09111 Abstracts Collection
Computational Geometry
— Dagstuhl Seminar —

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Abstract. From March 8 to March 13, 2009, the Dagstuhl Seminar 09111 "Computational Geometry " was held in Schloss Dagstuhl – Leibniz Center for Informatics. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

09111 Executive Summary – Computational Geometry

1 Computational Geometry Evolution

The field of computational geometry is concerned with the design, analysis, and implementation of algorithms for geometric problems, which arise in a wide range of areas, including computer graphics, CAD, robotics computer vision, image processing, spatial databases, GIS, molecular biology, and sensor networks. Since the mid 1980s, computational geometry has arisen as an independent field, with its own international conferences and journals.

In the early years mostly theoretical foundations of geometric algorithms were laid and fundamental research remains an important issue in the field. Meanwhile, as the field matured, researchers have started paying close attention to applications and implementations of geometric algorithms. Several software libraries for geometric computation (e.g. LEDA, CGAL, CORE) have been developed. Remarkably, these implementations emerged from the originally theoretically oriented computational geometry community itself, so that many researchers are concerned now with theoretical foundations as well as implementations.

Dagstuhl Seminar Proceedings 09111
Computational Geometry
http://drops.dagstuhl.de/opus/volltexte/2009/2034

2 Seminar Topics

The seminar focused on theoretical as well as practical issues in computational geometry. In the following, we list some of the currently most important topics in computational geometry, together with some of the leading researchers working in those areas whom were invited to this seminar:

- *Theoretical foundations* of computational geometry lie in combinatorial geometry and its algorithmic aspects. They are of an enduring relevance for the field, particularly the design and the analysis of efficient algorithms require deep theoretical insights. [Chazelle, Sharir, Welzl,. . .]
- Various *applications* such as robotics, GIS, or CAD lead to interesting variants of the *classical topics* originally investigated, including convex hulls, Voronoi diagrams and Delaunay triangulations, and geometric data structures. For example, pseudotriangulations, generalization of triangulations and developed in connection with visibility and shortest-path problems, have turned out to be useful for many other applications and are being investigated intensively. [van Kreveld, Mitchell, Streinu,. . .]
- Because of applications in molecular biology, computer vision, geometric databases, *shape analysis* has become an important topic. [Asano, Knauer]
- Another increasingly important application of computational geometry is *modeling and reconstruction of surfaces*. It brings about many interesting questions concerning fundamental structures like triangulations as well as new issues in *computational topology*. [Erickson,. . .]
- Massive geometric data sets are being generated by networks of sensors at unprecedented spatial and temporal scale. How to store, analyze, query, and visualize them has raised several algorithmic challenges. New computational models have been proposed to meet these challenges, e.g., streaming model, communication-efficient algorithms, and maintaining geometric summaries. [Arge, Efrat,. . .]
- *Implementation issues* have become an integral part of the research in computational geometry. Besides general software design questions especially *robustness* of geometric algorithms is important. Several methods have been suggested and investigated to make geometric algorithms numerically robust while keeping them efficient, which lead to interaction with the field of computer algebra, numerical analysis, and topology. [Everett, Lazard, Mehlhorn, Wolpert,. . .]

3 Participants

Dagstuhl seminars on computational geometry have been organized since 1990, lately in a two year rhythm, and always have been extremely successful and on a very high scientific level, possibly the highest of all meetings on computational geometry worldwide.

This year, 42 researchers from various countries and continents attended the meeting.

The feedback from participants was very positive.

Participants, especially junior researchers, appreciate the opportunity to meet leaders in the field and benefit from their expertise. Keeping the attendance small enough is a necessary condition for an easy communication and a good research atmosphere, but, having most leaders in the field still allows to invite some very promising younger people. This formula has been recognized as very successful for years.

The long breaks in the schedule are particularly useful for research in working groups.

The place itself is a great strength of the Seminar. Dagstuhl allows people to really meet and socialize, providing them with a wonderful atmosphere of a unique closed and pleasant environment, which is highly beneficial to interactions. Therefore, we warmly thank the scientific, administrative and technical staff at Schloss Dagstuhl!

Joint work of: Agarwal, Pankaj K.; Alt, Helmut; Teillaud, Monique

Kinetic spanners in $\mathbb{R}^d$

Mohammad Abam (Univ. of Aarhus, DK)

We present a new $(1+\epsilon)$-spanner for sets of n points in $\mathbb{R}^d$.

Our spanner has size $O(n/\epsilon^{d-1})$ and maximum degree $O(\log^d n)$.

The main advantage of our spanner is that it can be maintained efficiently as the points move: Assuming the trajectories of the points can be described by bounded-degree polynomials, the number of topological changes to the spanner is $O(n^2/\epsilon^{d-1})$, and using a supporting data structure of size $O(n \log^d n)$ we can handle events in $O(\log^{d+1} n)$ time. Moreover, the spanner can be updated in $O(\log n)$ time if the flight plan of a point changes.

This is the first kinetic spanner for points in $\mathbb{R}^d$ whose performance does not depend on the spread of the point set.

Keywords: Kinetic data structures, geometric spanners

Approximate Euclidean Shortest Paths amid Convex Obstacles

Pankaj Kumar Agarwal (Duke University, US)

We develop algorithms and data structures for the approximate Euclidean shortest path problem amid a set of convex obstacles in 2D and 3D. The running time of our algorithms is linear in the total complexity of obstacles and the size and query time of our data structure depends only on approximation factor and the number of obstacles. Our approach is to quickly compute a small sketch of the obstacles whose size is independent of the total complexity and then compute approximate shortest paths with respect to sketch.

Keywords: Navigation, shortest paths, geometric summaries

Joint work of: Agarwal, Pankaj Kumar; Sharathkumar, R., and Yu, Hai

Small empty monochromatic k-gons in colored point sets

Oswin Aichholzer (TU Graz, AT)

We consider bi-colored variations of problems stated by Erdös and Szekeres in 1935 and Erdös and Guy in 1973. The first question is about the existence of convex k-gons determined by any set S of at least n points in the plane, while the second problem deals with the number of convex k-gons determined by any set S of n points in the plane.

In our setting the points of S are colored and we say that a (not necessarily convex) spanned polygon is monochromatic, if all its points have the same color. Moreover a polygon is called empty if it does not contain any points of S in its interior.

We give an overview of existing results in this area and discuss open problems. We then present two recent results:

1) Any bi-colored set of n points in $\mathbb{R}^2$ in general position determines a super-linear number of empty monochromatic triangles, namely $\Omega(n^{5/4})$. Similar results for higher dimensions also exist.

2) Any bi-colored set of $n \geq 5044$ points in $\mathbb{R}^2$ in general position determines at least one empty, monochromatic quadrilateral (and thus linearly many).

Matching Terrains under a Linear Transformation

Boris Aronov (Polytechnic Institute of NYU, US)

We study the problem of matching two polyhedral terrains, where one can be changed vertically by a linear transformation of the third coordinate (scaling and translation). We give an algorithm that minimizes the maximum distance over all linear transformations in $O(n^{4/3} \operatorname{polylog} n)$ expected time. We also study matching two 1-dimensional terrains, and give a $(1+\varepsilon)$- approximation algorithm for minimizing the area in between that runs in $O(n/\sqrt{\varepsilon})$ time, for any fixed $\varepsilon > 0$.

Keywords: Terrains, matching, linear programming, approximation algorithms

Joint work of: Agarwal, Pankaj; Aronov, Boris; van Kreveld, Marc; Loffler, Maarten; Silveira, Rodrigo

Constant working space algorithms for geometric problems

Tetsuo Asano (JAIST - Ishikawa, JP)

A main interest in this talk is how to implement algorithms using only constant working space. More formally, it uses only constant amount of working space in addition to a read-only array storing input data and outputs are put on a write-only array. Such algorithms have been studied in the community of complexity theory under the name of log-space computation. In our case we are more interested in efficient implementation of those algorithms. In this talk I will describe some constant working space algorithms for several geometric problems, such as those of computing convex hull, triangulation, and Voronoi diagrams.

Keywords: Constant working space algorithm, geometric problem, triangulation, Voronoi diagram

Two-Convex Polygons

Franz Aurenhammer (TU Graz, AT)

We introduce a notion of k-convexity and explore some properties of polygons that have this property. In particular, 2-convex polygons can be recognized in $O(n \log n)$ time, and k-convex polygons can be triangulated in $O(kn)$ time.

Keywords: K-convexity, polygons, triangulation

Joint work of: Aurenhammer, Franz; Aichholzer, Oswin; Hurtado, Ferran; Ramos, Pedro; Urrutia, Jorge

Meshing manifolds embedded in higher dimensional spaces: an intrinsic dimension sensitive approach

Jean-Daniel Boissonnat (INRIA Sophia Antipolis - Mditerrane, FR)

We propose an algorithm to sample and mesh a k-manifold M embedded in $\mathbb{R}^d$. We assume that the dimension k of M is given and that the tangent space at any point of M can be obtained in constant time. Differently from most algorithms that have been developped for meshing surfaces of $\mathbb{R}^3$, we do not compute any subdivision of $\mathbb{R}^d$ (such as a grid or a triangulation of the sample points) but only local stars consisting of k-dimensional simplices around each sample point. By refining the sample, we can insure that all stars become coherent leading to a k-dimensional mesh of M. Our approach extends a similar technique developped recently by Boissonnat, Wormser and Yvinec for anisotropic mesh generation. We prove that the resulting mesh is a good approximation of M. All numerical operations are performed in $\mathbb{R}^k$ except two that are performed in $\mathbb{R}^d$, specifically: 1. projecting a point of $\mathbb{R}^d$ onto a k-dimensional flat; 2. deciding whether a $(d-k)$-flat intersects M and, in the affirmative, reporting an intersection point.

Keywords: Mesh generation, Delaunay refinement, manifold, intrinsic dimension

Joint work of: Boissonnat, Jean-Daniel; Arijit Ghosh

The Convergence of Bird Flocking

Bernard Chazelle (Princeton University, US)

We show how to bound the time it takes for bird flocks to reach steady state in the Vicsek-Cucker-Smale model.

Keywords: Bird Flocking, Tower-of-twos

Lines pinning lines

Otfried Cheong (KAIST - Daejeon, KR)

A line transversal to a family of convex objects in $\mathbb{R}^3$ is a line intersecting each member of the family. If ℓ is a line transversal to a family F but any perturbation of ℓ no longer is, we say that F is a pinning of ℓ. A pinning F of a line is minimal if no proper subset of F pins ℓ. We show any minimal pinning of a line by polytopes in $\mathbb{R}^3$ such that no face of a polytope is coplanar with the line has size at most eight.

Keywords: Line transversal, Helly-type theorem, convex polytopes

Joint work of: Aronov, Boris; Cheong, Otfried; Goaoc, Xavier; Rote, Guenter

A Pseudopolynomial Algorithms for Alexandrov's Theorem

Erik Demaine (MIT - Cambridge, US)

Alexandrov's Theorem states that every metric with the global topology and local geometry required of a convex polyhedron is in fact the intrinsic metric of some convex polyhedron. Recent work by Bobenko and Izmestiev describes a differential equation whose solution is the polyhedron corresponding to a given metric. We describe an algorithm based on this differential equation to compute the polyhedron given the metric, and prove a pseudopolynomial bound on its running time.

Keywords: Folding, metrics, pseudopolynomial, algorithms

Joint work of: Kane, Daniel; Price, Gregory; Demaine, Erik

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2009/2032

Full Paper:
http://arxiv.org/abs/0812.5030

Improved algorithms for joint routing and channel assignment in multi-channel wireless infrastructure networks

Alon Efrat (University of Arizona - Tucson, US)

Multi-channel wireless networks are increasingly being employed as infrastructure networks in metro areas. Recently Ahuja, Gopalan and Ramasubramanian ("Joint routing and channel assignment in multi-channel wireless infrastructure networks", BROADNETS 08) studies the following problem: Given source and destination nodes, the problem is to find a path and channel assignment along the path such that the path bandwidth is the same as that of the link bandwidth. This path must satisfy the constraint that no two consecutive links on the path are assigned the same channel, referred to as channel discontinuity constraint, and is called a CDC Path.

In this paper we propose an improved algorithm that finds such a path using $O(N^2)$ fixed-size messages. We also present an algorithm that generates a sparse network (t-spanner) such that between every possible source and destination the cost of the shortest CDC-path is almost unchanged.

Joint work of: Sankararaman, Swaminathan; Efrat, Alon; Ramasubramanian, Srinivasan; Agarwal, Pankaj K.

Homology Flows, Cohomology Cuts

Jeff Erickson (Univ. of Illinois - Urbana, US)

I will describe the first near-linear-time algorithms to compute maximum flows and minimum cuts in surface-embedded graphs. Specifically, given any undirected edge-weighted graph embedded on an orientable surface of genus g, with two specified vertices s and t, we can compute a minimum (s,t)-cut in $g^{O(g)} n \log n$ time, and a maximum (s,t)-flow in $O(g^7 n \log^2 n \log^2 C)$ time for integer capacities that sum to C, or in $g^{O(g)} n \log^{O(g)} n$ time for arbitrary real capacities. For any fixed surface, all our algorithms run in $O(n \operatorname{polylog} n)$ time. Algorithms to compute maximum flows and minimum cuts in undirected planar graphs in $O(n \log n)$ time have been known for more than 20 years. Except for this special case, the fastest previous algorithms for surface-embedded graphs have no better performance than for general sparse graph. The key insight in our maxflow algorithm is to optimize the relative homology class of the flow, rather than directly optimizing the flow itself.

Keywords: Computational topology, topological graph theory, combinatorial optimization

Joint work of: Chambers, Erin W.; Erickson, Jeff; Nayyeri, Amir

Full Paper:
http://www.cs.uiuc.edu/~jeffe/pubs/surflow.html
See also: STOC 2009 and SOCG 2009

The Voronoi diagram of three arbitrary lines in $\mathbb{R}^3$

Hazel Everett (University Nancy 2, FR)

In this paper we study the Voronoi diagram of lines in $\mathbb{R}^3$.

The Voronoi diagram of three lines in general position was studied in [everett07a].

In this paper we complete this work by presenting a complete characterization of the Voronoi diagram of three arbitrary lines in $\mathbb{R}^3$. As in the general case, we prove that the arcs of trisectors are always monotonic in some direction and we show how to separate the connected components and to sort points along each arc of a trisector using only rational linear semi-algebraic tests. These results are important for the robust computation of the Voronoi diagrams of polyhedra.

Small-size Epsilon-Nets for Axis-Parallel Rectangles and Boxes

Esther Ezra (Duke University, US)

We show the existence of ϵ-nets of size $O\left(\frac{1}{\epsilon}\log\log\frac{1}{\epsilon}\right)$ for planar point sets and axis-parallel rectangular ranges. The same bound holds for points in the plane with "fat" triangular ranges, and for point sets in 3-space and axis-parallel boxes; these are the first known non-trivial bounds for these range spaces. Our technique also yields improved bounds on the size of ϵ-nets in the more general context considered by Clarkson and Varadarajan. For example, we show the existence of ϵ-nets of size $O\left(\frac{1}{\epsilon}\log\log\log\frac{1}{\epsilon}\right)$ for the dual range space of "fat" regions and planar point sets (where the regions are the ground objects and the ranges are subsets stabbed by points). Plugging our bounds into the technique of Brönnimann and Goodrich, we obtain improved approximation factors (computable in randomized polynomial time) for the HITTING SET or the SET COVER problems associated with the corresponding range spaces.

Keywords: Geometric range spaces, randomized algorithms, ϵ-nets, set cover, hitting set

Joint work of: Aronov, Boris; Ezra, Esther; Sharir, Micha

Minimizing Absolute Gaussian Curvature Locally

Joachim Giesen (Universität Jena, DE)

One of the remaining challenges when reconstructing a surface from a finite sample is recovering non-smooth surface features like sharp edges.

There is practical evidence showing that a two step approach could be an aid to this problem, namely, first computing a polyhedral reconstruction isotopic to the sampled surface, and secondly minimizing the absolute Gaussian curvature of this reconstruction globally. The first step ensures topological correctness and the second step improves the geometric accuracy of the reconstruction in the presence of sharp features without changing its topology. Unfortunately it is computationally hard to minimize the absolute Gaussian curvature globally. Hence we study a local variant of absolute Gaussian curvature minimization problem which is still meaningful in the context of surface fairing. Absolute Gaussian curvature like Gaussian curvature is concentrated at the vertices of a polyhedral surface embedded into $\mathbb{R}^3$. Local optimization tries to move a single vertex in space such that the absolute Gaussian curvature at this vertex is minimized. We show that in general it is algebraically hard to find the optimal position of a vertex. By algebraically hard we mean that in general an optimal solution is not constructible, i.e., there exist no finite sequence of expressions starting with rational numbers, where each expression is either the sum, difference, product, quotient or k'th root of preceding expressions and the last expressions give the coordinates of an optimal solution. Hence the only option left is to approximate the optimal position. We provide an approximation scheme for the minimum possible value of the absolute Gaussian curvature at a vertex.

Keywords: Absolute Gaussian curvature, surface reconstruction, mesh smoothing

Joint work of: Giesen, Joachim; Madhusudan, Manjunath

On the Multi-Cover Problem in Geometric Settings

Sariel Har-Peled (Univ. of Illinois - Urbana, US)

We consider the set multi-cover problem in geometric settings. Given a set of points P and a collection of geometric shapes (or sets) F, we wish to find a minimum cardinality subset of F such that each point $p \in P$ is covered (contained in) at least $\mathrm{demand}(p)$ sets. Here $\mathrm{demand}(p)$ is an integer demand (requirement) for p. When the demands $\mathrm{demand}(p) = 1$ for all p, this is the standard set cover problem. The set cover problem in geometric settings admits an approximation ratio that is better than that for the general version. In this paper, we show that similar improvements can be obtained for the multi-cover problem as well. In particular, we obtain an $O(\log Vpt)$ approximation for set systems of bounded VC-dimension, and an $O(1)$ approximation for covering points by half-spaces in three dimensions and for some other classes of shapes.

Joint work of: Chkeuri, Chandra and Clarkson, Ken, and Har-Peled, Sariel;

Full Paper:
http://valis.cs.uiuc.edu/~sariel/papers/08/multi_cover/multi_cover.pdf

Counting solar rays

Ferran Hurtado (TU of Catalonia - Barcelona, ES)

Given a set S of n labeled points in $\mathbb{R}^2$, we study the number of different ways of shooting one ray from each point is such a way that the rays are pairwise non-crossing. Two such sets are different if the correpsonding circular permutation indiced at infinity is different.

If we denote by $r(S)$ this number, and call $r(n) = \max_{|S|=n} r(S)$, we prove that $\lim r(n)^{1/n} = 4$.

We also consider the related problem of counting in how many different ways the points in S can be connected to a given curve by means of pairwise non-crossing segments.

Keywords: Geometric graphs; combinatorial geometry; finite point sets

Joint work of: Garca-Olaverri, Alfredo; Hurtado, Ferran; Tejel, Javier; Urrutia, Jorge

A Complete Approximation Algorithm for Shortest Bounded-Curvature Paths

David G. Kirkpatrick (University of British Columbia - Vancouver, CA)

We address the problem of finding a polynomial-time approximation scheme for shortest bounded-curvature paths in the presence of obstacles. Given an arbitrary environment $\mathcal{E}$ consisting of polygonal obstacles, two feasible configurations, a length ℓ, and an approximation factor ϵ, our algorithm either (i) verifies that every feasible bounded-curvature path joining the two configurations is longer than ℓ or (ii) constructs such a path Π whose length is at most $(1 + \epsilon)$ times the length of the shortest such path.

The run time of our algorithm is polynomial in n (the total number of obstacle vertices and edges in $\mathcal{E}$), m (the bit precision of the input), ϵ^{-1}, and ℓ.

For general polygonal environments, there is no known upper bound on the length, or description, of a shortest feasible bounded-curvature path as a function of n and m. Furthermore, even if the length and description of a shortest path are known to be linear in n and m, finding such a path is known to be NP-hard.

Previous results construct $(1 + \epsilon)$ approximations to the shortest ϵ-robust bounded-curvature path in time that is polynomial in n and ϵ^{-1}. (Intuitively, a path is ϵ-robust if it remains feasible when simultaneously twisted by some small amount at each of its environment contacts.) Unfortunately, ϵ-robust solutions do not exist for all problem instances that admit bounded-curvature paths. Furthermore, even if a ϵ-robust path exists, the shortest bounded-curvature path may be arbitrarily shorter than the shortest ϵ-robust path. In effect, these earlier results confound two distinct sources of problem difficulty, measured by ϵ^{-1} and ℓ. Our result is not only more general, but it also clarifies the critical factors contributing to the complexity of bounded-curvature motion planning.

Keywords: Motion planning, bounded-curvature, shortest paths, approximation algorithms

Joint work of: Backer, Jonathan; Kirkpatrick, David G.

New results on Visibility in Simple Polygons

Rolf Klein (Universität Bonn, DE)

We show that (A), 14 points on the boundary of a Jordan curve, and (B), 16 points in convex position encircled by a Jordan curve, cannot be shattered by interior visibility domains. This means that there always exists a subset of the given points, for which no point of the curve's interior domain sees all points of the subset and no point of its complement. As a consequence, we obtain a new result on guarding art galleries. If each point of the art gallery sees at least an r-th part of the gallery's boundary, then the art gallery can be covered by $13 \cdot C \cdot r \log r$ guards placed on the boundary. Here, C is the constant from the ϵ-net theorem.

Keywords: Computational geometry, art galleries, guards, VC-dimension,

Joint work of: Gilbers, Alexander; Klein, Rolf

Algorithms for graphs of bounded treewidth via orthogonal range searching

Christian Knauer (FU Berlin, DE)

We show that, for any fixed constant $k \geq 3$, the sum of the distances between all pairs of vertices of an abstract graph with n vertices and treewidth at most k can be computed in $O(n \log^{k-1} n)$ time.

We also show that, for any fixed constant $k \geq 2$, the dilation of a geometric graph (i.e., a graph drawn in the plane with straight-line segments) with n vertices and treewidth at most k can be computed in $O(n \log^{k+1} n)$ expected time.

The dilation (or stretch-factor) of a geometric graph is defined as the largest ratio, taken over all pairs of vertices, between the distance measured along the graph and the Euclidean distance.

The algorithms for both problems are based on the same principle: data structures for orthogonal range searching in bounded dimension provide a compact representation of distances in abstract graphs of bounded treewidth.

Keywords: Treewidth, Geometric Graphs, Orthogonal Range Searching, Wiener Index, Dilation

Joint work of: Knauer, Christian; Cabello, Sergio

3D visibility and Lines in spaces: On the complexity of the space of maximal free line segments among objects in 3D

Sylvain Lazard (INRIA - Nancy, FR)

Computing visibility information in a 3D environment is crucial to many applications such as computer graphics, vision and robotics. Typical visibility problems include computing the view from a given point, determining whether two objects partially see each other, and computing the umbra and penumbra cast by a light source. In a given scene, two points are visible if the segment joining them does not properly intersect any obstacle in the scene. The study of visibility is thus intimately related to the study of the set of free line segments in a scene. In this talk, I will review some recent combinatorial and algorithmic results related to non-occluded segments tangent to up to four objects in three dimensional scenes.

I will show, in particular, that the worst-case combinatorial complexity of the set of maximal non-occluded line segments among n unit balls is in $\Theta(n^4)$. This improves on the trivial $\Omega(n^2)$ and $O(n^4)$ bounds and also on the $\Omega(n^3)$ lower bound for the restricted setting of arbitrary-size balls [Devillers and Ramos, 2001]. This also implies that the visibility complex of disjoint fat objects can be as large as for arbitrarily thin triangles. I will also present a new $\Omega(n^3)$ lower bound on the complexity of the set of non-occluded lines among n balls of arbitrary radii, improving on the trivial $\Omega(n^2)$ bound. This new bound is however not known to be tight as the only known upper bound is the trivial $O(n^4)$ bound.

Keywords: 3D visibility, visibility skeleton, visibility complex, fat objects

Dependable and Efficient Algorithms for Curves and Surfaces

Kurt Mehlhorn (MPI für Informatik - Saarbrücken, DE)

We report on recent advances in dependable and efficient algorithms for curves and surfaces.

Keywords: Geometric computing, efficiency, dependability

Open Problem Session

Joseph S. Mitchell (SUNY - Stony Brook, US)

This is a scribing of the open problems posed at the Tuesday evening open problem session. Posers of problems provided input after the session.

Keywords: Open problems, computational geometry

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2009/2030

Approximation Algorithms for Some Geometric Coverage and Connectivity Problems

Joseph S. Mitchell (SUNY - Stony Brook, US)

We present some recent results on approximation algorithms for relay placement in sensor networks, spanning trees and tours for regions in the plane, and the watchman route problem. We give improved constant-factor approximations and in some cases give PTAS's.

Keywords: Approximation algorithms, TSP, TSP with neighborhoods, watchman route, sensor networks, relay placement

Two Applications of Point Matching

Günter Rote (FU Berlin, DE)

The two following problems can be solved by a reduction to a minimum-weight bipartite matching problem (or a related network flow problem):

a) Floodlight illumination: We are given n infinite wedges (sectors, spotlights) that can cover the whole plane when placed at the origin. They are to be assigned to n given locations (in arbitrary order, but without rotation) such that they still cover the whole plane.

(This extends results of Bose et al. from 1997.)

b) Convex partition: Partition a convex m-gon into m convex parts, each part containing one of the edges and a given number of points from a given point set. (García and Tejel 1995, Aurenhammer 2008)

Keywords: Bipartite matching, least-squares

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2009/2029

Full Paper:
http://page.mi.fu-berlin.de/~rote/Papers/pdf/Two+applications+of+point+matching.pdf

See also: Abstracts of the 25th European Workshop on Computational Geometry (EuroCG'09), Brussels, March 2009, pp. 187-189.

Shortest paths amidst growing discs in the plane

Jörg-Rüdiger Sack (Carleton Univ. - Ottawa, CA)

For a robot between two points that avoids n discs growing in the plane. When the discs grow at the same speed, our algorithm runs in $O(n^2 \log n)$ time.

This improves upon the best previous solution by a factor of n. The complexity for the growing disc problem matches the known bound for the more restricted case when all discs are static. When the discs grow at different speeds, our algorithm runs in $O(n^3 \log n)$ time.

Part of this work was presented at ISAAC 2007 (see below link).

Keywords: Shortest paths, navigation, robotics, computational geometry

Joint work of: Maheshwari, Anil; Nussbaum, Doron; Sack, Jörg-Rüdiger; Yi, Jiehua

See also: A. Maheshwari, D. Nussbaum, J.-R. Sack, and Jihua Yi "An O(n**2log n) Time Algorithm for Computing Shortest Paths amidst growing Discs in the Plane" ISAAC 2007, December 17-19, 2007; Proceedings ISAAC 2007, Lecture Notes in Computer Science, Vol. 4835, pp. 668-680.

Measuring the similarity of geometric graphs

Daria Schymura (FU Berlin, DE)

We define a geometric graph distance that is inspired by the concept of edit distance. For two geometric graphs A and B, transform A into B by a sequence of the following operations (in that order!):

1. Edge deletion (cost: length of the edge)
2. Vertex deletion (free)
3. Vertex translation (cost: distance of translation + change in edge length)
4. Vertex insertion (free)
5. Edge insertion (cost: length of the edge)

The geometric graph distance $ggd(A, B)$ is the minimum cost over all such sequences that transform A into B.

This distance turns out to be a metric on the set of geometric graphs without isolated vertices and has an integer linear programming formulation.

Joint work of: Cheong, Otfried; Gudmundsson, Joachim; Kim, Hyo-Sil; Schymura, Daria; Stehn, Fabian

Sharing joints, in moderation

Micha Sharir (Tel Aviv University, IL)

We present the recent algebraic machinery of Guth and Katz, as applied to the problem of joints in arrangements of lines in 3-space (points incident to three non-coplanar lines). Using this machinery, they have shown that the number of such joints is $O(n^{3/2})$, thus settling a long-standing open problem. We review their ideas and extend them to show that the number of incidences between these joints and the given lines is also $O(n^{3/2})$.

Keywords: Incidences, Joints, Algebraic geometry, Lines in space

Joint work of: Sharir, Micha; Elekes, Gyorgy, Kaplan, Haim

Aggressive tetrahedral mesh improvement

Jonathan Shewchuk (Univ. of California - Berkeley, US)

We present a tetrahedral mesh improvement program called Stellar that locally optimizes finite element meshes so their worst tetrahedra have a level of quality substantially better than those produced by any previous method for tetrahedral mesh generation or "mesh clean-up." Mesh optimization methods often get stuck in bad local optima (poor-quality meshes) because their repertoire of mesh transformations is weak. We employ a broader palette of operations than any previous mesh improvement software. Alongside the best traditional topological and smoothing operations, we introduce a topological transformation that inserts a new vertex (sometimes deleting others at the same time). We describe a schedule for applying and composing these operations that rarely gets stuck in a bad optimum. We demonstrate that all three techniques–smoothing, vertex insertion, and traditional transformations–are substantially more effective than any two alone. Our implementation usually improves meshes so that all dihedral angles are between 34 and 131 degrees.

Keywords: Tetrahedral mesh improvement, triangulation, mesh generation

Joint work of: Klingner, Bryan; Shewchuk, Jonathan

Full Paper:
http://www.cs.berkeley.edu/∼jrs/papers/aggress.pdf

See also: Bryan Matthew Klingner and Jonathan Richard Shewchuk, Aggressive Tetrahedral Mesh Improvement, Proceedings of the 16th International Meshing Roundtable (Seattle, Washington), pages 3–23, October 2007.

Computing the reduced precision Voronoi diagram

Jack Snoeyink (University of North Carolina - Chapel Hill, US)

We compute the implicit Voronoi diagram of LPT99 using arithmetic predicates of degree 3.

Keywords: Voronoi, degree-driven algorithm design

Joint work of: Dave Millman, Jack Snoeyink

How far can you reach?

Ileana Streinu (Smith College - Northampton, US)

The talk announced the solution to a long-standing open question in robotics: characterizing, combinatorially, the maximum reach of a 3D robot arm with revolute joints; and raised some questions about other, far less controlled, reachability issues.

Joint work of: Streinu, Ileana; Borcea, Ciprian

Triangulating the 3D Periodic Space

Monique Teillaud (INRIA Sophia Antipolis - Mditerrane, FR)

This work is motivated by the need for software computing 3D periodic triangulations in numerous domains including astronomy, material engineering, biomedical computing, fluid dynamics etc.

We design an algorithmic test to check whether a partition of the 3D flat torus actually forms a triangulation (which subsumes that it is a simplicial complex). We propose an incremental algorithm that computes the Delaunay triangulation of a set of points in the 3D flat torus without duplicating any point, whenever possible. As far as we know, this is the first algorithm of this kind whose output is provably correct. Proved algorithms found in the literature are in fact always computing with 27 copies of the input points, while our algorithmic test detects when such a duplication can be avoided, which is usually possible in practical situations.

The algorithm was implemented and has been submitted to the CGAL Editorial Board. A video of this work was presented at SoCG'08 (http://www.computational-geometry.org/SoCG-videos/socg08video/).

Keywords: Delaunay triangulation

Joint work of: Caroli, Manuel; Teillaud, Monique

Full Paper:
http://hal.inria.fr/inria-00356871/

See also: Manuel Caroli and Monique Teillaud. Computing 3D Periodic Triangulations. Research Report 6823, INRIA, 2009.

On Decomposing Multiple Coverings

Kasturi R. Varadarajan (University of Iowa, US)

We present a result due to Matt Gibson that any k-fold cover in the plane using translates of a triangle can be decomposed into $c \cdot k$ covers, for some constant $c > 0$.

Keywords: Set cover, multiple coverings

Triangulations of Convex Polygons - A Historical Note

Emo Welzl (ETH Zürich, CH)

Catalan Numbers, $C_n := \frac{1}{n+1}\binom{2n}{n}$, are omnipresent when counting "things," one instance being the number of triangulations of a convex polygon with $n+2$ vertices. We briefly review the history, starting with the problem statement in a letter of Euler to Goldbach in 1751. By the end of the 1750's Segner derived the recurrence today known as Catalan recurrence. But it took some 80 years before Lamé resolved the problem, confirming a conjecture of Euler's. Interestingly, Catalan entered the picture only after that.

Keywords: Counting triangulations, Catalan numbers

Shortest Path Problems on a Polyhedral Surface

Carola Wenk (Univ. of Texas at San Antonio, US)

We develop algorithms to compute edge sequences, Voronoi diagrams, shortest path maps, the Fréchet distance, and the diameter for a polyhedral surface. Distances on the surface are measured either by the length of a Euclidean shortest path or by link distance. Our main result is a linear-factor speedup for computing all shortest path edge sequences on a convex polyhedral surface.

Keywords: Shortest paths, edge sequences

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2009/2033

Maintaining Exactly the Convex Hull of Points Moving along Circles

Nicola Wolpert (Hochschule f. Technik - Stuttgart, DE)

In this work we consider the problem of maintaining geometrically exact the convex hull of points moving along circles in the plane. The points are allowed to start with an arbitrary rational angle (in the radian system) on the circle. The main difficulty with respect to the aim of exact geometric computing in this setting is that we have to deal with transcendental numbers. The points in time where the convex hull change are not algebraic but transcendental. We show that, even in this non-algebraic setting, all certificates in the underlying kinetic convex hull algorithm of Razzazi and Sajedi can be evaluated mathematically exact. Our approach has been implemented prototypically.

Keywords: Exact geometric computing, kinetic algorithm, transcendental numbers

Joint work of: Becker, Paul-Georg; Bykbayrak, Elvir; Wolpert, Nicola

The complexity of bisectors and Voronoi diagrams on realistic terrains

Mark de Berg (TU Eindhoven, NL)

We prove tight bounds on the complexity of bisectors and Voronoi diagrams on so-called realistic terrains, under the geodesic distance. In particular, if n denotes the number of triangles in the terrain, we show the following two results.

(i) If the triangles of the terrain have bounded slope and the projection of the set of triangles onto the xy-plane has low density, then the worst-case complexity of a bisector is $\Theta(n)$.

(ii) If, in addition, the triangles have similar sizes and the domain of the terrain is a rectangle of bounded aspect ratio, then the worst-case complexity of the Voronoi diagram of m point sites is $\Theta(n + m\sqrt{n})$.

Keywords: Voronoi diagrams, geodesic distance, terrains, realistic input models

Joint work of: Aronov, Boris; de Berg, Mark; Thite, Shripad

Full Paper:
http://dx.doi.org/10.1007/978-3-540-87744-8_9

See also: Proc. ESA 2008, LNCS 5193, pages 100–111, 2008.

Preprocessing Imprecise Points and Splitting Triangulations

Marc van Kreveld (Utrecht University, NL)

Traditional algorithms in computational geometry assume that the input points are given precisely. In practice, data is usually imprecise, but information about the imprecision is often available. In this context, we investigate what the value of this information is. We show here how to preprocess a set of disjoint regions in the plane of total complexity n in $O(nlogn)$ time so that if one point per set is specified with precise coordinates, a triangulation of the points can be computed in linear time. In our solution, we solve another problem which we believe to be of independent interest. Given a triangulation with red and blue vertices, we show how to compute a triangulation of only the blue vertices in linear time.

Keywords: Data imprecision, triangulation

Joint work of: van Kreveld, Marc; Loffler, Maarten; Mitchell, Joseph S.B.

Full Paper:
http://dx.doi.org/10.1007/978-3-540-92182-0_49

See also: Algorithms and Computation, 19th International Symposium, ISAAC 2008