09302 Abstracts Collection

New Developments in the Visualization and Processing of Tensor Fields

— Dagstuhl Seminar —

Bernhard Burgeth[1] and David H. Laidlaw[2]

[1] Saarland University, DE
burgeth@math.uni-sb.de
[2] Brown University, USA
dhl@cs.brown.edu

Abstract. From 19.07. to 24.07.2009, the Dagstuhl Seminar 09302 "New Developments in the Visualization and Processing of Tensor Fields " was held in Schloss Dagstuhl – Leibniz Center for Informatics. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Visualization, image processing, tensor fields, diffusion tensor imaging (DT-MRI), partial differential equations (PDEs), tensor decomposition, feature extraction, segmentation, fiber tracking, elastography, fluid dynamics, structural mechanics

09302 Summary – New Developments in the Visualization and Processing of Tensor Fields

This Dagstuhl Seminar was concerned with the visualization and processing of tensor fields, like its two predecessors: seminar 04172 organized by Hans Hagen and Joachim Weickert in April 2004, and the follow-up seminar 07022 in January 2007 with David Laidlaw and Joachim Weickert as organizers. Both earlier meetings were successful, resulting in well received books and triggering fruitful scientific interaction and exchange of experience across interdisciplinary boundaries. We believe that the 2009 seminar will prove to have been equally successful.

Keywords: Visualization, image processing, tensor fields, diffusion tensor imaging (DT-MRI), partial differential equations (PDEs), tensor decomposition, feature extraction, segmentation, fiber tracking, elastography, fluid dynamics, structural mechanics

Joint work of: Burgeth, Bernhard; Laidlaw, David H.

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2009/2237

Some Mathematical Aspects of Tensor Sampling

Philip Batchelor (King's College London, GB)

In this talk I discuss issues relating to what is the best way to sample a tensor or a multivariate polynomial. This means we concentrate on symmetric tensors.

I show how polynomial/tensor sampling is a linear problem, and error propagation can be studied from that point. In particular we can look at the condition number of the design matrix. This was proposed by Skare from the point of view of DT-MRI, where the diffusion tensor is sampled assuming the Stejskal-Tanner equation. Stefan Skare found a minimum condition number of 1.3228

I show how this is actually likely to be sqrt(7)/2 in 3D, and how we can generalise such questios, on one hand to large/infinite number of directions, and to higher numbers of variables. I estimate the answer in 2D, and from this ask what the asymptotics is in any dimension.

Keywords: Condition number, sampling schemes

Numerical methods non-local operators

Steffen Boerm (Universität Kiel, DE)

Non-local interactions play an important role in many applications, e.g., the gravitational field of a sun can be observed even over large distances, not just in a local neighbourhood. Other examples include population dynamics and the modeling of stock prices.

The non-local behaviour couples essentially all degrees of freedom of a problem to each other and means that simple algorithms will have a prohibitively large runtime.

In order to handle problems of this type efficiently, the long-range interactions are usually approximated. A particularly efficient approximation is provided by $\mathcal{H}^2$-matrices, which combine concepts of multilevel techniques with higher-order approximation to handle the n^2 interactions of n degrees of freedom in $\sim nk$ operations, where k is a parameter controlling the accuracy.

The presentation gives an introduction to typical non-local interactions and the solution approach based on $\mathcal{H}^2$-matrices.

Keywords: Non-local operators, data-sparse approximation, $\mathcal{H}^2$-matrices

Anisotropic Evolution Processes for Matrix Fields

Bernhard Burgeth (Universität des Saarlandes, DE)

There are some well-established anisotropic filters for scalar images based on partial differential equations (PDEs), notably, edge enhancing and coherence enhancing diffusion filters, but also adaptive morphological filters induced by anisotropic dilation and erosion.

In this talk we discuss a general framework that allows for an efficient transfer of these filters to fields of symmetric matrices.

This encompasses an appropriate matrix-valued PDE-formulation as well as proper numerical solution schemes.

In numerical experiments on real and synthetic 3D data we explore the performance of the matrix-valued counterparts of the aforementioned filters.

Decompositions of Higher-Order Tensors and Applications: an Overview

Lieven De Lathauwer (K.U. Leuven, BE)

This lecture is meant as an introduction to recent developments concerning decompositions of tensors of order three or higher.

We discuss the following basic decompositions: (i) Tucker decomposition or multilinear singular value decomposition, (ii) decomposition in rank-1 terms, known as canonical or parallel factor decomposition, and (iii) decomposition in block terms. Decompositions of higher-order tensors have led to new methods for signal processing and data analysis.

We discuss three basic approaches for blind signal separation: (i) parallel factor analysis, (ii) independent component analysis, and (iii) block component analysis.

Keywords: Tensor decomposition, higher-order tensors

Manifold ways of coloring diffusion tensor fields

Cagatay Demiralp (Brown University - Providence, US)

I will talk about two surfaces, Boy's immersion of the real projective plane (RP2) and the flat torus, for coloring diffusion tensor fields and their integral curves.

Processing Tensor Data with Higher Order PDEs

Stephan Didas (Fraunhofer ITWM - Kaiserslautern, DE)

Higher order partial differential equations offer the possibility to denoise data without the stair-casing effect known from classical second-order nonlinear diffusion filtering.

We investigate matrix-valued extensions of the underlying PDEs and their practical properties. Special interest lies in the preservation of piecewise polynomial data with such methods.

Keywords: Image denoising, Partial Differential Equations

Harmonic Vector Field Analysis

Hans Hagen (TU Kaiserslautern, DE)

Harmonic Analysis Techniques is a technique that is known for a long time now and is very successfull in different application areas with the Fourier Decomposition as one famous representative. We will show some possible ways to use Harmonic Analysis on vector fields in a global way and address some of the modelling and calculation issues for them. We will care especially about the issue of huge computational costs arising from global approaches and introduce a local framework for vector field analysis. Finally, we want to start a discussion how these techniques can be transfered to tensor fields.

Keywords: Harmonic Analyisis, Vector Field Analysis

Joint work of: Hagen, Hans; Wagner, Christian

Another kind of diffusion tensor imaging: directional dependence in the anomalous diffusion exponent

Matt Hall (University College London, GB)

We introduce the concept of the "Anomalous exponent tensor" in diffusion imaging by demonstrating that the usual assumptions of regular diffusion in diffusion MRI are broken by the presence of resriction and disorder in the diffusion environment. We show that this leads to a stretched exponential form for diffusion weighted signal decay and that this curve has dependence on orientation of diffusion-sensitising gradients.

We postulate a Gaussian form for this dependence and construct images of quantites analgous to Fractional Anisotropy and mean diffusivity.

Keywords: Anomalous Diffusion, disorder, non-monoexponential

Joint work of: Hall, Matt; Barrick, Thomas

Features in Uncertain Tensor Fields

Hans-Christian Hege (ZIB - Berlin, DE)

Referencing to another lecture, we began by discussing a method for 3D visualization of tensor fields.

In this method (which has first been presented at the first tensor Dagstuhl Workshop in 2004) semi-transparent splats that are are oriented in space are drawn. The shape of the splats is determined by the linear, planar and spherical anisotropy.

Since tensor data are used to to infer information and take decisions that sometimes far-reaching, it is a very important requirement that the image analysis and visualization techniques are reliable. However, all image data are affected by uncertainty. Therefore, conclusions have to be drawn from uncertain information. Error estimation therefore should be ubiquitous and visualization tools should show which information is reliable and which is less trustworthy.

For this it is necessary to estimate the error in the input data and to control the error propagation during the reconstruction, regularizing, feature extraction and visualization step. The (algorithm-independent) condition numbers of the functions computed in these steps determine how errors in the input data are amplified.

This is illustrated for one of the most simple features in fields: iso-contours in scalar fields. Two measures are introduced that represent the probability of the presence of a iso-contour (for a given iso-level) in space. Various visualization techniques are discussed and applications are sketched.

Finally it is discussed how, in the future, these results could be generalized to tensorial data.

Keywords: Tensor data, uncertainty, condition number, error propagation, iso-contours

Challenges in Tensor Field Visualization for Mechanical Engineering

Ingrid Hotz (ZIB - Berlin, DE)

Tensor fields occur in engineering and scientific simulations, either as intermediate product or as final result. Mostly, the analysis of the resulting data is based on scalar fields derived from these tensor fields. Since this approach often is insufficient to understand the entire physical process, there is an increasing interest in the analysis of tensor data itself. The wealth of information contained in tensor data, however, makes visualization, analysis, and finally understanding of the data a challenging problem. In addition, the scientists are often not used to looking at the tensor data itself, which leads to a lack of specific questions. Thus, the first goal is to provide an overview of the data without missing important details and without overwhelming the observer at the same time.

An essential step to reduce the amount of information is a segmentation that separates the field into regions of similar characteristic behavior. Such segmentation can be considered as a basis for visualization techniques using textures and glyphs. We developed a topology-based method for generating a segmentation of 2D symmetric tensor fields using directional as well as eigenvalue characteristics of the underlying field.

Keywords: Stress tensor, segmentation

Joint work of: Hotz, Ingrid; Auer, Cornelia; Sreevalsan Nair, Jaya

Sampling the Scale-Space behavior of Tensor Invariants

Gordon Kindlmann (University of Chicago, US)

This talk will explore the intersection between two otherwise unrelated topics: 1) one of the simplest (but scientifically most useful) aspects of diffusion tensor imaging, scalar invariants such as fractional anisotropy, and 2) scale-space treatment of images undergoing continuously increasing blurring. Aside from bulk mean diffusivity (also referred to as "apparent diffusion coefficient"), interesting invariants such as norm, FA, and mode are all non-linear: F(A+B) != F(A) + F(B). Therefore, the behavior of the invariant as the image is blurred is also non-linear. I will describe early results in using particle systems to sample scale-space features in tensor fields, and propose a self-contained research project that uses manifold learning to quantify the information content of the invariants in tensor-field scale space.

Keywords: Tensor invariants, scale space, particle systems

Towards Closure on Structure Tensor Estimation: Introducing Monomial Quadrature Filter Sets

Hans Knutsson (Linköping University Hospital, SE)

Description and estimation of local spatial structure has a long history. The central concept "structure tensor" has a fairly broad and unspecific meaning. The first part of this chapter is intended to clarify the different flavours of commonly used local structure descriptors. The starting point of this brief historic expose is image descriptors such as local moments and edge detectors. We continue to give a short account of the developments leading up to the local structure tensor concept and summarize the current state of the art.

The second part presents a new method for structure tensor estimation which is a generalization of many of it's predecessors. The method uses pairs of filter sets having Fourier directional responses in the form of monomials, one odd order set and one even order set. It is shown that such filter sets allow for a particularly simple way of attaining phase invariant, positive semidefinte, local structure tensor estimates. Further, we argue that with a particular choice of filter set follows a simple natural generalization of the concept of phase.

Keywords: Structure tensor quadrature monomial filter estimation

Joint work of: Knutsson, Hans; Westin, Carl-Fredrik; Andersson, Mats

How are we making a**ses of ourselves?

David H. Laidlaw (Brown University - Providence, US)

Implicit and unquestioned assumptions can be perilous. We will examine together some of the assumptions researchers in our areas make and try to establish what the costs of the assumptions may be.

(Symmetric) Stress Tensor Visualization in Turbulent Flow Calculations

Georgeta Elisabeta Marai (University of Pittsburgh, US)

Symmetric stress tensors can help mechanical engineers design faster computational solutions to the simulation of turbulent gaseous flow. In this talk I will discuss the challenges to symmetric stress tensor visualization in fluid dynamics, and sketch a solution to the pervasing problems of occlusion and clutter. The targetted result is a visual analysis tool to assist combustion researchers in debugging computational solutions to the simulation of gaseous flow under dynamic conditions.

Keywords: Turbulent flow calculations, tensor visualization and interaction

Adaptation of Tensor Voting to Image Structure Estimation

Rodrigo Moreno (Universität des Saarlandes, DE)

The tensor voting framework is a well-known robust technique for structure estimation in a cloud of points. In this talk we discuss how the tensor voting framework can also be adapted to structure estimation in grey-scale and color images. We use the structural relationships between the tensor voting framework and the structure tensor to propose such an adaptation. Experiments in the specific applications of edge and corner detection show that the tensor voting has a better performance than the classical structure tensor.

Why do tensors swell?

Ofer Pasternak (Tel Aviv University, IL)

Interpolating two tensors using a Euclidean metric and Cartesian coordinates may yield a “swollen" tensor with a determinant (or volume) larger than either of the original tensors.

Replacing the Euclidean metric with an affine-invariant metric reduces the swelling. We show that in the most general case the swelling of diffusion tensors is predicted by Johnson noise. We show that when restricting the analysis to white matter, the affine-invariant metric introduces other artifacts. We conclude by introducing a framework for tensor operations which solves the swelling effect in white matter.

Keywords: DTI, Riemannian metric, invariance

An approach to visualise tensor fields in engineering

Bernhard Peters (University of Luxembourg, LU)

Tensorial quantities are widely employed in engineering such as in continuum mechanics, electrodynamics or characterisation of material properties. Therefore, the objective of this contribution is to introduce approaches that visualise tensorial quantities with a particular emphasis on engineering applications. Tensors in mechanical engineering emerged from stress and strain analysis of structures because it requires to charcterise properties on a higher level than vectors. Furthermore, tensors are invariant to coordinate transformation. Thus, tensors are a very versatile approach to deal with engineering applications.

However, visualisation of tensors or tensor fields emanating from an analysis of various structures is not as simple as that of vectors or scalars. In principle, tensors of rank 2 as applied in engineering contain already 9 independent elements. In order to reduce the data, a transformation involving eigen-values and eigen-vectors is carried out, which still characterises the state of stress or strain meaningfully. The resulting 3 eigenvalues represent the semi-axis of an ellipsoid, that may be visualised as a spheroid in space. Its colour may refer to the maximal shear stress, upon which failure of a structure is usually assessed.

Keywords: Stresstensor, visualisaton

Non-Local Regularisation and Interpolation of Matrix-Valued Images

Luis Pizarro (Universität des Saarlandes, DE)

We introduce a discrete variational framework for the joint regularisation and interpolation of matrix-valued images. Unlike classical variational methods which operate (semi-)locally, our approach exploits structural information contained in non-local neighbourhoods to produce more reliable and accurate estimations. An important characteristic of the proposed framework is that it offers several degrees of freedom that allow us to obtain related models known from literature as special cases. Experiments on synthetic and real world (DT-MRI) data demonstrate the capabilities of our approach for the joint regularisation and interpolation of tensor fields.

Keywords: Regularisation, interpolation, tensor field, DT-MRI

Joint work of: Pizarro, Luis; Burgeth, Bernhard; Didas, Stephan; Weickert, Joachim

Magnetic Resonance Elastography: From palpation to tissue structure

Ingolf Sack (Charité Berlin, DE)

The mechanical behavior of biological tissue is determined by rigidity and compliance of cells as well as their geometrical organization and intercellular connectivity. Such tissue microstructural properties are important diagnostic markers and thus it is desirable to establish medical imaging techniques based on tissue mechanics. Traditionally, the elasticity of living tissue is assessed by palpation in order to 'feel' suspicious elasticity changes associated with disease. A technical "palpation", known as magnetic resonance elastography (MRE), combines shear waves with MR imaging (MRI). Shear wave fields are introduced inside the body by surface drivers vibrating at frequencies of a few tens of Hertz. The propagation of wave field components is captured by motion-sensitive MRI; in principal the entire field is accessible. Finally, field maps are analyzed using appropriate wave-inversion algorithms and image processing tools for calculating spatially resolved maps of wave speed and wave damping. MRE uniquely allows measuring the viscoelasticity of brain tissue in its intact physiological environment. First MRE studies performed on volunteers and patients show the effect of aging and multiple sclerosis on the biomechanical properties of the brain. MRE has also been applied on skeletal muscle for measuring elasticity as a synonym of muscle function in various muscle-related diseases. Moreover, new experiments demonstrate the feasibility of MRE to detect myocardial elasticity changes between diastole and systole, which are related to intraventricular pressure changes. This allows for the first time the noninvasive measurement of cardiac work in the human heart which is potentially useful for assessing pathologies associated with increased myocardial stiffness such as diastolic dysfunction.

Keywords: Elastography, shear waves, tissue mechanics, viscoelasticity, brain, muscle, heart

Towards Resolving Fiber Crossings with Higher Order Tensor Inpainting

Thomas Schultz (MPI für Informatik - Saarbrücken, DE)

Fiber tracking in higher-order tensor fields allows one to resolve fiber crossings. However, estimation of the higher-order tensors currently requires acquisition of high angular resolution diffusion imaging (HARDI) data, which is very time consuming.

The goal of our work is to investigate the potential of estimating higher-order tensor fields that can be used for tractography from a smaller number of measurements, by integrating directional information from local neighborhoods.

Our strategy resembles techniques for image inpainting which are based on partial differential equations (PDEs): We mask areas in which only insufficient directional information is present, and try to infer the most likely dominant orientations from the surrounding voxels. Our framework involves a higher-order tensor voting scheme and anisotropic image diffusion with multiple diffusion tensors.

Joint work with Stephan Didas.

Keywords: Higher-Order Tensors, Diffusion-based Image Processing, DT-MRI, Tensor Voting

Visualizing the Structure of 3D Tensor Fields: Topology, Invariant Creases, and Lagrangian Metaphor

Xavier Tricoche (Purdue University, US)

This talk is concerned with the basic problem of characterizing salient structures in 3d symmetric tensor fields independently of a particular application. Topology has been proposed as a mathematical formalism to do just that but its practical shortcomings have been documented in certain applications such as Diffusion Tensor MRI. Another approach that has been increasingly studied in recent years consists in characterizing structures as ridge manifolds of tensor invariants. Our recent work has shown that the two approaches are in fact linked in that certain ridges of a particular invariant called mode offer a partial topological picture of 3d tensor fields.

I will discuss a method that applies a ridge-based structure characterization to a non-local processing of a direction field to identify salient structures. This approach can be used to recover the full topological structure and is fundamentally more robust than the standard topology in noisy data sets. Conceptually, it can also be seen as a special case of a more general ridge-based framework in which the tensor field's orientation information plays a distinguished role. Early results will be presented for synthetic, medical, and engineering datasets.

Hybrid HARDI/DTI visualization

Anna Vilanova (Eindhoven University of Technology, NL)

The diffusion weighted MRI (DW-MRI) technique High Angular Resolution Diffusion Imaging (HARDI) is able to model complex fiber configurations. This overcomes one of the clear limitations of Diffusion Tensor Imaging (DTI) that is not able to follow. However, one has to keep in mind all the drawbacks that come along: complex modeling of the data, non-intuitive and computationally heavy visualization, as well as the inability to interactively explore and transform the data.

A framework is presented that tries to select and benefit from both DTI and HARDI. We have a framework where several classical DTI visualization have been implemented. We also have a fast GPU based HARDI glyph visualization based on models defined by Spherical Harmonics. We apply a classification scheme based on HARDI anisotropy measures calculated from spherical harmonics. The threshold is selected semiautomatically that facilitates the inconvenient threshold tuning for the purpose of classification.

This classification allows simplification of the data representation in areas with single fiber bundle coherence. The resulting interactive, multi-field framework, allows data exploration of fused HARDI and DTI data. This is our first attempt to combine the advantages of HARDI and DTI for visualization, better classification and ways to efficiently combine both representations still need to be investigated.

Full Paper:
http://www.bmia.bmt.tue.nl

Representation and Estimation of Tensor-pairs

Carl-Fredrik Westin (Harvard Medical School - Boston, US)

In many applications, there is a need to represent and process pairs of events. An example of this is the analysis of diffusion MRI data where the traditional diffusion tensor model fail in complex neighborhoods of crossing fiber bundles. One remedy is to estimate more than one tensor in each data point. However, after doing this, three is no inherent ordering of the these events, and no natural way to separate them into more simple fields. Being able to process the data without the need to separate the events is thus desirable. In this work we present a way of representing pairs of tensors. The method uses a symmetrized tensor outer product as a representation, and an eigenvector solution for recovering the events.

Keywords: Tensors

Joint work of: Westin, Carl-Fredrik; Knutsson, Hans

2D Asymmetric Tensor Field Topology

Eugene Zhang (Oregon State University, US)

The gradient of a velocity vector field is an asymmetric tensor field which can provide critical insight that is difficult to infer from traditional trajectory-based vector field visualization techniques.

We describe the structures in the eigenvalue and eigenvector fields of the gradient tensor and how these structures can be used to infer the behaviors of the velocity field that can represent either a 2D compressible flow or the projection of a 3D compressible or incompressible flow onto a two-dimensional manifold.

In addition, we visualize the structures in the eigenvector fields by using a combination of hyperstreamline tracing and glyph packing.

Keywords: Asymmetric tensor field analysis, flow visualization, tensor field visualization, hyperstreamline placement, glyph packing

Visualizing DTI fibers as 2D points

Song Zhang (Mississippi State University, US)

Visual exploration is essential to the visualization and analysis of densely sampled 3D DTI fibers, due to the high geometric, spatial and anatomical complexity of fiber tracts. Previous methods for DTI fiber visualization use zooming, color-mapping, selection, and abstraction to deliver the characteristics of the fibers. However, these schemes mainly focus on the optimization of visualization in the 3D space where cluttering and occlusion make it difficult to grasp even a few thousand fibers. This talk introduces a novel interaction method that augments the 3D visualization with a 2D representation containing a low-dimensional embedding of the DTI fibers. This embedding preserves the relationship between the fibers and removes the visual clutter that is inherent in 3D renderings of the fibers. This new interface allows the user to manipulate the DTI fibers as both 3D curves and 2D embedded points, and easily compare or validate their results in both domains. The implementation of the framework is GPU-based to achieve real-time interaction.

The framework was applied to several tasks and the results show that our method reduces the users' workload in recognizing 3D DTI fibers and permits quick and accurate DTI fiber selection.

Keywords: DTI fiber, dimension reduction, projection, interface, interaction

Tensor field segmentation for white matter analysis

Rodrigo de Luis Garcia (University of Valladolid, ES)

We present a framework for white matter analysis from DTI based on the automatic 3D segmentation of the structures of interest. As opposed to most white matter studies, which are usually performed through volumetric analysis on images registered to a common template (VBM), along fiber tracts (TBSS) or on manually-delineated regions of interest (typically 2D), here we propose to analyze 3D regions of interest in the space of the original image. To that end, the segmentation aproach that we describe is based on two key elements:

(a) the use of a DTI atlas of human white matter to automatically obtain initial contours and a spatial prior for the segmentation process, thus enabling a completely automatic segmentation.

(b) the use of tensor invariants, together with the orientation of the tensor, as features to drive the segmentation, instead of other commonly used tensor distances.

Using this segmentation approach, we performed a white matter study comparing FA in healthy controls and schizophrenia patients, showing significant differences in the corpus callosum and the corona radiata, which is consistant with other findings in the literature.

Also, we address the problem of selecting the most appropriate tensor distance for DTI segmentation, proposing a comparison of the segmentation on a DTI atlas as a feasible approach to it.

Keywords: DTI, tensor analysis, segmentation, Geodesic Active Regions, schizophrenia studies