Semilattices, Domains, and Computability (Invited Talk)

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As everyone knows, one popular notion of Scott domain is defined as a bounded complete algebraic cpo. These are closely related to algebraic lattices: (i) A Scott domain becomes an algebraic lattice with the adjunction of an (isolated) top element. (ii) Every non-empty Scott-closed subset of an algebraic lattice is a Scott domain. Moreover, the isolated (= compact) elements of an algebraic lattice form a semilattice (under join). This semilattice has a zero element, and, provided the top element is isolated, it also has a unit element. The algebraic lattice itself may be regarded as the ideal completion of the semilattice of isolated elements. This is all well known. What is not so clear that is that there is an easy-to-construct domain of countable semilattices giving isomorphic copies of all countably based domains. This approach seems to have advantages over both "information systems" or more abstract lattice formulations, and it makes definitions of solutions to domain equations very elementary to justify. The "domain of domains" also has an immediate computable structure.

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