

Dagstuhl Seminar 09041

Group 1: EMO and Robustness

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1 Robustness

1.1 Uncertainty in the decision space

The main challenge is that the preference relationship \prec defined by

$$x \prec y \quad \text{iff} \quad \phi(x) < \phi(y),$$

where ϕ is some merit function mapping decisions onto real-valued vectors, has to be generalized if x and y are sets instead of elements.

Possible workarounds include the following:

- use representatives of both sets. The question here is, of course, how to choose the corresponding representing members.
- compare all members with each other. The cardinality of the sets involved might be the main bottleneck here. Other issues arise if some members are incomparable, etc.
- Let \bar{x} and σ be expected value and variance of the set x . We can compare these two measures among different sets in a bi-objective fashion.

1.2 Uncertainty in objectives and constraints

We propose the following tentative definition of robustness with respect to the objectives:

a solution x is called robust if small changes in x incur small changes in $f(x)$.

Here, "small" corresponds to a prespecified threshold of the DM.

Uncertainty (especially noise) in the constraints gives rise to the issue of *reliability*. We explain reliability in the last section, where we will also propose to solve uncertainty in the constraints by chance-constrained optimization.

1.3 Further Issues

Finally we would like to mention two further issues.

- Inverse problems: in this scenario, the Pareto-front is given, and the DM would not like to go beyond a certain distance from the front. What is the maximum perturbation allowed in the uncertainty?
- Uncertainty in the cone the DM is using leads to the necessity to find the corresponding "robust" part of the Pareto-front.

2 Flexibility

How to define flexibility:

$$\min_x E_\alpha(f(T(\alpha, x)))$$

x : decision α : info from environment (random) T : transformation function (taking care of flexibility) f : quality function

Robustness: $T = \text{id}$. (no flexibility)

Also interesting: $T \in \mathcal{T}$.

Typical examples:

1. Network optimization with technological changes. A network (to be designed) needs to be robust wrt to technological changes, i.e. protocol changes, routing changes etc. These changes are encoded in the transformation function T .

2. TSP. New customer arrives at some place, solution already found for the TSP needs to be adapted in a flexible way.
3. Path-finding for autonomous robots. Moving obstacles leads to the necessity to update the optimal route found.

Research challenges:

1. iterate T (build up a time-dependent path of solutions). What is the optimization criterion? How to measure optimality? What about the constraints over time?
2. Trade-off between flexibility and solution quality?

3 Approaches to Handle Uncertainty

3.1 Uncertainty in the problem formulation

The basic concept of uncertainty arises by a set-valued mapping from the decision space into the value space: points are mapped into sets, whose shape and form is not necessarily a priori known by the user of the model or the decision maker. Likewise, the mapping that maps points from the decision space into the constraint space (where feasibility resp. infeasibility is measured) is also set-valued.

Uncertainty in the cost (i.e. in measuring the fitness) can usually be estimated by sampling the cost several times at a particular point. One can then either take the mean or factor the samples into the multiobjective problem by using the concept of stochastic dominance. Note that sampling can be time-consuming.

Vincke distinguished between four different types of robustness. In a similar vein, we distinguish between three different types of criteria that have to be taken into account when handling uncertainty. These criteria can, for example, be added to the given list of criteria of a multiobjective optimization problem. Each criterion operates on the decision space, i.e. each criterion maps a decision on a numerical value. The criteria are

- risk: roughly defined as the conditional expectation of becoming infeasible under random perturbations of the given decision. More precisely, probabilities of perturbations are multiplied with a measure of infeasibility.

- reliability: while similar to risk, the measure of infeasibility used here is simply a value of 1 for infeasibility and a value of 0 for feasibility. As such, reliability measures the volume of space that can be reached by perturbations that lead to infeasibility.
- robustness: while risk and reliability measure a quality corresponding to feasibility, robustness measures a quality corresponding to the given objectives of a multiobjective optimization problem. Roughly, 100% robustness is not always necessary. Robustness with respect to the preferences of the decision maker can also be taken into account here.

Note that we omit a formal definition of risk, reliability, and robustness—and this on purpose. While we highly value examples enlightening the reader in what precisely is meant by a particular term we believe it to be too early to constrain oneself to a particular definition at the present state of affairs. Instead, we would like to reiterate again that there are various ways to measure the quality of a particular approach to handle uncertainty.

There is a clear trade-off between optimality of a given point and its robustness: quite often, it seems that one can gain a certain amount of robustness by giving up Pareto-optimality and moving to a dominated point instead. We believe that decision makers should be aware of this.

3.2 Uncertainty in reformulating the model

The two main problems with uncertainty when reformulating the model is underestimation resp. overestimation. There is a clear tradeoff here between accuracy of a model and the complexity of it. Again, the user of the model and the decision maker need to be aware of it.

3.3 Typical algorithmic approaches

A general algorithmic framework can be as follows. The problem

$$\begin{aligned} \min \quad & f(x, \theta) \\ \text{s.t.} \quad & g(x, \theta) \leq 0 \end{aligned}$$

with random parameter θ can be replaced by a problem with objective

$$\min R(f(x, \theta), x, U(x))$$

where R measures risk, reliability, and robustness and U is a function handling uncertainty.

Generally, approaches for handling uncertainty seem to be much better developed in single-objective optimization than in multi-objective optimization. Typical approaches include:

- Chance-constrained optimization: convert uncertainty in the constraints into constraints of the form "the point x will be feasible with probability $1 - \epsilon$ " (where $\epsilon > 0$ is chosen by the user / decision maker).
- Substitute means into variables/parameters: replace the problem "minimize $f(x, \theta)$ " where uncertainties are contained in θ , by "minimize $f(x, \bar{\theta})$ ", where $\bar{\theta}$ is the mean of the random variable θ .
- Single-objective robust programming ("worst case approach"): replace the problem "minimize $f(x, \theta)$ " where uncertainties are contained in the parameters θ , by "minimize $\max_{\theta} f(x, \theta)$ ", where the inner maximization goes over a suitably defined set of θ 's.
- Single-objective stochastic programming (with and without recourse) ("average case approach")
- Interval arithmetic (for the decision variables)

3.4 Typical examples

- job shop scheduling (new jobs arriving in time, see Branke et al)
- robust TSP (See Knowles et al)
- robust car design (uncertainties in parameters, see Deb et al)
- trajectory optimization (Flight time & fuel expenditure to be minimized. Accepting epsilon-dominant solutions lead to more robust trajectories. See Schuetze et al)
- portfolio optimization (many authors)