Revealed preference, iterated belief revision and dynamic games

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Abstract

In previous work (G. Bonanno, Rational choice and AGM belief revision, *Artificial Intelligence*, 2009) a semantics for AGM belief revision was proposed based on choice frames, borrowed from the rational choice literature. In this paper we discuss how to use choice frames to analyze extensive-form games. Given an extensive form with perfect recall, a choice frame can be used to represent a player's initial beliefs and her disposition to change those beliefs when she is informed that it is her turn to move. When some players move more than once along some play of the game, the issue of iterated belief revision arises. We provide a semantics for iterated belief revision in terms of choice frames and provide an outline of how to use choice frames to analyze solution concepts for extensive-form games.

Keywords: Choice function, AGM belief revision, extensive-form game, iterated belief revision

1 Introduction

In [5] the notion of AGM-consistent choice frame was proposed as a semantics for the theory of one-stage belief revision put forward by Alchourrón, Gärdenfors and Makinson [1]. In this paper we discuss how to use choice frames to analyze extensive-form games. Given an extensive form with perfect recall, a choice frame can be used to represent a player's initial beliefs and her disposition to change those beliefs when she is informed that it is her turn to move. When some players move more than once along some play of the game, the issue of

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iterated belief revision arises. We provide a semantics for iterated belief revision in terms of choice frames and provide an outline of how to use choice frames to analyze solution concepts for extensive-form games.

2 AGM belief revision and choice frames

In this section we briefly review the AGM theory of belief revision ([1], [8]) and its semantics based on choice frames ([5]).

Let Φ be the set of formulas of a propositional language based on a countable set of atoms. Given a subset $K \subseteq \Phi$, its deductive closure, denoted by $[K]$, is defined as follows: $\psi \in [K]$ if and only if there exist $\phi_1, ..., \phi_n \in K$ (with $n \geq 0$) such that $(\phi_1 \wedge ... \wedge \phi_n) \rightarrow \psi$ is a tautology. A set $K \subseteq \Phi$ is *deductively closed* if $K = [K]$ and it is *consistent* if $[K] \neq \Phi$. Let K be a consistent and deductively closed set representing the agent's initial beliefs and let $\Psi \subseteq \Phi$ be a set of formulas representing possible items of information. A *belief revision function based on* K is a function $B_K : \Psi \rightarrow 2^{\Phi}$ (where 2^{Φ} denotes the set of subsets of Φ) that associates with every formula $\phi \in \Psi$ (thought of as new information) a set $B_K(\phi) \subseteq \Phi$ (thought of as the revised beliefs). If $\Psi \neq \Phi$ then B_K is called a *partial* belief revision function, while if $\Psi = \Phi$ then B_K is called a *full* belief revision function.

Let $B_K : \Psi \rightarrow 2^{\Phi}$ be a (partial) belief revision function and $B_K^* : \Phi \rightarrow 2^{\Phi}$ a full belief revision function. We say that B_K^* is an *extension* of B_K if, for every $\phi \in \Psi$, $B_K^*(\phi) = B_K(\phi)$.

A *full* belief revision function is called an *AGM function* if it satisfies the following properties, known as the AGM postulates: $\forall \phi, \psi \in \Phi$,

(AGM1) $B_K(\phi) = [B_K(\phi)]$.
(AGM2) $\phi \in B_K(\phi)$.
(AGM3) $B_K(\phi) \subseteq [K \cup \{\phi\}]$.
(AGM4) if $\neg\phi \notin K$, then $[K \cup \{\phi\}] \subseteq B_K(\phi)$.
(AGM5) $B_K(\phi) = \Phi$ if and only if ϕ is a contradiction.
(AGM6) if $\phi \leftrightarrow \psi$ is a tautology then $B_K(\phi) = B_K(\psi)$.
(AGM7) $B_K(\phi \wedge \psi) \subseteq [B_K(\phi) \cup \{\psi\}]$.
(AGM8) if $\neg\psi \notin B_K(\phi)$, then $[B_K(\phi) \cup \{\psi\}] \subseteq B_K(\phi \wedge \psi)$.

AGM1 requires the revised belief set to be deductively closed. AGM2 requires that the information be believed. AGM3 says that beliefs should be revised minimally, in the sense that no new formula should be added unless it can be deduced from the information received and the initial beliefs. AGM4 says that if the information received is compatible with the initial beliefs, then any formula that can be deduced from the information and the initial beliefs should be part of the revised beliefs. AGM5 requires the revised beliefs to be consistent, unless the information ϕ is a contradiction (that is, $\neg\phi$ is a tautology). AGM6 requires that if ϕ is propositionally equivalent to ψ then the result of revising by ϕ be identical to the result of revising by ψ. AGM7 and AGM8 are a generalization of AGM3 and AGM4 that requires $B_K(\phi \wedge \psi)$ to be the same as the expansion of $B_K(\phi)$ by ψ, as long as ψ is compatible with $B_K(\phi)$.

Choice frames provide a set-theoretic semantics for belief revision functions.

Definition 1 *A* choice frame *is a triple* $\langle \Omega, \mathcal{E}, f \rangle$ *where*

Ω *is a non-empty set of* states*; subsets of* Ω *are called* events*.*

$\mathcal{E} \subseteq 2^{\Omega}$ *is a collection of events such that* $\varnothing \notin \mathcal{E}$ *and* $\Omega \in \mathcal{E}$*.*

$f : \mathcal{E} \to 2^{\Omega}$ *is a function that associates with every event* $E \in \mathcal{E}$ *an event* $f(E)$ *satisfying the following properties: (1)* $f(E) \subseteq E$ *and (2)* $f(E) \neq \varnothing$*.*

In rational choice theory a set $E \in \mathcal{E}$ is interpreted as a set of available alternatives and $f(E)$ is interpreted as the subset of E which consists of the chosen alternatives (see, for example, [16] and [17]). In our case, we think of the elements of $\mathcal{E}$ as possible items of information and the interpretation of $f(E)$ is that, if informed that event E has occurred, the agent considers as possible all and only the states in $f(E)$. The set $f(\Omega)$ is interpreted as the set of states that are initially considered possible.

In order to interpret a choice frame $\langle \Omega, \mathcal{E}, f \rangle$ in terms of belief revision we need to add a *valuation* $V : A \to 2^{\Omega}$ that associates with every atomic formula $p \in A$ the set of states at which p is true. The quadruple $\langle \Omega, \mathcal{E}, f, V \rangle$ is called a *model* (or an *interpretation*) *of* $\langle \Omega, \mathcal{E}, f \rangle$. Given a model $\mathcal{M} = \langle \Omega, \mathcal{E}, f, V \rangle$, truth of an arbitrary formula at a state is defined recursively as follows ($\omega \models_{\mathcal{M}} \phi$ means that formula ϕ is true at state ω in model $\mathcal{M}$):

(1) for $p \in A$, $\omega \models_{\mathcal{M}} p$ if and only if $\omega \in V(p)$,

(2) $\omega \models_{\mathcal{M}} \neg\phi$ if and only if $\omega \not\models_{\mathcal{M}} \phi$ and

(3) $\omega \models_{\mathcal{M}} (\phi \vee \psi)$ if and only if either $\omega \models_{\mathcal{M}} \phi$ or $\omega \models_{\mathcal{M}} \psi$ (or both).

The truth set of formula ϕ in model $\mathcal{M}$ is denoted by $\|\phi\|_{\mathcal{M}}$, that is, $\|\phi\|_{\mathcal{M}} = \{\omega \in \Omega : \omega \models_{\mathcal{M}} \phi\}$.

Given a model $\mathcal{M} = \langle \Omega, \mathcal{E}, f, V \rangle$ we say that

- the agent *initially believes that* ψ if and only if $f(\Omega) \subseteq \|\psi\|_{\mathcal{M}}$,
- the agent *believes that* ψ *upon learning that* ϕ if and only if (1) $\|\phi\|_{\mathcal{M}} \in \mathcal{E}$ and (2) $f(\|\phi\|_{\mathcal{M}}) \subseteq \|\psi\|_{\mathcal{M}}$.

Accordingly, we can associate with every model a (partial) belief revision function as follows. Let

$$\begin{aligned}
&K_{\mathcal{M}} = \{\phi \in \Phi : f(\Omega) \subseteq \|\phi\|_{\mathcal{M}}\}, \\
&\Psi_{\mathcal{M}} = \{\phi \in \Phi : \|\phi\|_{\mathcal{M}} \in \mathcal{E}\}, \\
&B_{K_{\mathcal{M}}} : \Psi_{\mathcal{M}} \to 2^{\Phi} \text{ be given by } B_{K_{\mathcal{M}}}(\phi) = \{\psi \in \Phi : f(\|\phi\|_{\mathcal{M}}) \subseteq \|\psi\|_{\mathcal{M}}\}.
\end{aligned} \tag{1}$$

What properties must a choice frame satisfy in order for it to be the case that the (typically partial) belief revision function associated with an *arbitrary* interpretation of it can be extended to a full AGM belief revision function? This question motivates the following definition.

Definition 2 *A choice frame $\langle \Omega, \mathcal{E}, f \rangle$ is* AGM-consistent *if, for every model $\mathcal{M} = \langle \Omega, \mathcal{E}, f, V \rangle$ based on it, the (partial) belief revision function $B_{K_{\mathcal{M}}}$ associated with $\mathcal{M}$ (see (1)) can be extended to a full belief revision function that satisfies the AGM postulates.*

Recall that a binary relation $\succsim$ on Ω is a *total pre-order* if it is complete ($\forall \omega, \omega' \in \Omega$, either $\omega \succsim \omega'$ or $\omega' \succsim \omega$) and transitive ($\forall \omega, \omega', \omega'' \in \Omega$, if $\omega \succsim \omega'$ and $\omega' \succsim \omega''$ then $\omega \succsim \omega''$).

Definition 3 *A choice frame $\langle \Omega, \mathcal{E}, f \rangle$ is* rationalizable *if there exists a total pre-order $\succsim$ on Ω such that, for every $E \in \mathcal{E}$,*

$f(E) = \{\omega \in E : \omega \succsim \omega', \forall \omega' \in E\}$.

The interpretation of $\omega \succsim \omega'$ is that state ω is at least as plausible as state ω'. The following proposition is proved in [5]:

Proposition 4 *Let $\langle \Omega, \mathcal{E}, f \rangle$ be a choice frame where Ω is finite. Then the following are equivalent:*

(a) $\langle \Omega, \mathcal{E}, f \rangle$ *is AGM-consistent,*

(b) $\langle \Omega, \mathcal{E}, f \rangle$ *is rationalizable.*

On the basis of Proposition 4, rationalizable choice frames can be viewed as providing a semantics for one-stage belief revision functions that obey the AGM postulates. In the next section we turn to iterated belief revision.

3 Iterated belief revision

Iterated belief revision, that is, the evolution of beliefs over time in response to sequences of informational inputs, has been investigated extensiveley in the literature (see, for example, [6], [7], [10]). AGM belief revision functions map a belief set $K \subseteq \Phi$ and an informational input $\phi \in \Phi$ into a new belief set $B_K(\phi) \subseteq \Phi$. While such functions are sufficient for modeling one-stage belief revision, it has been argued (see, for example, [10] and [15]) that, in the context of iterated belief revision, one should model the evolutions of *belief states* (or epistemic states), rather than simply of belief sets. A belief state is a pair (K, B_K), consisting of a belief set togehter with a disposition to revise one's beliefs, as captured by the belief revision function B_K. Thus iterated belief revision should be construed as a function that maps a belief state (K, B_K) and an informational input ϕ into a new belief state (K', B'_K). In particular, one should allow for the possibility that, after learning that ϕ one changes his disposition to revise one's beliefs; in other words, in general it is possible that $B'_K \neq B_K$.

Fix the set of states Ω. A choice frame $\langle \Omega, \mathcal{E}, f \rangle$ incorporates both the initial beliefs and the disposition to revise those beliefs; thus it can be regarded as representing a belief state. In accordance with the view expressed above, iterated belief revision can be captured semantically by a pair $\langle \mathbb{C}, \mathbb{B} \rangle$ where $\mathbb{C}$ is a set of choice frames and $\mathbb{B}$ is a function that maps a pair consisting of a choice frame $\langle \Omega, \mathcal{E}, f \rangle \in \mathbb{C}$ and an information input $E \in \mathcal{E}$ into a new choice frame in $\mathbb{C}$. We call the pair $\langle \mathbb{C}, \mathbb{B} \rangle$ an *iterated choice structure*. We require that, for all $\langle \Omega, \mathcal{E}, f \rangle \in \mathbb{C}$, $\mathbb{B}\left(\langle \Omega, \mathcal{E}, f \rangle, \Omega\right) = \langle \Omega, \mathcal{E}, f \rangle$, that is, the trivial information Ω does not change the agent's belief state. Furthermore, if $\langle \Omega, \mathcal{E}', f' \rangle = \mathbb{B}\left(\langle \Omega, \mathcal{E}, f \rangle, E\right)$, consistency requires that

$$f'(\Omega) = f(E) \tag{2}$$

In fact, if athe agent's initial beliefs are $f(\Omega)$ and he learns that E, then his revised beliefs are $f(E)$ and these constitute the initial beliefs in the new belief state $\langle \Omega, \mathcal{E}', f' \rangle$, namely $f'(\Omega)$.

Definition 5 *An iterated choice structure $\langle \mathbb{C}, \mathbb{B} \rangle$ is* AGM-consistent *if every choice frame $\langle \Omega, \mathcal{E}, f \rangle \in \mathbb{C}$ is AGM-consistent (see Definition 2).*

From now on we shall restrict attention to AGM-consistent iterated choice structures.

Iterated choice structures can be represented by means of rooted trees. Let t_0 be the root of the tree. Associate with it the initial belief state $\langle \Omega, \mathcal{E}, f \rangle$. For every $E \in \mathcal{E}$ draw an arrow out of t_0 leading to a new node t and associate with t the choice frame $\langle \Omega, \mathcal{E}', f' \rangle = \mathbb{B}\left(\langle \Omega, \mathcal{E}, f \rangle, E\right)$ and proceed similarly for every $E' \in \mathcal{E}'$.

If $\langle \Omega, \mathcal{E}', f' \rangle = \mathbb{B}\left(\langle \Omega, \mathcal{E}, f \rangle, E\right)$ (with $E \in \mathcal{E}$) with abuse of notation we shall denote f' by $\mathbb{B}_{f,E}$ (thus $\mathbb{B}_{f,E} : \mathcal{E}' \to 2^{\Omega}$). The following lemma highlights a property of AGM-consistent iterated choice structures.[1]

Lemma 6 *Let $\langle \Omega, \mathcal{E}, f \rangle$ be a choice frame and $E, F \in \mathcal{E}$ be such that $F \subseteq E$ and $f(E) \cap F \neq \varnothing$. Let $\langle \Omega, \mathcal{E}', f' \rangle = \mathbb{B}\left(\langle \Omega, \mathcal{E}, f \rangle, E\right)$ and suppose that $F \in \mathcal{E}'$. Then if both $\langle \Omega, \mathcal{E}, f \rangle$ and $\langle \Omega, \mathcal{E}', f' \rangle$ are AGM-consistent, $f'(F) = f(F)$. More succintly:*

$$\text{if } F \subseteq E \text{ and } f(E) \cap F \neq \varnothing \text{ then } \mathbb{B}_{f,E}(F) = f(F). \tag{3}$$

Lemma 6 says that, when $F \subseteq E$, if the agent is first informed that E and in his revised beliefs he does not rule out F, then, if he is next informed that F, the propositions that he believes are the same as the ones that he would have believed had he been informed that F to start with. Lemma 6 has no bite (and thus is trivialy satisfied) if the condition $f(E) \cap F \neq \varnothing$ does not hold. We

[1] The proof of Lemma 6 is omitted. It can be found in the extended version of this paper, which will be posted on the author's web page (http://www.econ.ucdavis.edu/faculty/bonanno/).

shall assume the following strenghtening of (3) (obtained by dropping the clause $f(E) \cap F \neq \varnothing$):

$$\text{if } F \subseteq E \text{ then } \mathbb{B}_{f,E}(F) = f(F). \tag{4}$$

According to (4) the agent will hold the same beliefs no matter whether he is informed that F right away or whether he is first informed that E and then that F, whenever $F \subseteq E$. Note that this principle is implied by the best-known theories of iterated belief revision (see, for example, [6], [7], [10]).

We shall now restrict attention to iterated choice structures that satisfy the following property, which we call *information refinement*:

$$\text{if } \langle \Omega, \mathcal{E}', f' \rangle = \mathbb{B}\left(\langle \Omega, \mathcal{E}, f \rangle, E\right) \text{ then, for every } S \in \mathcal{E}', \;\; S \subseteq E. \tag{5}$$

Information refinement says that if the agent is first informed that E and, later on, is informed that F, then $F \subseteq E$. Hence the agent never receives information that contradicts earlier information. Note, however, that (5) does not rule out the possibility that every new piece of information contradicts the agent's previous beliefs.

It will be shown in the next section that the property of information refinement is satisfied in extensive-form games with perfect recall.

Applying the iterated belief revision principle (4) to iterated choice structures that satisfy information refinement we obtain the following property. Let $\succsim$ be the total pre-order on Ω that rationalizes the choice frame $\langle \Omega, \mathcal{E}, f \rangle$ (see Proposition 4), that is, for every $S \in \mathcal{E}$, $f(S) = \{\omega \in S : \omega \succsim x, \forall x \in S\}$. Let $E \in \mathcal{E}$ and $\langle \Omega, \mathcal{E}', f' \rangle = \mathbb{B}\left(\langle \Omega, \mathcal{E}, f \rangle, E\right)$. Then

$$\forall T \in \mathcal{E} \cap \mathcal{E}', \quad f'(T) = f(T) = \{\omega \in T : \omega \succsim x, \forall x \in T\} \tag{6}$$

that is, the same total pre-order $\succsim$ rationalizes both $f(T)$ and $f'(T)$. The above considerations motivate the following definition.

Definition 7 *An iterated choice structure with information refinement* $\langle \mathbb{C}, \mathbb{B} \rangle$ *is* rationalizable *if there exists a total pre-order* $\succsim$ *of* Ω *such that, for every choice frame* $\langle \Omega, \mathcal{E}, f \rangle \in \mathbb{C}$ *and for every* $E \in \mathcal{E}$*,* $f(E) = \{\omega \in E : \omega \succsim x, \forall x \in E\}$*.*

Thus a rationalizable iterated choice structure with information refinement $\langle \mathbb{C}, \mathbb{B} \rangle$ *is equivalent to a one-stage choice frame* $\langle \Omega, \mathcal{E}, f \rangle$ where $\mathcal{E}$ is the union of the domains of the choice frames that belong to $\mathbb{C}$. For example, let $\langle \mathbb{C}, \mathbb{B} \rangle$ be the following iterated choice structure: $\mathbb{C} = \{\langle \Omega, \mathcal{E}_1, f_1 \rangle, \langle \Omega, \mathcal{E}_2, f_2 \rangle, \langle \Omega, \mathcal{E}_3, f_3 \rangle\}$ with $\mathcal{E}_1 = \{\Omega, E, F\}$, $\mathcal{E}_2 = \{\Omega, E_1, E_2\}$, $\mathcal{E}_3 = \{\Omega, F_1, F_2\}$ and, for $k = 1, 2$, $E_k \subseteq E$ and $F_k \subseteq F$; $\mathbb{B}\left(\langle \Omega, \mathcal{E}_1, f_1 \rangle, E\right) = \langle \Omega, \mathcal{E}_2, f_2 \rangle$ and $\mathbb{B}\left(\langle \Omega, \mathcal{E}_1, f_1 \rangle, F\right) = \langle \Omega, \mathcal{E}_3, f_3 \rangle$. Then $\langle \mathbb{C}, \mathbb{B} \rangle$ is equivalent to the choice frame $\langle \Omega, \mathcal{E}, f \rangle$ where $\mathcal{E} = \mathcal{E}_1 \cup \mathcal{E}_2 \cup \mathcal{E}_3$ and f is defined by $f(S) = \{\omega \in S : \omega \succsim x, \forall x \in S\}$ (for every $S \in \mathcal{E}$),where $\succsim$ is the total pre-order that rationalizes $\langle \mathbb{C}, \mathbb{B} \rangle$.

4 Choice frames and extensive-form games

We shall use the history-based definition of extensive-form game (see [11]).[2] Let A be a set of actions and A^* the set of finite sequences in A. If $h = \langle a_1, ..., a_k \rangle \in A^*$ and $j \leq k$, the sequence $h' = \langle a_1, ..., a_j \rangle$ is called a *prefix* of h. If $h = \langle a_1, ..., a_k \rangle \in A^*$ and $a \in A$, we denote the sequence $\langle a_1, ..., a_k, a \rangle$ by ha.

A *finite extensive form* is a tuple $\left\langle H, N, P, p_0, \{\approx_i\}_{i \in N} \right\rangle$:[3]

- A finite set of histories $H \subseteq A^*$ which is closed under prefixes (that is, if $h \in H$ and $h' \in A^*$ is a prefix of h, then $h' \in H$) and contains the empty sequence $\emptyset$. A history $h \in H$ such that, for every $a \in A$, $ha \notin H$, is called a *terminal history*. The set of terminal histories is denoted by Z. Let $D = H \setminus Z$ denote the set of nonterminal or *decision* histories. For every history $h \in H$, we denote by $A(h)$ the set of actions available at h, that is, $A(h) = \{a \in A : ha \in H\}$. Thus $A(h) \neq \varnothing$ if and only if $h \in D$.

- A finite set $N = \{1, ...n\}$ of players. An additional player, denoted by 0 and called *chance* or *nature*, might also be added.

- A function $P : D \to N \cup \{0\}$ that assigns a player to each nonterminal history. Thus $P(h)$ is the player who moves at history h. A game is said to be *without chance moves* if $P(h) \neq 0$ for every $h \in D$. For every $i \in N \cup \{0\}$, let $D_i = P^{-1}(i)$ be the histories assigned to player i. Thus $\{D_0, D_1, ..., D_n\}$ is a partition of D.

- A function p_0 that associates with every $h \in D_0$ a probability distribution over $A(h)$.

- For each player $i \in N$, an equivalence relation $\approx_i$ on D_i. The interpretation of $h \approx_i h'$ is that, when choosing an action at history $h \in D_i$, player i does not know whether she is moving at h or at h'. The equivalence class of $h \in D_i$ is denoted by $I_i(h)$ and is called an *information set of player* i; thus $I_i(h) = \{h' \in D_i : h \approx_i h'\}$. The following restriction applies: if $h' \in I_i(h)$ then $A(h') = A(h)$, that is, the set of actions available to a player is the same at any two histories that belong to the same information set of that player.

- The following property, known as *perfect recall*, is satisfied: for every player $i \in N$, if $h^1, h^2 \in D_i$, $a \in A(h^1)$ and $h^1 a$ is a prefix of h^2 then for every $h' \in I_i(h^2)$ there exists an $h \in I_i(h^1)$ such that ha is a prefix of h'.

[2] Similar structures were introduced in the computer science literature by Parikh and Ramanujam ([13], [14]; see also [12]). These sructures are more general than extensive-form games in that they specify a player's information at every node, that is, not only at nodes where the player himself has to move. Hoewever, as shown in [2] and [3], it is possible to extend the definition of extensive-form game by specifying, for every node, the information that every player has at that node.

[3] Given an extensive form, one obtains an *extensive game* by adding, for every player $i \in N$ a *utility* or *payoff function* $U_i : Z \to \mathbb{R}$ (where $\mathbb{R}$ denotes the set of real numbers and Z the set of terminal histories).

Intuitively, perfect recall requires a player to remember what he knew in the past and what actions he took previously.[4]

Given an extensive form, we can associate with every player i a (possibly iterated) choice frame $\langle \Omega, \mathcal{E}_i, f_i \rangle$ as follows: $\Omega = H$ (the set of histories), $E \in \mathcal{E}_i$ if and only if either $E = H$ or E consists of an information set of player i . Recall that, if $h \in D_i$, player i's information set that contains h is denoted by $I_i(h)$.
Thus

$$\mathcal{E}_i = \{H\} \cup \{I_i(h) : h \in D_i\}. \tag{7}$$

Finally, the function f_i provides conditional beliefs about past and future moves.

If we assume that the (iterated) choice frame of player i is rationalizable, then there exists a total pre-order $\succsim_i$ such that, r for every $E \in \mathcal{E}_i$, $f_i(E) = \{h \in E : h \succsim_i h', \forall h' \in E\}$.

What are natural properties to impose on these choice frames, that is, on the associated (conditional) beliefs? We shall propose three properties.

The first property says that the continuation of a history h cannot be more plausible than h itself:

$$\forall h \in D, \quad \forall a \in A(h), \quad h \succsim_i ha. \tag{$P2$}$$

The second property says that every history has some continuation which is at least as plausible as the history itself and, furthermore, such action must be the same for any two histories that belong to the same information set:

$$\begin{array}{c} \forall h \in D, \exists a \in A(h) : \\ (1)\ ha \succsim_i h \text{ and } (2) \text{ if } h' \in I_j(h) \text{ for some } j \in N \text{ then } h'a \succsim_i h'. \end{array} \tag{$P3$}$$

The third property says that the relative plausibility of past moves is not reversed by the observation of a new move:

$$\begin{array}{c} \forall h, h' \in D, \text{ if } h' \in I_j(h) \text{ for some } j \in N, \text{ then,} \\ \forall a \in A(h),\ h \succsim_i h' \iff ha \succsim_i h'a. \end{array}, \tag{$P4$}$$

In the extended version of this paper these properties are used, in conjunction with other properties, to characterize solutions concepts for extensive-form games, in particular the notion of sequential equilibrium introduced in [9].

[4] For an in-dept analysis on the notion of perfect recall see [4].

References

[1] Alchourrón, Carlos, Peter Gärdenfors and David Makinson, On the logic of theory change: partial meet contraction and revision functions, *The Journal of Symbolic Logic*, 1985, 50: 510-530.

[2] Battigalli, Pierpaolo and Giacomo Bonanno, Synchronic information, knowledge and common knowledge in extensive games, in: M. Bacharach, L.A. Gérard-Varet, P. Mongin and H. Shin (editors), *Epistemic logic and the theory of games and decisions,* Kluwer Academic, 1997, pp. 235-263. Reprinted in: *Research in Economics*, 1999, 53: 77-99.

[3] Bonanno, Giacomo, Players' information in extensive games, *Mathematical Social Sciences*, 1992, 24: 35-48.

[4] Bonanno, Giacomo, Memory and perfect recall in extensive games, *Games and Economic Behavior*, 2004, 47: 237-256.

[5] Bonanno, Giacomo, Rational choice and AGM belief revision, *Artificial Intelligence*, 2009, 173: 1194–1203.

[6] Boutilier, Craig, Iterated revision and minimal change of conditional beliefs, *Journal of Philosophical Logic*, 1996, 25: 263,305.

[7] Darwiche, Adnan and Judea Pearl, On the logic of iterated belief revision, *Artificial Intelligence,* 1997, 89: 1-29.

[8] Gärdenfors, Peter, *Knowledge in flux: modeling the dynamics of epistemic states*, MIT Press, 1988.

[9] Kreps, David and Robert Wilson, Sequential equilibrium, *Econometrica,* 1982, 50: 863-894.

[10] Nayak, Abhaya, Maurice Pagnucco and Pavlos Peppas, Dynamic belief revision operators, *Artificial Intelligence*, 2003, 146: 193-228.

[11] Osborne, Martin and Ariel Rubinstein, *A course in game theory,* MIT Press, 1994.

[12] Pacuit, Eric, Some comments on history based structures, *Journal of Applied Logic*, 2007, 5: 613–624.

[13] Parikh, Rohit and R. Ramanujam, Distributed processes and the logic of knowledge, in: *Logic of Programs*, *Lecture Notes in Computer Science*, vol.193, Springer, 1985, pp.256–268.

[14] Parikh, Rohit and R. Ramanujam, A knowledge based semantics of messages, *Journal of Logic, Language and Information*, 2003, 12: 453–467.

[15] Rott, Hans, Coherence and conservatism in the dynamics of belief, *Erkenntnis, 1999, 50: 387-412.*

[16] Rott, Hans, *Change, choice and inference*, Clarendon Press, 2001.

[17] Suzumura, Kotaro, *Rational choice, collective decisions and social welfare*, Cambridge University Press, 1983.