properties. For signals that admit sparse representations over a known dictionary, a commonly used denoising technique is to seek the sparsest representation that synthesizes a signal close enough to the corrupted one. As this problem is too complex in general, approximation methods, such as greedy pursuit algorithms, are often employed. In this line of reasoning, we are led to believe that detection of the sparsest representation is key in the success of the denoising goal. Does this means that other competitive and slightly inferior sparse representations are meaningless? Suppose we are served with a group of competing sparse representations, each claiming to explain the signal differently. Can those be fused somehow to lead to a better result? Surprisingly, the answer to this question is positive; merging these representations can form a more accurate, yet dense, estimate of the original signal even when the latter is known to be sparse. In this talk we demonstrate this behavior, propose a practical way to generate such a collection of representations by randomizing the Orthogonal Matching Pursuit (OMP) algorithm, and produce a clear analytical justification for the superiority of the associated Randomized OMP (RandOMP) algorithm. We show that while the Maximum a-posterior Probability (MAP) estimator aims to nd and use the sparsest representation, the Minimum Mean-Squared-Error (MMSE) estimator leads to a fusion of representations to form its result. Thus, working with an appropriate mixture of candidate representations, we are surpassing the MAP and tending towards the MMSE estimate, and thereby getting a far more accurate estimation, especially at medium and low SNR.

Keywords: OMP, MAP, MMSE, Sparse Representation, Denoising

Joint work of: Elad, Michael; Yavneh, Irad

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2009/1882

Beyond Nyquist: Compressed Sensing of Analog Signals

Yonina Eldar (Technion - Haifa, IL)

A traditional assumption underlying most data converters is that the signal should be sampled at a rate which exceeds twice the highest frequency. This statement is based on a worst-case scenario in which the signal occupies the entire available bandwidth. In practice, many signals posses a sparse structure so that a large part of the bandwidth is not exploited. In this talk, we consider a framework for utilizing this sparsity in order to sample such analog signals at a low rate, by relying on results developed in the context of compressed sensing (CS) of finite-length vectors. The distinguishing feature of our results is that in contrast to the problems treated in the context of CS, here we consider sampling of analog-signals for which no underlying finite-dimensional model exists. Our approach combines ideas from analog sampling in a subspace with a recently developed block diagram that converts an infinite set of sparse equations to a finite counterpart. Using these two components we formulate our problem within the framework of finite CS and then rely on efficient and stable algorithms developed in that context.

A Banach Gelfand Triple Framework for Regularization and Approximation

Hans Georg Feichtinger (Universität Wien, AT)

So far the setting of Banach Gelfand Triples (S0,L2,SO') over Rd was used to describe mappings such as the Fourier transform, to represent operators commuting with translation (time-invariant linear systems) as convolution operators or as Fourier multipliers, or to have a convenient setting for a kernel theorem resp. describe the transition from the kernel of an operator to its Kohn-Nirenberg symbol or the spreading distribution and back.

In the present talk we will try to explain, that at least from the functional analytic point of view this setting is also quite well suited to describe the transition between continuous and disrete setting. In contrast to the wavelet setting the context of time-frequency analysis provides a natural finite/discrete model, and one can hope (and in many cases realize computationally) that the general questions of e.g. Gabor analysis can be numerically well approximated in the finite-dimensional context.

We will try to point out what the properties of the BGTR (S0,L2,SO') are which allow for such explanation resp. numerical approximations.

Keywords: Banach Gelfand Triples, regularization, approximation, FFT, Fourier transform

Full Paper:
http://univie.ac.at/nuhag-php/bibtex/open_files/cofelu07_Cime-Final.pdf

Subspace decomposition method for very large scale sparse optimizations

Massimo Fornasier (RICAM - Linz, AT)

This talk is concerned with the numerical minimization of energy functionals in Hilbert spaces involving convex constraints coinciding with a semi-norm for a subspace.

The optimization is realized by alternating minimizations of the functional on a sequence of orthogonal subspaces. On each subspace an iterative proximity-map algorithm is implemented via oblique thresholding, which is the main new tool introduced in this work. We provide convergence conditions for the algorithm in order to compute minimizers of the target energy. Analogous results are derived for a parallel variant of the algorithm. Applications are presented in nonoverlapping domain decomposition methods for singular elliptic PDEs arising in total variation minimization. We present also variations for overlapping domain decompositions in this setting. We include numerical examples which show efficient solutions to classical inverse problems in signal and image processing.

This is a joint work with Andreas Langer and Carola B. Schoenlieb.

Keywords: Domain decomposition, subspace correction methods, total variation minimization, l1 minimization, large scale optimization

Full Paper:
http://www.ricam.oeaw.ac.at/people/page/fornasier/DDsparse.pdf

Full Paper:
http://www.ricam.oeaw.ac.at/people/page/fornasier/subsp.pdf

See also: Domain decomposition methods for linear inverse problems with sparsity constraints, Inverse Problems, Vol. 23, 2007, pp. 2505-2526, Subspace correction methods for total variation and l1-minimization, (with C. Schoenlieb), submitted to SIAM J. Numer. Anal., December 2007, 33 pp.

Pseudospectral Fourier reconstruction with IPRM

Karlheinz Gröchenig (Universität Wien, AT)

We generalize the Inverse Polynomial Reconstruction Method (IPRM) for mitigation of the Gibbs phenomenon by reconstructing a function as an algebraic polynomial of degree $n-1$ from the function's m lowest Fourier coefficients ($m \geq n$). We compute approximate Legendre coefficients of the function by solving a linear least squares problem, and we show that the condition number of the problem does not exceed $\sqrt{\frac{m}{m-\alpha_0 n^2}}$, where $\alpha_0 = \frac{4\sqrt{2}}{\pi^2} = 0.573\ldots$. Consequently, whenever $m \geq n^2$, the convergence rate of the modified IPRM for an analytic function is root exponential on the whole interval of definition. Stability and accuracy of the proposed algorithm are validated with numerical experiments.

Keywords: IPRM, Fourier series, inverse methods, pseudospectral methods

Joint work of: Gröchenig, Karlheinz; Hrycak, Tomasz

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2009/1883

Sparse Decompositions Applied: From image and video compression to GPU-based video processing

Onur G. Guleryuz (DoCoMo USA Labs - Palo Alto, US)

Statistical models based on sparse decompositions have recently become popular in image/video applications. Focusing on video compression and processing, I will summarize some of our recent work that relies on sparse decompositions and accomplishes:

- Surprisingly good inter-picture predictions in video that is corrupted with structured interference signals with statistics similar to the pictures being predicted. (Simply put, the algorithm I will discuss performs denoising when noise is the interference and automatically switches to "de-barbara-ing" when the interfering signal is the image barbara.)

- Image compression with data-driven, sparse, orthonormal transform designs.

- Denoising and zooming of video with significant quality improvements.

This portfolio of work provides results that are competitive with (or better than) established literature over large sets of simulation examples. I will highlight the underlying statistical models and provide examples that showcase the techniques. Motivated by the desire to obtain high performance within computationally attractive form factors, I will also highlight some implementation related trade-offs that allow these algorithms to be deployed on platforms ranging from cell phones to graphics processing units (GPUs)

Keywords: Sparse representations, prediction of sparse signals, zooming, super-resolution

Algorithm and Applications of Matrix Extension with Symmetry

Bin Han (University of Alberta, CA)

The matrix extension problem with symmetry in dimension one cane be stated as follows: For two $n \times 1$ column vectors $p(z)$ and $\tilde{p}(z)$ of Laurent polynomials with coefficients in F, where $F = R$ or C, such that p and $\tilde{p}$ have the same symmetry pattern, find two $n \times n$ matrices $A(z)$ and $\tilde{A}(z)$ of Laurent polynomials with coefficients in F and with an induced symmetry pattern such that $A(z)^*\tilde{A}(z) = I_n$ and the first columns of A and $\tilde{A}$ are p and $\tilde{p}$, respectively. Moreover, when $\tilde{p} = p$, we require $\tilde{A} = A$.

The matrix extension problem without symmetry is guaranteed by the Quillen-Suslin Theorem and algorithms have been developed in the literature. However, despite the importance of symmetry in applications, there are little known results on matrix extension problem with symmetry even in dimension one. In this talk, we show that the matrix extension problem with symmetry in dimension one is always solvable and a constructive algorithm is available to realize it.

We shall discuss two applications of the matrix extension with symmetry: symmetric biorthogonal multiwavelets and symmetric complex orthonormal wavelets with a general dilation factor. Combining with the CBC algorithm and the algorithm for matrix extension with symmetry, we now have a complete algorithm for constructing all biorthogonal multiwavelets with symmetry and arbitrarily high vanishing moments. With the help of the matrix extension with symmetry, we now also have a complete picture on symmetric orthonormal complex wavelets for a general dilation matrix.

Keywords: Matrix extension with symmetry, symmetric biorthogonal multiwavelets, symmetric orthonormal complex wavelets, dilation factor

Full Paper:
http://www.ualberta.ca/∼bhan

The Bregman Method for Image Denoising: Convergence Analysis and Modified Algorithms

Rong-Qing Jia (University of Alberta, CA)

The Total Variation model of Rudin, Osher, and Fatemi for image denoising is considered to be one of the best denoising models. But there were two serious issues about the ROF model. First, it was very complicated to compute the solutions of the optimization problems induced by the variational method. Second, it was difficult to extract textures from images by using the ROF model. For the first issue, Goldstein and Osher recently introduced the split Bregman method for L1 regularized problems. The Bregman method gave rise to very efficient algorithms for solutions of the ROF model. In this talk, we will give a rigorous proof for the convergence of the Bregman method. For the second issue, we will propose some modified algorithms based on the combination of the Bregman method with wavelet packet decompositions. It will be demonstrated that our algorithms have better performance in texture preservation.

Keywords: Riesz bases, wavelets

l1-Minimization and the Geometric Separation Problem

Gitta Kutyniok (Universität Osnabrück, DE)

Consider an image mixing two (or more) geometrically distinct but spatially overlapping phenomena - for instance, pointlike and curvelike structures in astronomical imaging of galaxies. This raises the Problem of Geometric Separation, taking a single image and extracting two images one containing just the pointlike phenomena and one containing just the curvelike phenomena.

Although this seems impossible - as there are two unknowns for every datum - suggestive empirical results have already been obtained.

We develop a theoretical approach to the Geometric Separation Problem in which a deliberately overcomplete representation is chosen made of two frames. One is suited to pointlike structures (wavelets) and the other suited to curvelike structures (curvelets or shearlets). The decomposition principle is to minimize the l1 norm of the analysis (rather than synthesis) frame coefficients. This forces the pointlike objects into the wavelet components of the expansion and the curvelike objects into the curvelet or shearlet part of the expansion. Our theoretical results show that at all sufficiently fine scales, nearly-perfect separation is achieved.

Our analysis has two interesting features. Firstly, we use a viewpoint deriving from microlocal analysis to understand heuristically why separation might be possible and to organize a rigorous analysis. Secondly, we introduce some novel technical tools: cluster coherence, rather than the now-traditional singleton coherence and l1-minimization in frame settings, including those where singleton coherence within one frame may be high.

Our general approach applies in particular to two variants of geometric separation algorithms. One is based on frames of radial wavelets and curvelets and the other uses orthonormal wavelets and shearlets.

This is joint work with David L. Donoho (Stanford University).

Approximation of smooth multivariate functions is intractable

Erich Novak (Universität Jena, DE)

We prove that L_∞-approximation of C^∞-functions defined on $[0,1]^d$ is intractable. The computational cost is exponential in d.

This holds despite the fact that the rate of convergence is infinite.

Keywords: Rate of convergence, tractability, curse of dimensionality

Joint work of: Novak, Erich; Wozniakowski, Henryk

Uncertainty in time-frequency representations and applications to sparse matrix identification

Götz Pfander (Jacobs University - Bremen, DE)

Classical and recent results on uncertainty principles for functions on finite Abelian groups relate the cardinality of the support of a function to the cardinality of the support of its Fourier transform. We obtain corresponding results relating the support sizes of functions and their short-time Fourier transforms. We discuss consequences of our findings to the theory of recovering and storing signals with sparse time-frequency representations.

Keywords: Gabor frames, sparse operator representation, identification

See also: Uncertainty in time-frequency representations on finite Abelian groups and applications, Applied and Computational Harmonic Analysis Volume 25, Issue 2, September 2008, Pages 209-225

An open question on the existence of Gabor frames in general linear

Götz Pfander (Jacobs University - Bremen, DE)

Uncertainty principles for functions defined on finite Abelian groups generally relate the cardinality of a function to the cardinality of its Fourier transform. We examine how the cardinality of a function is related to the cardinality of its short–time Fourier transform. We illustrate that for some cyclic groups of small order, both, the Fourier and the short–time Fourier case, show a remarkable resemblance. We pose the question whether this correspondence holds for all cyclic groups.

Keywords: Gabor systems, erasure channels, time–frequency dictionaries, short–time Fourier transform, uncertainty principle.

Joint work of: Krahmer, Felix; Pfander, Götz E.; Rashkov, Peter

The Easy Path Wavelet Transform: A New Adaptive Wavelet Transform for Sparse Representation of Two-dimensional Data

Gerlind Plonka-Hoch (Universität Duisburg-Essen, DE)

We introduce a new locally adaptive wavelet transform, called **Easy Path Wavelet Transform** (EPWT), that works along pathways through the array of function values and exploits the local correlations of the data in a simple appropriate manner. The usual discrete orthogonal and biorthogonal wavelet transform can be formulated in this approach.

The EPWT can be incorporated into a multiresolution analysis structure and generates data dependent scaling spaces and wavelet spaces.

Numerical results show the enormous efficiency of the EPWT for representation of two-dimensional data.

Keywords: Wavelet transform along pathways, data compression, adaptive wavelet bases, directed wavelets

Sparse Reconstructions for Inverse PDE Problems

Thorsten Raasch (Universität Marburg, DE)

We are concerned with the numerical solution of linear parameter identification problems for parabolic PDE, written as an operator equation $Ku = f$.

The target object u is assumed to have a sparse expansion with respect to a wavelet system $\Psi = \{\psi_\lambda\}$ in space-time, being equivalent to a priori information on the regularity of $u = \mathbf{u}^\top \Psi$ in a certain scale of Besov spaces $B^s_{p,p}$.

For the recovery of the unknown coefficient array $\mathbf{u}$, we miminize a Tikhonov-type functional

$$\min_{\mathbf{u}} \|K\mathbf{u}^\top \Psi - f^\delta\|^2 + \alpha \sum_\lambda \omega_\lambda |u_\lambda|^p$$

by an associated thresholded Landweber algorithm, f^δ being a noisy version of f.

Since any application of the forward operator K and its adjoint involves the numerical solution of a PDE, perturbed versions of the iteration have to be studied. In particular, for reasons of efficiency, adaptive applications of K and K^* are indispensable.

By a suitable choice of the respective tolerances and stopping criteria, also the adaptive iteration could recently be shown to have regularizing properties for $p > 1$. Moreover, the sequence of iterates linearly converges to the minimizer of the functional, a result which can also be proved for the special case $p = 1$, see [DaFoRa08].

We illustrate the performance of the resulting method by numerical computations for one- and two-dimensional inverse heat conduction problems.

References:

[BoMa08a] T. Bonesky and P. Maass, Iterated soft shrinkage with adaptive operator evaluations, Preprint, 2008

[DaFoRa08] S. Dahlke, M. Fornasier, and T. Raasch, Multiscale Preconditioning for Adaptive Sparse Optimization, in preparation, 2008

[Ra07] T. Raasch, Adaptive wavelet and frame schemes for elliptic and parabolic equations, Dissertation, Philipps-Universität Marburg, 2007

Keywords: Adaptivity, sparse reconstructions, l1 minimization, parameter identification

Sparsity in Time-Frequency Representations

Holger Rauhut (Universität Bonn, DE)

We consider signals and operators in finite dimension which have sparse time-frequency representations. As a main result an S-sparse Gabor representation in C^n with respect to a random unimodular window can be recovered by l_1 minimization with high probability provided that $S < Cn/log(n)$.

This is applicable to the channel estimation problem in wireless communications.

Keywords: Compressed sensing

Redundancy, Sparsity, and Algorithm

Zuowei Shen (National University of Singapore, SG)

Efficient algorithms in image restoration and data recovery are derived by exploring sparse approximations of the underlying solutions by redundant systems. Several algorithms and numerical simulation results for deblurring, compressed sensing, and matrix completion will be presented in this talk.

Keywords: Redundancy, Compressed Sensing, matrix completion

Optimal Approximation of Elliptic Problems by Linear and Nonlinear Mappings

Winfried Sickel (Universität Jena, DE)

We study the optimal approximation of the solution of an operator equation $\mathcal{A}(u) = f$ by different types of mappings: a) linear and nonlinear mappings based on linear information about the right hand side; b) n-term approximation with respect to an optimal Riesz basis and with respect to frames; c) continuous mappings; d) approximation based on function values of the right hand side.

We consider worst case errors, where f is an element of the unit ball of a Sobolev or Besov space $B^r_q(L_p(\Omega))$ and $\Omega \subset \mathbb{R}^d$ is a bounded Lipschitz domain.

In our earlier papers we only considered the error with respect to the energy norm or an equivalent norm, i.e., with respect to the H^s-norm where s was given by the operator since we assumed that $\mathcal{A} : H^s_0(\Omega) \to H^{-s}(\Omega)$ is an isomorphism.

The most typical case is $s = 1$.

It is well known that for certain regular problems the order of convergence is improved if one takes the L_2-norm.

In this paper we study error bounds with respect to such a weaker norm, i.e., we assume that $H^s_0(\Omega)$ is continuously embedded into a space X and we measure the error in the norm of X.

A major example is $X = L_2(\Omega)$ or $X = H^r(\Omega)$ with $r < s$.

Keywords: Elliptic operator equations, worst case error, best n-term approximation, approximation numbers, Riesz basis, wavelets

Joint work of: Sickel, Winfried; Dahlke, Stephan, Novak, Erich

Split Bregman Algorithm, Douglas-Rachford Splitting and Frame Shrinkage

Gabriele Steidl (Universität Mannheim, DE)

In recent years variational models were successfully applied in image restoration. These methods came along with various computational algorithms. Interestingly, the roots of many restoration algorithms can be found in classical algorithms from convex analysis dating back more than 40 years.

To discover these relations is useful from different points of view:

Classical convergence results carry over to the restoration algorithms at hand and ensure their convergence. On the other hand, earlier mathematical results have found new applications and should be acknowledged. The present paper fits into this context. Our aim is twofold: First, we show that the *alternating Split Bregman Algorithm* proposed by Goldstein and Osher for image restoration and compressed sensing can be interpreted as a *Douglas-Rachford Splitting Algorithm*. In particular, this clarifies the convergence of the algorithm.

Second, we show that for denoising tasks with an L_2 data fitting term and a discrete versions of an $B^1_{1,1}$ regularization term involving Parseval frames the alternating Split Bregman Algorithm can be seen as an application of a *Forward-Backward Splitting Algorithm*. The later is also related to the *Gradient Descent Reprojection Algorithm* proposed without convergence proof by Chambolle.

Finally, we apply our findings to create an efficient frame-based minimization algorithm for the Rudin-Osher-Fatemi model.

Joint work of: Steidl, Gabriele; Setzer, Simon

Divergence-free wavelet bases on the hypercube

Rob Stevenson (University of Amsterdam, NL)

Given a biorthogonal pair of multiresolution analyses on the interval, by integration or differentiation, we build a new biorthogonal pair of multiresolution analyses. Using both pairs, isotropic or, as we focus on, anisotropic divergence-free wavelet bases on the hypercube are constructed. Our construction generalizes the one from [*Rev. Mat. Iberoamericana*, 8 (1992), pp. 221–237] by P.G. Lemarié-Rieusset for stationary multiresolution analyses on *IR*. It turns out that this generalization requires a judicious choice of boundary conditions.

Euclidean Sparse Recovery with Optimal Measurement (in progress)

Martin J. Strauss (University of Michigan, US)

A Euclidean approximate sparse recovery system consists of parameters epsilon, k, d, an n-by-d measurement matrix, A, and a decoding algorithm, D.

Given a vector, x, the system approximates x by y=D(Ax), where y is a support-k approximation to x with L2 error no worse than (1+epsilon) times that of the best support-k approximation.

For each vector x, the system must succeed with probability at least 3/4.

Many previous works have addressed this problem. Among the goals are minimizing the number n of measurements and the runtime of the decoding algorithm, D.

In this talk, we give a system with n=O(k log(d/k)/epsilon**2) measurements—matching a lower bound, up to constants—and decode time k*poly(log(d)/epsilon), matching a lower bound up to factors of log(d)/epsilon.

Keywords: Sparse recovery, sublinear-time algorithms

Joint work of: Gilbert, Anna C.; Li, Yi; Porat, Ely; Strauss, Martin J.

From Helmholtz to Heisenberg: Sparse Remote Sensing

Thomas Strohmer (University of California - Davis, US)

I will present recent advances in applying sparse representations and compressed sensing to the area of remote sensing. By means of this concrete application I will highlight benefits, but also fundamental mathematical challenges as well as current limitations of compressed sensing.

Keywords: Sparse representations, compressed sensing, Helmholtz equation, radar

Tight Frames for Subdivision Schemes with Irregular Vertices

Joachim Stöckler (TU Dortmund, DE)

Wavelet decompositions can be combined with subdivision algorithms for several tasks of multilevel geometry processing, such as surface editing or compression.

In the spirit of the time-domain approach of Chui et al. [1] for the construction of tight frames, we develop tight frames for the Loop subdivision scheme in the presence of irregular vertices. The underlying approach has two main steps: first, a "global" semi-definite matrix is defined based on the refinement stencils of the subdivision method. Secondly, local factorization techniques are employed.

By means of the time-domain approach, the frame decomposition is symmetric even around irregular vertices.

Adding frame elements to the "pure" subdivision algorithm can be used in order to improve the smoothness of the surface around irregular vertices. This is joint work with Maria Charina.

Keywords: Tight frame, subdivision, irregular vertex, surface editing

See also: [1] C. Chui, W. He, J. Stöckler, Nonstationary tight wavelet frames I: Bounded intervals, ACHA 17(2004), 141–197

Time–frequency analysis and PDE's

Anita Tabacco (Politecnico di Torino, IT)

We study the action on modulation spaces of Fourier multipliers with symbols $e^{i\mu(\xi)}$, for real-valued functions μ having unbounded second derivatives. We show that if μ satisfies the usual symbol estimates of order $\alpha \geq 2$, or if μ is a positively homogeneous function of degree α, the corresponding Fourier multiplier is bounded as an operator between the weighted modulation spaces $\mathcal{M}^{p,q}_{\delta}$ and $\mathcal{M}^{p,q}$, for every $1 \leq p, q \leq \infty$ and $\delta \geq d(\alpha - 2)|\frac{1}{p} - \frac{1}{2}|$.

Here δ represents the loss of derivatives. The above threshold is shown to be sharp for *all* homogeneous functions μ whose Hessian matrix is non-degenerate at some point.

Keywords: Fourier multipliers, modulation spaces, short-time Fourier transform

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2009/1879

A Compressive Landweber Iteration for Solving Ill-Posed Inverse Problems

Gerd Teschke (Hochschule Neubrandenburg, DE)

In this talk we shall be concerned with the construction of an adaptive Landweber iteration for solving linear ill-posed and inverse problems. Classical Landweber iteration schemes provide in combination with suitable regularization parameter rules order optimal regularization schemes. However, for many applications the implementation of Landweber's method is numerically very intensive. Therefore we propose an adaptive variant of Landweber's iteration that significantly may reduce the computational expense, i.e. leading to a compressed version of Landweber's iteration.

We lend the concept of adaptivity that was primarily developed for well-posed operator equations (in particular, for elliptic PDE's) essentially exploiting the concept of wavelets (frames), Besov regularity, best N-term approximation and combine it with classical iterative regularization schemes.

As the main result we define an adaptive variant of Landweber's iteration. In combination with an adequate refinement/stopping rule (a-priori as well as a-posteriori principles) we prove that the proposed procedure is a regularization method which converges in norm for exact and noisy data. The proposed approach is verified in the field of computerized tomography imaging.

Keywords: Compressive Landweber iteration, Sparsity, adaptive operator evaluation

Joint work of: Teschke, Gerd; Ramlau, Ronny; Zhariy, Mariya

Full Paper:
http://stacks.iop.org/0266-5611/24/065013

Sparsity with dependent coefficients

Bruno Torresani (Université de Provence, FR)

We consider sparse regression problems in situations where dependence relationships between the regression coefficients are to be expected.

Two approaches will be discussed: - variational approaches using mixed norm based coefficient priors, and corresponding optimization algorithms - Bayesian approaches, based upon probabilistic modeling of the regression (synthesis) coefficients.

Applications to a couple of signal processing problems will be discussed.

Geometric properties related to compressed sensing

Przemyslaw Wojtaszczyk (University of Warsaw, PL)

We describe some geometric properties of measurement matrix and show how they can be used to show instance optimality in probability and stability of ℓ_1 minimization decoder. (joint work with R. DeVore and G.Petrova)

Keywords: Compressed sensing

SESOP: Sequential Subspace Optimization Method for very-large-scale optimization problems

Michael Zibulevsky (Technion - Haifa, IL)

SESOP is a method for large-scale smooth unconstrained optimization. At each iteration it searches for a minimum of the objective function over a subspace spanned by the current gradient and by directions of few previous steps. We also include into this subspace the direction from the starting point to the current point, and a weighted sum of all previous gradients, following [Nemirovski,1982]. This safeguard measure provides an optimal worst case error decay of order $1/N^2$ (for convex problems), where N is the iteration count. In the case of quadratic objective, the method is equivalent to the conjugate gradients method.

We identify an important class of problems, where subspace optimization can be implemented extremely fast. This happens when the objective function is a combination of expensive linear mappings with computationally cheap non-linear functions. This is a typical situation in many applications, like tomography, signal and image denoising with Basis Pursuit, pattern recognition with Support Vector Machines, and many others.

Combining SESOP with other methods, like Parallel Coordinate Descent or Truncated Newton, boosts their efficacy. The MATLAB code is available at http://ie.technion.ac.il/ mcib/

Full Paper:
http://iew3.technion.ac.il/~mcib/slides_sesop1.pdf