Executive Summary of Dagstuhl-Seminar

Circuits, Logic, and Games

07/02/10–12/02/10

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Classification: data structures/algorithms/complexity

Keywords: computational complexity theory, finite model theory, Boolean circuits, regular languages, finite monoids, Ehrenfeucht-Fraïssé-games.

Description of the Seminar's Topic

Starting with the seminal paper by Furst, Saxe and Sipser, the last two decades of the previous century saw an immense interest in the computational model of Boolean circuits. Emerging powerful lower bound techniques promised progress towards solutions of major open problems in computational complexity theory. Within a very short time, further progress was made in papers by Fagin et al., Gurevich and Lewis, Håstad, Razborov, Smolensky, and Yao, to mention only a few. The just mentioned result by Furst et al. was obtained independently by Ajtai making use of model-theoretic arguments, and many further lower bounds in complexity have been obtained afterwards making use of inexpressibility results in logic, very often making use of model-theoretic games. After a decade of active research in this direction things slowed down considerably.

In the same way as during the first seminar on *Circuits, Logic, and Games* (Nov. 2006, 06451), the organizers aimed to bring together researchers from the areas of finite model theory and computational complexity theory, since they felt that perhaps not all developments in circuit theory and in logic had been explored fully in the context of lower bounds. In fact, the interaction between the areas has flourished a lot in the past 2-3 years, as can be exemplified by the following lines of research.

Results of Barrington, Straubing and Thérien show that most circuit classes, if they can be separated at all, can be separated by regular languages – which means that *algebraic properties* of such languages could be used in

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lower bound proofs. Recent results prove almost linear upper bounds on the size of circuits for regular languages in many important constant-depth circuit classes, implying that an $\Omega(n^{1+\varepsilon})$ lower bound suffices to separate such classes from NC^1. Interesting connections to *communication complexity* have been obtained in the past two years, showing, e. g., that languages with bounded multiparty communication complexity can be recognized by programs over commutative monoids and thus have very small depth circuit complexity.

While inexpressibility results in finite model theory have been used since the 1980s to obtain circuit lower bounds, recent results make use of circuit lower bounds to separate different logics: The result that number of-variable hierarchy in first-order logic over finite ordered structures is strict was obtained by showing that a certain clique-problem cannot be solved by constant-depth circuits of a certain size.

Further connections between logic and circuits concern *uniformity conditions for Boolean circuits*: It was proved that in a quite general context, when a circuit based language class is characterized using first- order descriptive complexity, the circuit uniformity conditions spring up in the logic in the form of restrictions on the set of numerical predicates allowed. So-called "extensional uniformity conditions" have been studied: Intersecting a non-uniform constant-depth circuit class with a uniform class $\mathcal{L}$ (e.g., a formal language class) in some contexts results in a circuit class that can be characterized by first-order logic with $\mathcal{L}$-numerical predicates. (Intuitively, $\mathcal{L}$-numerical predicates are those predicates definable in first-order logic with one "oracle call" to a language from $\mathcal{L}$, i.e., more precisely, with one appearance of a generalized quantifier for such a language.)

While this in principal points out new ways to separate uniformity conditions via logical means, another line of results goes in the opposite direction: It is shown that for a specific arithmetical problem (division), circuits can be constructed that are uniform under much stricter requirements than was anticipated before. Again, the proofs make heavy use of finite model theory.

A further area of investigation is the structural complexity of dynamic algorithmic problems. There are, so far, no techniques available to prove that a problem does not have AC^0 update complexity. Recent (and forthcoming) work therefore started an investigation of the fine structure of the class of problems with AC^0 updates, yielding lower bound results and uncovering (yet another) surprising characterization of the regular languages as those that can be maintained with quantifier-free formulas.

These results demonstrate the impressive growth of interest and activity in the intersection of finite model theory and Boolean circuit complexity, and as will have become apparent from our above description of recent research, many of these developments rely strongly on game-based methods.

Organization of the Seminar and Activities

The workshop brought together 46 researchers from different areas of logic and complexity theory with complementary expertise. The participants consisted of both senior and junior researchers, including a number of postdocs and a few advanced graduate students.

Participants were invited to present their work and to communicate state-of-the-art advances. Twenty-seven talks of various lengths took place over the five days of the workshop. Around half of these talks were scheduled prior to workshop, including most of the longer morning talks and tutorials. The remaining slots were filled as the workshop commenced.

The morning schedule consisted of survey talks of between 60 and 90 minutes. The presenters and topics were:

- Meena Mahajan, Valiant's classes
- Jean-Eric Pin, A topological approach to recognition
- Jouko Väänänen, Dependence logic
- Kousha Etessami, The computational complexity of Nash and other equilibria and fixed points of algebraic functions

There were an additional eight one-hour talks, as well as fifteen shorter talks of 30 or 45 minutes. These talks covered a wide range of topics in circuits, logic, games and the nexus of these areas. The different approaches discussed above in the seminar description were all very well represented by the different talks given during the five days of the seminar.

- Finite model theory and descriptive complexity
  - Lauri Hella, Regular representations of uniform TC^0
  - Jan van den Busche, Expressiveness hierarchy within FO^3 through relation algebra
  - Nicole Schweikardt, Addition-invariant FO and regularity
  - Wolfgang Thomas, Separating weak from strong monadic theories
  - Luc Segoufin, Characterization of logics on trees
  - Pavel Pudlak, First-order sentences and polynomial search problems

- Thomas Schwentick, The dynamic complexity of regular languages
- Michael Thomas, Complexity of fragments of autoepistemic logic

◦ The algebraic approach

- Klaus-Jörn Lange, An algebraic approach: the infinite case
- Andreas Krebs, Infinite algebras: the 2-variable case

◦ Circuits and computational complexity

- Kristoffer Arnsfelt Hansen, Exact threshold circuits
- Samir Datta, Topological restrictions of various graph/circuit problems
- Raghavendra Rao B.V., Faster algorithms for finding and counting subgraphs
- Srikanth Srinivasan, The help functions problem for Boolean circuits
- Michal Koucky, A new characterization of ACC^0 and probabilistic CC^0
- Arkadev Chattopadhyay, Linear systems over composite moduli
- Benjamin Rossman, Lower bounds for k-clique on random graphs

◦ Proof complexity

- Olaf Beyersdorff, Proof complexity of default logic
- Jan Johannsen, Lower bounds for width-restricted clause learning
- Nicola Galesi, The strength of treelike parameterized resolution

◦ Game-theoretic methods

- Peter Bro Miltersen, Winning concurrent reachability games requires doubly-exponential patience

◦ Dependence logic

- Juha Kontinen, Definability in dependence logic
- Peter Lohmann, Satisfiability in modal dependence logic

This classification is necessarily rough, as several talks crossed the boundaries between these areas, in keeping with the theme of the workshop. The broad scope of the talks extended even to areas not anticipated by the organizers, such as dependence logic. The workshop thus achieved its aim of bringing together researchers from various related communities to share state-of-the-art research across the areas of circuits, logic and games.

Concluding Remarks and Future Plans

The organizers regard the workshop as a great success. The weeklong format was well-suited to a workshop of so broad a scope. Bringing together researchers from different areas of theoretical computer science and logic fostered valuable interactions and led to fruitful collaborations. Feedback from the participants was very positive as well.

Finally, the organizers wish to express their gratitude toward the Scientific Directorate of the Center for its support of this workshop, and hope to continue this series of workshops on *Circuits, Logic, and Games* into the future.