Speech acts as announcements

Nadine Guiraud and **Andreas Herzig** and **Emiliano Lorini**

Universit Paul Sabatier
IRIT-LILaC
118 route de Narbonne
F-31062 Toulouse Cedex 9
France

Abstract

Our aim is to use the logic of public announcements and more generally dynamic epistemic logics as a logic of speech acts. To that end we start from a simple multimodal logic of beliefs and goals (without common belief), and add public announcements. We suppose that announcements do not modify goals. We then consider several variants of speech acts of assertive and directive force and provide a modeling in terms of speech acts.

1. Introduction

In this paper, we consider the domain of communication between agents, in particular we describe the dynamics of mental states due to communication by means of speech acts. Starting from the representation of speech acts by (Grice 1989), we consider the mental states which define the success preconditions of a speech act: intentions and belief. Thus Alice can say to Ben *I like cookies*. The mental state corresponding to this action, if Alice is honest, is that Alice *wants that Ben believes that she likes cookies*. This is the well-known assertive speech act. On this way, Grice defines other kinds of speech acts we consider in this paper: *assert, inform , confirm , request, yes-no question*. We do not only consider the mental states of the speaker, but also we represent the mental states of the hearer. In particular, we represent how the speech act influences beliefs and intentions of a locutor.

In agreement with Grice's theory, we suppose that when an agent speaks, he informs about his mental states. In the previous example, when Ben hears Alice's utterance *I like cookies*, whether he thinks Alice is honest or not, he starts to *believe that Alice wants him to beliefs that she likes cookies*. Thus, in case of a successfull communication, the hearer will understand the mental states of the speaker.

The success preconditions of speech acts are formalized in this work by means of a modal logic of beliefs and goals. Similar formalisms have been used in (Herzig and Longin 2002; david Sadek 2000). The logic is extended with modal operators for announcements of Dynamic Epistemic Logic (DEL) (van Ditmarsch, van der Hoek, and Kooi 2007) in order to model the dynamics speech acts. Our formalism

enables to represent how a speech act of the speaker modifies the beliefs of the hearer and so to capture the dynamic dimension of a dialogue between agents.

The paper is organized as follows: we first present the static modal logic of beliefs and goals. Then we provide a definition of model update by an announcement. Finally, we formally characterize the success preconditions of several kinds of speech acts and we develop an analysis of speech act dynamics.

2. A logic of beliefs and goals

Let $PRP = \{p, q, \ldots\}$ be a countable set of propositional letters, and let $AGT = \{i, j, \ldots\}$ be a countable set of agents. The set of formulas of our logic of beliefs and goals is defined by the following BNF:

$$\phi ::= p \mid \top \mid \neg\phi \mid \phi \wedge \phi \mid Bel_i\phi \mid Goal_i\phi$$

where p ranges over PRP and i ranges over AGT. $Bel_i\phi$ reads "agent i believes that ϕ" and $Goal_i\phi$ reads "agent i wants that ϕ". The others boolean operators $\bot, \vee, \rightarrow$ and $\leftrightarrow$ are defined in the standard way.

Models for that language are of the form $M = \langle W, \mathcal{B}, \mathcal{G}, V \rangle$ where

- W is a nonempty set of possible worlds;
- $\mathcal{B} : AGT \longrightarrow 2^{W \times W}$ associates a belief accessibility relation to every agent;
- $\mathcal{G} : AGT \longrightarrow 2^{W \times W}$ associates a goal accessibility relation to every agent;
- $V : W \longrightarrow 2^{PRP}$ associates to every possible world a valuation.

We use $\mathcal{B}_i(w)$ to denote the set $\{w' \mid \langle w, w' \rangle \in \mathcal{B}_i\}$, $\mathcal{G}_i(w)$ to denote the set $\{w' \mid \langle w, w' \rangle \in \mathcal{G}_i\}$. $\mathcal{B}_i(w)$ is the set of worlds that are compatible with agent i's beliefs at world w and $\mathcal{G}_i(w)$ is the set of worlds that are compatible with agent i's goals at world w .

The truth conditions are as usual:

- $M, u \models Bel_i\phi$ iff $M, v \models \phi$ for every v such that $u\mathcal{B}_iv$;
- $M, u \models Goal_i\phi$ iff $M, v \models \phi$ for every v such that $u\mathcal{G}_iv$;

The logic of that class of models is fairly standard. It can be axiomatized by the principles of K45 for the belief operator Bel_i, the principles of KD for the goal modal $Goal_i$,

plus the following axiom schemas of positive and negative introspection for goals:

$$(1) \qquad Goal_i\phi \rightarrow Bel_i Goal_i\phi$$

$$(2) \qquad \neg Goal_i\phi \rightarrow Bel_i\neg Goal_i\phi$$

Axioms 4 and 5 for beliefs correspond together to the following semantic constraint **(C.1)** on models, Axiom D for goals corresponds to the constraint **(C.2)** and, finally, the axioms of positive and negative introspection for goals correspond together to the semantic constraint **(C.3)**.

(C.1) $w' \in \mathcal{B}_i(w)$ implies $\mathcal{B}_i(w) = \mathcal{B}_i(w')$

(C.2) $\mathcal{G}_i(w) \neq \emptyset$

(C.3) $w' \in \mathcal{B}_i(w)$ implies $\mathcal{G}_i(w) = \mathcal{G}_i(w')$

3. Adding announcements

We suppose that announcements are public, and that they only modify the agents' beliefs, but not their goals. Technically, our logic combines Kooi's version of public announcement logic (Kooi 2007) with Gerbrandy's logic of private updates (Gerbrandy 1999; Gerbrandy and Groeneveld 1997). The latter allows us to update beliefs without updating goals, and is a particular dynamic epistemic logic (Baltag, Moss, and Solecki 1998; Baltag and Moss 2004; van Ditmarsch, van der Hoek, and Kooi 2007).[1]

We extend our language by modal operators of public announcement by adding $[\phi!]\phi$ to the above BNF. The formula $[\psi!]\phi$ reads "ϕ holds after the public announcement of ψ".

In order to give semantics to announcements we define the *update* of a model by an announcement. The update of $M = \langle W, \mathcal{B}, \mathcal{G}, V \rangle$ by $\phi!$ is the model $M^{\phi!} = \langle W^{\phi!}, \mathcal{B}^{\phi!}, \mathcal{G}^{\phi!}, V^{\phi!} \rangle$ where

- $W^{\phi!} = \{u_b : u \in W\} \cup \{u_g : u \in W\}$;

- $\mathcal{B}^{\phi!} = \{\langle u_b, v_b \rangle : \langle u, v \rangle \in \mathcal{B} \text{ and } M, v \models \phi\} \cup \{\langle u_g, v_g \rangle : \langle u, v \rangle \in \mathcal{B}\}$;

- $\mathcal{G}^{\phi!} = \{\langle u_b, v_g \rangle : \langle u, v \rangle \in \mathcal{G}\} \cup \{\langle u_g, v_g \rangle : \langle u, v \rangle \in \mathcal{G}\}$;

- $V^{\phi!}(u_b) = V^{\phi!}(u_g) = V(u)$.

Basically, the effect of an announcement is to shrink the set of belief accessible worlds, while keeping constant the set of goal accessible worlds.

Proposition 1. *For every ϕ, if M is a model of our logic of beliefs and goals then $M^{\phi!}$ is still a model of our logic of beliefs and goals.*

Proof. The proof consists in verifying the conservation of semantic properties seen ahead.

- For **C.1**: There are two cases for belief after announcement: either $w_{2b} \in \mathcal{B}_i^{\psi!}(w_{1b})$ or $w_{2g} \in \mathcal{B}_i^{\psi!}(w_{1g})$. In each case, it implies by definition that $w_2 \in \mathcal{B}_i(w_1)$. Then $\mathcal{B}_i(w_1) = \mathcal{B}_i(w_2)$, because M satisfies constraint **C.1**. And also assume $w_3 \in \mathcal{B}_i(w_2)$. Then $w_3 \in \mathcal{B}_i(w_1)$.

---

[1] For readers who are familiar with these logics: our event models have two possible events b and g, where $\mathcal{B}_i = \{\langle b, b \rangle, \langle g, g \rangle\}$ and $\mathcal{G}_i = \{\langle b, g \rangle, \langle g, g \rangle\}$.

In case $w_{2b} \in \mathcal{B}_i^{\psi!}(w_{1b})$: if $w_3 \models \psi$ then $w_{3b} \in \mathcal{B}_i^{\psi!}(w_{2b})$ and $w_{3b} \in \mathcal{B}_i^{\psi!}(w_{1b})$. And if $w_{3b} \not\models \psi$ then $w_{3b} \notin \mathcal{B}_i^{\psi!}(w_{2b})$ and $w_{3b} \notin \mathcal{B}_i^{\psi!}(w_{1b})$ and $w_{3g} \notin \mathcal{B}_i^{\psi!}(w_{2b})$ and $w_{3g} \notin \mathcal{B}_i^{\psi!}(w_{1b})$.

Then $\mathcal{B}_i^{\psi!}(w_{1b}) = \mathcal{B}_i^{\psi!}(w_{2b})$.

In case $w_{2g} \in \mathcal{B}_i^{\psi!}(w_{1g})$: whether or not $w_3 \models \psi$ then $w_{3g} \in \mathcal{B}_i^{\psi!}(w_{2g})$ and $w_{3g} \in \mathcal{B}_i^{\psi!}(w_{1g})$.

Then $\mathcal{B}_i^{\psi!}(w_{1g}) = \mathcal{B}_i^{\psi!}(w_{2g})$.

Therefore, $M^{\psi!}$ satisfies constraint **C.1**.

- For **C.2**: Let us suppose $\mathcal{G}_i^{\psi!}(w_b) = \emptyset$. Then $\mathcal{G}_i(w) = \emptyset$ by the semantic definition, and it is contradictory with the constraints on M. Then $\mathcal{G}_i^{\psi!}(w_b) \neq \emptyset$. It is the same proof for $\mathcal{G}_i^{\psi!}(w_g) \neq \emptyset$. Therefore $M^{\psi!}$ satisfies constraint **C.2**.

- For **C.3**: There are two cases for belief after announcement: either $w_{2b} \in \mathcal{B}_i^{\psi!}(w_{1b})$ or $w_{2g} \in \mathcal{B}_i^{\psi!}(w_{1g})$. In each case, it implies by definition that $w_2 \in \mathcal{B}_i(w_1)$. Then $\mathcal{G}_i(w_1) = \mathcal{G}_i(w_2)$, because M satisfies constraint **C.3**. And also assume $w_3 \in \mathcal{G}_i(w_2)$. Then $w_3 \in \mathcal{G}_i(w_1)$.

In case $w_{2b} \in \mathcal{B}_i^{\psi!}(w_{1b})$: $w_{3g} \in \mathcal{G}_i^{\psi!}(w_{2b})$ and $w_{3g} \in \mathcal{G}_i^{\psi!}(w_{1b})$. And in case $w_{2g} \in \mathcal{B}_i^{\psi!}(w_{1g})$: $w_{3g} \in \mathcal{G}_i^{\psi!}(w_{2g})$ and $w_{3g} \in \mathcal{G}_i^{\psi!}(w_{1g})$.

Then $\mathcal{G}_i^{\psi!}(w_1) = \mathcal{G}_i^{\psi!}(w_2)$ in both cases.

Therefore, $M^{\psi!}$ satisfies constraint **C.3**.

Items 1, 2 and 3 together imply that $M^{\psi!}$ is a model of our logic of belief and goal. $\square$

The truth condition for announcement is:

- $M, u \models [\psi!]\phi$ iff $M^{\psi!}, u_b \models \phi$

We suppose the usual definitions of validity and satisfiability. Then the following equivalences are valid:

$$(3) \qquad [\psi!]p \leftrightarrow p \quad \text{if } p \in PRP$$

$$(4) \qquad [\psi!]\neg\phi \leftrightarrow \neg[\psi!]\phi$$

$$(5) \qquad [\psi!](\phi \wedge \chi) \leftrightarrow ([\psi!]\phi \wedge [\psi!]\chi)$$

$$(6) \qquad [\psi!]Bel_i\phi \leftrightarrow Bel_i(\psi \rightarrow [\psi!]\phi)$$

$$(7) \qquad [\psi!]Goal_i\phi \leftrightarrow Goal_i\phi$$

Proof.
(3) $\langle W, \mathcal{B}, \mathcal{G}, V \rangle, w \models [\psi!]p$
iff $\langle W^{\psi!}, \mathcal{B}^{\psi!}, \mathcal{G}^{\psi!}, V^{\psi!} \rangle, w_b \models p$
iff $w \in V^{\psi!}(p)$
iff $w \in V(p)$
iff $\langle W, \mathcal{B}, \mathcal{G}, V \rangle, w \models p$.

(4) $\langle W, \mathcal{B}, \mathcal{G}, V \rangle, w \models [\psi!]\neg\varphi$
iff $\langle W^{\psi!}, \mathcal{B}^{\psi!}, \mathcal{G}^{\psi!}, V^{\psi!} \rangle, w_b \models \neg\varphi$
iff $\langle W^{\psi!}, \mathcal{B}^{\psi!}, \mathcal{G}^{\psi!}, V^{\psi!} \rangle, w_b \not\models \varphi$
iff $\langle W, \mathcal{B}, \mathcal{G}, V \rangle, w \not\models [\psi!]\varphi$
iff $\langle W, \mathcal{B}, \mathcal{G}, V \rangle, w \models \neg[\psi!]\varphi$.

(5) $\langle W, \mathcal{B}, \mathcal{G}, V \rangle, w \models [\psi!](\varphi_1 \wedge \varphi_2)$
iff $\langle W^{\psi!}, \mathcal{B}^{\psi!}, \mathcal{G}^{\psi!}, V^{\psi!} \rangle, w_b \models \varphi_1 \wedge \varphi_2$

iff $\langle W^{\psi!}, \mathcal{B}^{\psi!}, \mathcal{G}^{\psi!}, V^{\psi!}\rangle, w_b \models \varphi_1$
and $\langle W^{\psi!}, \mathcal{B}^{\psi!}, \mathcal{G}^{\psi!}, V^{\psi!}\rangle, w_b \models \varphi_2$
iff $\langle W, \mathcal{B}, \mathcal{G}, V\rangle, w \models [\psi!]\varphi_1$ and $\langle W, \mathcal{B}, \mathcal{G}, V\rangle, w \models [\psi!]\varphi_2$
iff $\langle W, \mathcal{B}, \mathcal{G}, V\rangle, w \models [\psi!]\varphi_1 \wedge [\psi!]\varphi_2$.

(6) We show that the equivalent formula
$$\neg[\psi!]Bel_i\varphi \leftrightarrow \neg Bel_i(\psi \to [\psi!]\varphi)$$
is valid:

$\langle W, \mathcal{B}, \mathcal{G}, V\rangle, w \models \neg[\psi!]Bel_i\varphi$
iff $\langle W, \mathcal{B}, \mathcal{G}, V\rangle, w \models [\psi!]\neg Bel_i\varphi$, by **(4)**,
iff $\langle W^{\psi!}, \mathcal{B}^{\psi!}, \mathcal{G}^{\psi!}, V^{\psi!}\rangle, w_b \models \neg Bel_i\varphi$
iff $\exists w_b' \in \mathcal{B}^{\psi!}(w_b)$ s.t. $\langle W^{\psi!}, \mathcal{B}^{\psi!}, \mathcal{G}^{\psi!}, V^{\psi!}\rangle, w_b' \models \neg\varphi$
iff $\exists w' \in \mathcal{B}(w)$ s.t. $\langle W, \mathcal{B}, \mathcal{G}, V\rangle, w' \models \psi$ and
$\langle W, \mathcal{B}, \mathcal{G}, V\rangle, w' \models [\psi!]\neg\varphi$
iff $\exists w' \in \mathcal{B}(w)$ s.t. $\langle W, \mathcal{B}, \mathcal{G}, V\rangle, w' \models \psi \wedge \neg[\psi!]\varphi$, by **(4)**,
iff $\exists w' \in \mathcal{B}(w)$ s.t. $\langle W, \mathcal{B}, \mathcal{G}, V\rangle, w' \models \neg(\psi \to [\psi!]\varphi)$
iff $\langle W, \mathcal{B}, \mathcal{G}, V\rangle, w \models \neg Bel_i(\psi \to [\psi!]\varphi)$.

(7) We show that the equivalent formula
$$\neg[\psi!]Goal_i\varphi \leftrightarrow \neg Goal_i\varphi$$
is valid:

$\langle W, \mathcal{B}, \mathcal{G}, V\rangle, w \models \neg[\psi!]Goal_i\varphi$
iff $\langle W, \mathcal{B}, \mathcal{G}, V\rangle, w \models [\psi!]\neg Goal_i\varphi$, by **(4)**,
iff $\langle W^{\psi!}, \mathcal{B}^{\psi!}, \mathcal{G}^{\psi!}, V^{\psi!}\rangle, w_b \models \neg Goal_i\varphi$
iff $\exists w_g' \in \mathcal{G}^{\psi!}(w_b)$ s.t. $\langle W^{\psi!}, \mathcal{B}^{\psi!}, \mathcal{G}^{\psi!}, V^{\psi!}\rangle, w_g' \models \neg\varphi$
iff $\exists w' \in \mathcal{G}(w)$ s.t. $\langle W, \mathcal{B}, \mathcal{G}, V\rangle, w' \models \neg\varphi$
iff $\langle W, \mathcal{B}, \mathcal{G}, V\rangle, w \models \neg Goal_i\varphi$. $\square$

The following rules of equivalence for announcements

(8) $\qquad$ from $\phi \leftrightarrow \phi'$ infer $[\psi!]\phi \leftrightarrow [\psi!]\phi'$

(9) $\qquad$ from $\psi \leftrightarrow \psi'$ infer $[\psi!]\phi \leftrightarrow [\psi'!]\phi$

preserve validity.

The above equivalences 3-7 and rules of inference 8-9 are so-called *reduction axioms*: they allow to 'move' announcement operators across the other logical operators 'inside' formulas, and to eliminate them once they face a propositional letters. Rewriting formulas starting from the innermost announcements, we can in this way eliminate announcement operators from formulas, resulting in an equivalent formula of the underlying logic of beliefs and goals.

This provides a completeness result for our logic of announcements.

Theorem 1. *Our logic of beliefs and goals is sound and complete.*

4. Speech acts as announcements

In dynamic epistemic logics announcements are usually viewed as communication actions performed by an agent that is 'outside the system', i.e. that is not part of the set of agents AGT under consideration. Our aim is to investigate how communication actions performed by agents from AGT can be modelled in our logic of announcements. To that end we consider a particular subset of announcements: we say that formulas of the form $Bel_i\psi!$ or $Goal_i\psi!$ are about the mental state of agent i. We identify announcements about the mental state of i, with actions performed by i. We follow Grice's idea that speech acts are expressions of

intentions by the speaker, and consider that such expressions take the form $Goal_i\psi!$ of announcements of goals. Such announcements of goals are directed towards an addressee.

4.1 Assertive speech acts

We consider the basic assertive as the primitive speech act and we suppose that the other kinds of speech acts are defined from this primitive speech act. We identify the expression "i *asserts* to j that ψ" with the announcement of $Goal_iBel_j\psi$:

$$Assert_{i,j}\psi \stackrel{\text{def}}{=} Goal_iBel_j\psi!$$

One might express a more complex notion of assertive speech act whose content is communicative intention in the sense of Grice (Grice 1989) of the form $Goal_iBel_j Goal_iBel_j\psi$, i.e. agent i wants that j believes that i wants that j believes ψ. In the classical Gricean view of linguistic communication, a communicative intention of the speaker is aimed at the recognition by the hearer of the speaker's goal of making the hearer to believe something.

Starting from the basic assertive, we define the two speech acts *inform* and *confirm*. The speech act *inform* is a speech act *assert* under the precondition that i believes that j has no opinion about ψ. We identify it therefore with the announcement of $Bel_i(\neg Bel_j\psi \wedge \neg Bel_j\neg\psi) \wedge Goal_iBel_j\psi$:

$$Inform_{i,j}\psi \stackrel{\text{def}}{=} Bel_i(\neg Bel_j\psi \wedge \neg Bel_j\neg\psi) \wedge Goal_iBel_j\psi!$$

The speech act *confirm* is a speech act *assert* under the precondition that i believes that j believes ψ. We identify it therefore with the announcement of $Bel_iBel_j\psi \wedge Goal_iBel_j\psi$:

$$Confirm_{i,j}\psi \stackrel{\text{def}}{=} Bel_iBel_j\psi \wedge Goal_iBel_j\psi!$$

In both cases one might add the sincerity condition of assertives to the announcement: the speaker believes what he asserts. We then would write $Bel_i\psi \wedge Goal_iBel_j\psi$ instead of $Goal_iBel_j\psi$ for the basic assertive of the form $Assert_{i,j}\varphi$, etc.

4.2 Directive speech acts

The speech act *request* is the basic speech act of the directive kind. We define it from the assertive. We identify "i *requests* j that ψ" with "i *asserts* that he wants that j has the goal that ψ":

$$Request_{i,j}\psi \stackrel{\text{def}}{=} Assert_{i,j} Goal_i Goal_j\psi$$

The yes-no query "i *asks* j whether ψ" is a particular case of a request:

$$Ask_{i,j}\psi \stackrel{\text{def}}{=} Request_{i,j}(Bel_i\psi \vee Bel_i\neg\psi)$$

Note that if we had the sincerity condition $Bel_i\psi$ for the assertive speech act, under the condition $\neg Bel_i\bot$, the act of request would imply $Goal_i Goal_j\psi$ and the yes-no query would imply $Goal_i Goal_j(Bel_i\psi \vee Bel_i\neg\psi)$.

Suppose j is cooperative. This could be expressed by the formula

$$Bel_j\,Goal_i\,Bel_j\,Goal_i\,Goal_j p \rightarrow Goal_j p$$

Under this hypothesis the satisfaction conditions can be guaranteed, i.e. we have

Theorem 2.
$\vdash [Request_{i,j}p](Bel_j\,Goal_i\,Bel_j\,Goal_i\,Goal_j p$
$\qquad\qquad\qquad \rightarrow Goal_j p) \rightarrow [Request_{i,j}p]Goal_j p$

Proof. We have the theorems that
$\quad\vdash [Request_{i,j}p]Bel_j\,Goal_i\,Bel_j\,Goal_i\,Goal_j p$
and $\vdash [Request_{i,j}p]Goal_j p \leftrightarrow Goal_j p$

so by K $\vdash[Request_{i,j}p](Bel_j\,Goal_i\,Bel_j\,Goal_i\,Goal_j p$
$\qquad\qquad \rightarrow Goal_j p) \rightarrow [Request_{i,j}p]Goal_j p$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

4.3 Moore sentences in speech acts

In our logic one can study Moore-like sentences for beliefs. We can prove that the assertion of $p \wedge \neg Bel_i p$ leads to inconsistencies of the (higher-order) beliefs:

Theorem 3.
$\vdash Bel_i(p \wedge \neg Bel_i p) \leftrightarrow Bel_i \bot$
$\vdash [Assert_{i,j}Bel_i(p \wedge \neg Bel_i p)!]Bel_j\,Goal_i\,Bel_j\,Bel_i \bot$

Proof.

- $\vdash \neg Bel_i p \rightarrow Bel_i \neg Bel_i p$ (axiom 5)
  so $\vdash \neg Bel_i \neg Bel_i p \rightarrow Bel_i p$
  and $\vdash Bel_i Bel_i p \wedge \neg Bel_i \bot \rightarrow \neg Bel_i \neg Bel_i p$
  so $\vdash Bel_i Bel_i p \wedge \neg Bel_i \bot \rightarrow Bel_i p$
  and so $\vdash Bel_i Bel_i p \wedge \neg Bel_i \bot \leftrightarrow Bel_i p \wedge \neg Bel_i \bot$
  $Bel_i(p \wedge \neg Bel_i p)$
  $\leftrightarrow Bel_i p \wedge Bel_i \neg Bel_i p \wedge (Bel_i \bot \vee \neg Bel_i \bot)$
  $\leftrightarrow (Bel_i p \wedge Bel_i \neg Bel_i p \wedge Bel_i \bot) \vee (Bel_i p \wedge Bel_i \neg Bel_i p \wedge \neg Bel_i \bot)$
  $\leftrightarrow Bel_i(p \wedge \neg Bel_i p \wedge \bot) \vee (Bel_i Bel_i p \wedge \neg Bel_i \bot \wedge Bel_i \neg Bel_i p)$
  $\leftrightarrow Bel_i \bot$
  Therefore $\vdash Bel_i(p \wedge \neg Bel_i p) \leftrightarrow Bel_i \bot$

- So $[Assert_{i,j}Bel_i(p \wedge \neg Bel_i p)!]Bel_j\,Goal_i\,Bel_j\,Bel_i \bot$
  iff $Bel_j(Goal_i\,Bel_j\,Bel_i(p \wedge \neg Bel_i p) \rightarrow$
  $[Goal_i\,Bel_j\,Bel_i(p \wedge \neg Bel_i p)!]Goal_i\,Bel_j\,Bel_i \bot)$ by **(6)**
  iff $Bel_j(Goal_i\,Bel_j\,Bel_i(p \wedge \neg Bel_i p) \rightarrow$
  $Goal_i\,Bel_j\,Bel_i \bot)$ by **(7)**
  iff $Bel_j(Goal_i\,Bel_j\,Bel_i \bot \rightarrow Goal_i\,Bel_j\,Bel_i \bot)$
  iff $Bel_j \top$
  which is a tautology.
  Therefore
  $\vdash [Assert_{i,j}Bel_i(p \wedge \neg Bel_i p)!]Bel_j\,Goal_i\,Bel_j\,Bel_i \bot$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

On the same way, we can prove that

$\vdash [Assert_{i,j}Bel_i(p \wedge \neg Bel_i p)!]Bel_i\,Goal_i\,Bel_j\,Bel_i \bot$

5. Discussion and conclusion

An immediate extension of our logic is to add announcements modifying goals, similar to (van Benthem 2007; Roy 2008; van Benthem, Girard, and Roy to appear). Such announcements would leave an agent's beliefs unaltered. In order to model this form of goal change, a product construction symmetric to the one for beliefs can be used.

Another issue we intend to investigate in the future is the theory of speech acts in the context of indirected communication. The idea is to extend the analysis presented in this paper to the case in which agents make assertions, requests, queries, etc. to other agents by using a shared blackboard which can be observed by other agents connected to it. In this case, an agent's speech act (defined in terms of announcements) modify and update the contents of the blackboard which is accessible to other agents, rather than directly modifying the beliefs and goals of other agents. In order to model this notion of blackboard, as a form of artefact used for mediating communication between agents, we intend to use the logic of acceptance recently proposed in (Lorini et al. 2009).

We also leave open the problem of integrating belief revision mechanisms into our logic, and refer to (Aucher 2003; 2008; van Ditmarsch, van der Hoek, and Kooi 2007; Baltag and Smets 2007) for possible solutions.

References

Aucher, G. 2003. A combined system for update logic and belief revision. Master's thesis, University of Amsterdam.

Aucher, G. 2008. *Perspectives on belief and change*. Ph.D. Dissertation, Université de Toulouse.

Baltag, A., and Moss, L. S. 2004. Logics for epistemic programs. *Synthese* 139(2):165–224.

Baltag, A., and Smets, S. 2007. From conditional probability to the logic of doxastic actions. In *Proc. TARK 2007*, 52–61. Presses Universitaires de Louvain.

Baltag, A.; Moss, L. S.; and Solecki, S. 1998. The logic of public announcements, common knowledge, and private suspicions. In *Proc. TARK'98*, 43–56. Morgan Kaufmann.

david Sadek. 2000. Dialogue acts are rational plans. In Martin M. Taylor, F. N., and Bouwhuis, D. G., eds., *The Structure of Multimodal Dialogue II*, 167–187. Amsterdam / Philadelphia: John Benjamins.

Gerbrandy, J., and Groeneveld, W. 1997. Reasoning about information change. *J. of Logic, Language and Information* 6(2).

Gerbrandy, J. 1999. *Bisimulations on Planet Kripke*. Ph.D. Dissertation, University of Amsterdam.

Grice, H. P. 1989. *Studies in the way of words*. USA: Harvard University Press, 3rd edition.

Herzig, A., and Longin, D. 2002. A logic of intention with cooperation principles and with assertive speech acts as communication primitives. In Castelfranchi, C., and Johnson, W. L., eds., *Proc. of the first Int. Joint Conf. on Autonomous Agent and Multi-Agent System (AAMAS 2002), Bologna, Italy, July 15–19*, volume 2, 920–927. ACM Press.

Kooi, B. 2007. Expressivity and completeness for public update logic via reduction axioms. *Journal of Applied Non-Classical Logics* 17(2):231–253.

Lorini, E.; Longin, D.; Gaudou, B.; and Herzig, A. 2009. The logic of acceptance: grounding institutions on agents' attitudes. *Journal of Logic and Computation* DOI: 10.1093/logcom/exn103.

Roy, O. 2008. *Thinking before acting: intentions, logic, rational choice*. Ph.D. Dissertation, University of Amsterdam, ILLC dissertation series.

van Benthem, J.; Girard, P.; and Roy, O. to appear. Everything else being equal: A modal logic for ceteris paribus preferences. *Journal of Philosophical Logic*.

van Benthem, J. 2007. Dynamic logic of preference upgrade. *Journal of Applied Non-Classical Logics* 17(2):157–182.

van Ditmarsch, H. P.; van der Hoek, W.; and Kooi, B. 2007. *Dynamic Epistemic Logic*. Kluwer Academic Publishers.