Online Traveling Salesman Problems with Flexibility

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Abstract. The Traveling Salesman Problem (TSP) is a well-known combinatorial optimization problem. We are concerned here with online versions of a generalization of the TSP on metric spaces where the server doesn't have to accept all requests. Associated with each request (to visit a point in the metric space) is a penalty (incurred if the request is rejected). Requests are revealed over time to a server, initially at a given origin, who must decide which requests to serve in order to minimize the time to serve all accepted requests plus the sum of the penalties associated with the rejected requests. In a first online version of this problem (basic version), we assume that the server's decision to accept or reject a request can be made any time after its release date. In a second online version of this problem (real-time version), we assume that the server's decision to accept or reject a request must be made exactly at its release date. After reviewing prior results on the online TSP, we first provide an optimal 2-competitive online algorithm for the basic version of the problem in a general metric space, improving prior results from the literature. We then consider the real-time version of the problem and show that there can't be any finite c-competitive online algorithm in a general metric space.

Keywords. online TSP, service flexibility, rejection options

The literature for the TSP is vast. The interested reader is referred to the books by Lawler et al. [1] and Korte and Vygen [2] for comprehensive coverage of results concerning the classical TSP.

A systematic study of online algorithms is given by Sleator and Tarjan [3], who suggest comparing an online algorithm with an optimal offline algorithm. Karlin et al. [4] introduce the notion of a *competitive ratio*. Online algorithms have been used to analyze paging in computer memory systems, distributed data management, navigation problems in robotics, multiprocessor scheduling, etc. (e.g. see the survey paper of Albers [5] and the books of Borodin and El-Yaniv [6] and Fiat and Woeginger [7] for more details and references.)

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Research concerning online versions of the TSP have been more recent but has been growing steadily. Kalyanasundaram and Pruhs [8] examine a unique version where new cities are revealed locally during the traversal of a tour (i.e., an arrival at a city reveals any adjacent cities that must also be visited). Angelelli et al. [9,10] study related online routing problems in a multi-period setting.

More related in spirit with our presentation is the stream of works which started with the paper by Ausiello et al. [11]. In this paper, the authors study the online version of the TSP with release dates (but with no service flexibility); they analyze the problem on the real line and on general metric spaces, developing online algorithms for both cases and achieving an optimal online algorithm for general metric spaces, with a competitive ratio of 2. They also provide a polynomial-time online algorithm, for general metric spaces, which is 3-competitive. Subsequently, the paper by Ascheuer et al. [12] implies the existence of a polynomial-time algorithm, for general metric spaces, which is 2.65-competitive as well as a $(2+\epsilon)$-competitive ($\epsilon > 0$) algorithm for Euclidean spaces. Lipmann [13] develops an optimal online algorithm for the real line, with a competitive ratio of 1.64. Blom et al. [14] give an optimal online algorithm for the non-negative real line, with a competitive ratio of $\frac{3}{2}$, and also consider different adversarial algorithms in the definition of the competitive ratio. Jaillet and Wagner [15] introduce the notion of a disclosure date, which is a form of advanced notice for the online salesman, and quantify the improvement in competitive ratios as a function of the advanced notice. A similar approach was taken by Allulli et al. [16] in the form of a *lookahead*.

There has also been work on generalizing the basic online TSP framework. The paper by Feuerstein and Stougie [17] considers the online dial-a-ride problem, where each city is replaced by an origin-destination pair. The authors consider both the uncapacitated case, giving an optimal 2-competitive algorithm, and the capacitated case, giving a 2.5-competitive algorithm. The previously cited paper by Ascheuer et al. [12] also gives a 2-competitive online algorithm and a $(1+\sqrt{1+8\rho})/2$-competitive polynomial-time online algorithm for the uncapacitated online dial-a-ride problem (ρ being the approximation ratio of a simpler but related offline problem). Their algorithm is generalizable to the case where there are multiple servers with capacities; this generalization is also 2-competitive. Jaillet and Wagner [18] consider the (1) online TSP with precedence and capacity constraints and the (2) online TSP with m salesmen. For both problems they give a 2-competitive online algorithms (optimal in case of the m-salesmen problem), consider polynomial-time online algorithms, and then consider resource augmentation, where the online servers are given additional resources to offset the powerful offline adversary advantage. Finally, they study online algorithms from an asymptotic point of view, and show that, under general stochastic structures for the problem data, *unknown and unused by the online player*, the online algorithms are almost surely asymptotically optimal.

Most related to this presentation are recent works dealing with online routing problems which do not require the server to visit every revealed request. Ausiello et al. [19] analyze the online Quota TSP, where each city to be visited has a weight associated with it and the server is given the task to find the shortest sub-tour through cities in such a way to collect a given quota of weights by visiting the chosen cities. They present an optimal 2-competitive algorithm for a general metric space. In Ausiello et al. [20], the authors provide a competitive analysis of the "prize-collecting TSP", a generalization of the quota problem where penalties for not visiting cities are also included, beyond meeting a given quota. They provide a 7/3-competitive algorithm and a lower bound on any competitive ratios of 2 for a general metric space, and refer to a 2-competitive algorithm and a lower bound of 1.89 on the non-negative real line. More generally, assuming a ρ-approximation algorithm for the offline problem, they show that their online algorithm is a $(2\rho + \frac{\rho}{1+2/\rho})$-competitive polynomial time algorithm. In Jaillet and Lu [21], we provide a competitive analysis of the "TSP with flexible service", a special case of the online prize-collecting TSP with no quotas. On the half-line, we provide and prove the optimality of a 2-competitive polynomial time online algorithm based on reoptimization subroutines, and extend it to an optimal 2-competitive online algorithm on the real line. Finally we consider the case of a general metric space and propose an original c-competitive online algorithm, where $c = \frac{\sqrt{17}+5}{4} \approx 2.28$. We also give a polynomial-time $(1.5\rho + 1)$-competitive online algorithm which uses a polynomial-time ρ-approximation for the offline problem.

In our most recent work, Jaillet and Lu [22], we first provide an optimal 2-competitive online algorithm for the online prize collecting TSP, improving the 7/3-competitive online algorithm of [20], and the 2.28-competitive online algorithm of [21]. We then consider the real-time version of the online TSP with rejection options. We show that this problem is significantly harder to tackle in an online setting. We first prove the optimality of a 2.5-competitive polynomial time online algorithm on the non-negative real line. We then consider the case of the real line, where we provide a 3-competitive online algorithm and prove a general lower bound of 2.64 and a tighter lower bound of 2.73 among a restricted family of online algorithms. Finally we consider the case of a general metric space and show that there can't be any finite c-competitive online algorithm. We show a $\Omega(\sqrt{\ln n})$ lower bound on any competitive ratios, and we finally describe an asymptotically optimal $O(\sqrt{\ln n})$-competitive online algorithm.

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