

# Node Degree based Improved Hop Count Weighted Centroid Localization Algorithm

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## Abstract

Hop-count based weighted centroid localization is a simple and straightforward localization algorithm, which uses anchors with known positions and the hop count to these anchors to estimate the real position of nodes. Especially in sensor networks, where energy restrictions prevent more complex algorithms, this fast and simple algorithm can be used. Unfortunately the localization error of the algorithm can hinder the practical usage.

In this paper we will improve the weighted centroid algorithm for hop count based localization by adding the node degree on the paths to the referenced anchors into the weights. After an analysis to obtain theoretically optimal coefficients we will show by means of simulation that for longer hop counts to the anchors and areas with different node degrees the proposed ND-WCL algorithm outperforms the known hop count based weighted centroid localization algorithm.

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## 1 Introduction

Fast and adaptive algorithms for distributing messages are needed in ad hoc networks, as these may be mobile, fast changing or even partially disrupted[6]. Geographic Routing may be one of the enabling technologies to ensure communication in these networks [2][4]. Unfortunately Geographic Routing is based on the knowledge of real location information. GPS equipment is expensive as well as energy consuming and relies on line of sight to the GPS satellites. The Ad Hoc Positioning System [5] extended GPS on a hop by hop behavior for usage in networks where only a fraction of nodes have the capability to detect their real location. A novel approach was made in [7] to construct virtual coordinates and use these as base for geographic routing. The proposed algorithm managed to construct virtual coordinates without any knowledge of the real coordinates of the network nodes. But still all nodes need to have a logical position assigned to work with geographic routing.

Our paper will improve the known general centroid algorithm (CL) [3] as well as the hop count based weighted centroid algorithm (WCL) [1]. The aim is to improve the performance of the localization in terms of localization error without using decentralized or global information or introducing additional communication messages for gathering data. The proposed novel algorithm ND-WCL will use the average node degree on the shortest paths to the anchors in the network, which is an easily obtainable information.

The remainder of this paper is organized as follows: Section 2 analyses the theoretic connections between node degree, distance between node pairs and n-hop-neighborhoods in



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ad-hoc networks with uniformly distributed nodes. Supported by simulations we obtain in section 3 approximations for the average distance between n-hop-neighbors, which are used in section 4 to define a novel node degree based weighted centroid algorithm for localization. Simulation in section 5 show the performance of the novel algorithm in relation to already known centroid algorithms.

## 2 Analysis of n-hop-Neighborhoods

For the analytic investigation we assume a static two dimensional plain scenario with  $N$  random uniformly distributed nodes which leads to a constant average node degree. Each node uses the same wireless communication module with a fixed unidirectional communication range. Nodes within the communication range  $R$  may communicate with each other and are denoted as 1-hop-neighbors. For the analytic investigation we place one node as central node (Node Zero) in the middle of the observance area.

Node density  $ND$  in the scenario is strongly linked to the node degree  $deg(N)$ , which is the number of neighbors.

$$ND = \frac{deg(N)}{\pi \cdot R^2} \quad (1)$$

Two nodes are denoted as n-hop-neighbors, if the shortest communication path, by means of shortest hop count, is equal to  $n$ .

In the following we observe the probability  $P_n(d)$  of a random node pair with distance  $d$  to each other to be n-hop-neighbors. We also observe the mean distance  $\bar{d}_n$  between n-hop-neighbors. As all nodes are independently and randomly placed with a uniform distribution the mean value of the distance of all possible n-hop-neighbors and the expected distance of randomly chosen n-hop-neighbors is the same. The communication range  $R$  will be set to 1 (distance unit) as simple scaling, resulting in a unit-disc-graph.

The probability  $P_1(d)$  that two nodes with distance  $d$  to each other are 1-hop-neighbors is

$$P_1(d) = \begin{cases} 0 & \text{for } d \leq 0 \\ 1 & \text{for } 0 < d \leq 1 \\ 0 & \text{for } d > 1 \end{cases} \quad (2)$$

as two nodes are 1-hop-neighbors if and only if they have a positive distance less equal to the communication range.

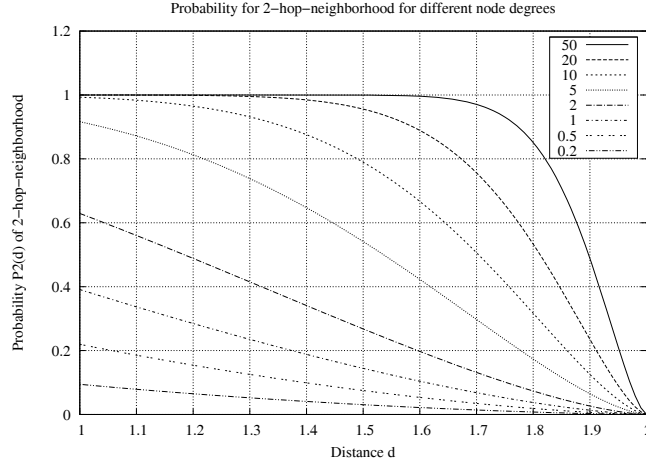
The average distance  $\bar{d}_1$  between 1-hop-neighbors can be computed and simplified as

$$\bar{d}_1 = \frac{1}{A} \int_A P_1(d) d dx dy = \frac{2}{3} \quad (3)$$

with  $A$  being the unit circle as all nodes in the unit circle around Node Zero are a 1-hop-neighbor of it. For 1-hop-neighbors  $\bar{d}_1$  is independent of the node degree.

Two nodes with a distance  $d$  equal less to 1 are already 1-hop-neighbors. If the distance  $d$  is greater than 2 no common neighbor can be found for the node pair. Therefore only node pairs with a distance  $1 < d \leq 2$  are potential 2-hop-neighbors. The probability  $P_2(d)$  is equal to the probability to find a third node to establish a 2-hop-neighborhood. This third node must be placed in the intersection area  $A$  of the communication range circles of the first two nodes. This area  $A$  can be computed for communication range  $r = 1$  as

$$A = 2 \arccos\left(\frac{d}{2}\right) - \frac{d}{2} \sqrt{4 - d^2} \quad (4)$$



■ **Figure 1** 2-hop-neighbors

The probability  $p_2(d)$  that one random but specific neighbor node is placed in the intersection of the two communication areas is

$$p_2(d) = \frac{A}{\pi r^2} \text{ for } 1 < d \leq 2 \quad (5)$$

and  $P_2(d)$  can be computed as

$$P_2(d) = 1 - \left(1 - \frac{2 \arccos(\frac{d}{2}) - \frac{d}{2} \sqrt{4 - d^2}}{\pi}\right)^{deg(N)} \quad (6)$$

The probability for two nodes with distance  $d$  to be 2-hop-neighbors is shown in Fig. 1 for different average node degrees. Note that even in scenarios with an average node degree less than 1 a small probability exists to find a third node for establishing a 2-hop-neighborhood. As expected a high node degree offers more possibilities to find a third node and an increased probability to find this third node even for node distances  $d$  close to the maximum of 2.

The average distance of 2-hop-neighbors can be computed using polar coordinates again

$$\bar{d}_2 = \frac{1}{A} \int_A P_2(d) dx dy = \frac{2}{3} \int_{r=1}^2 \left(1 - \left(1 - \frac{2 \arccos(\frac{r}{2}) - \frac{r}{2} \sqrt{4 - r^2}}{\pi}\right)^{deg(N)}\right) r^2 dr \quad (7)$$

which is unfortunately not independent of the node degree.

For node degrees close to infinity we may assume that all nodes within an annulus with a large radius  $R = n$  and a smaller radius  $r = n - 1$  are  $n$ -hop-neighbors if they are not placed on the inner circle. For example all nodes within the centered annulus  $A_{1,2}$  with  $r = 1$  and  $R = 2$  are 2-hop-neighbors of the central node at  $(0, 0)$ . Therefore the probability  $\hat{P}_n(d)$  of 2 nodes with distance  $d$  to be  $n$ -hop-neighbors for high node degrees is

$$\hat{P}_n(d) = \begin{cases} 0 & \text{for } d \leq n - 1 \\ 1 & \text{for } n - 1 < d \leq n \\ 0 & \text{for } d > n \end{cases} \quad (8)$$

The average node distance  $\bar{d}_n$  for an almost infinite high node degree can be computed as

$$\bar{d}_n = \frac{2n^3 - (n-1)^3}{3n^2 - (n-1)^2} \quad (9)$$

which equals  $2/3$  for  $n = 1$  as expected and converges to  $n - 1/2$  for  $n \rightarrow \infty$ .

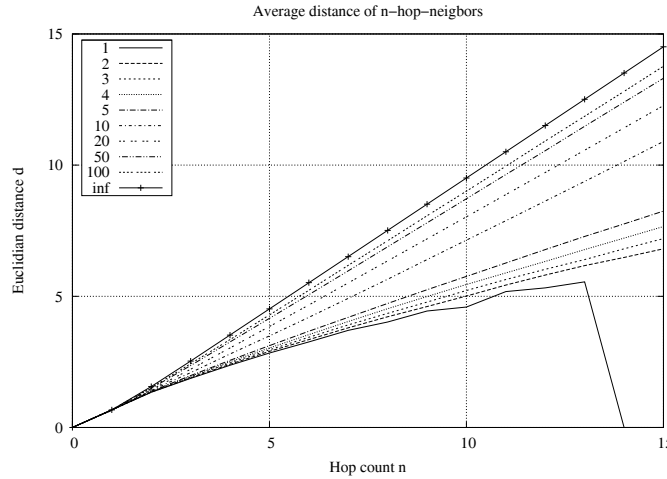


Figure 2 n-hop-neighbors

### 3 Simulation of n-hop-Neighborhoods

For higher n-hop-neighborhoods we used simulations to achieve information about the average distance between n-hop-neighbors. We simulated up to one million random scenarios for different node degrees. Therefore nodes were randomly placed with a uniform distribution around the Node Zero. The observance area was larger than the maximum observed hop count to prevent border effects. The average distance of n-hop-neighbors is shown in Figure 2 as well as the previously theoretically found boundary values for infinite node degrees. Note the insufficient simulation results for a node degree of 1, as with an average of just one neighbor per node, long paths between nodes are only to obtain by very long or rare event simulations.

To achieve one simple formula for the average distance of n-hop-neighbors we made linear approximations. The achieved linear coefficients can be also linear approximated for node degrees from 1 to 10 with an additional cut off at the theoretical values of node degrees towards infinity. This cut off is necessary as the linear approximation exceeds the theoretical bounds for higher node degrees. This approximation is used to achieve the average distance of n-hop-neighbors

$$\bar{d}_n(deg(n)) = (0.0391 \cdot deg(N) + 0.3338) * n + -0.1108 \cdot deg(N) + 0.9917 \tag{10}$$

for  $n \geq 2$ . For  $n = 1$  we assume a node degree independent  $\bar{d}_1 = \frac{2}{3}$ .

### 4 Node Degree based Weighted Centroid Localization Algorithm

As base for our improvement we assume that a node  $N_i$  in a network, which wants to localize itself, has only chances to send messages to its direct neighbors. By flooding a localization request with a hop counter through the network all anchors are reached. These send back their position as well as the hop count from  $N_i$  to the anchor. After receiving this messages from the anchors,  $N_i$  may use different localization algorithms.

In our improved algorithm ND-WCL the average node degree  $deg(N)$  on the shortest path to an anchor is also send to node  $N_i$ . This information is easily obtainable, as all nodes know through neighbor detection and listening to hello messages it's own node degree, which is the number of direct neighbors. No additional message is needed here.

As first reference algorithms we used the centroid algorithm (CL) [3], taking the mean of the positions of the  $n$  anchors to estimate the own position. The second algorithm observed is the weighted centroid localization algorithm (WCL) [1], taking the weighted mean of the positions of the  $n$  anchors to estimate the own position. The weight for each anchor is computed as the reciprocal of the hop count to the anchor.

Our algorithm uses the reciprocal of the formerly computed average distances to the anchors based on hop count and node degree as weights  $w_i$ .

$$w_i = \frac{1}{\bar{d}_{HopCount_i}(deg(n))} \quad (11)$$

$$Pos_{ND-WCL} = \frac{1}{\sum_{i=1}^n w_i} \sum_{i=1}^n w_i \cdot Pos_{Anchor}(i) \quad (12)$$

This algorithm is easily implementable and does not need any further messages. It can be computed locally on the node to localize as only one additional step with a linear equation must be computed. The computation complexity is close to the normal hop count based weighted centroid algorithm.

## 5 Simulation of Localization

To compare the performance of the new algorithm with the two existing ones we used extensive simulations in MATLAB. Up to 1000 nodes were placed in a two dimensional square of 50x50m with 4 or 9 anchors. Simulation details are shown in the left part of Table 1 with the number of nodes, communication range and number of placed anchors. To have paths to anchor nodes with rather different node degrees, we shifted half of the nodes from one side of the simulation area to the other in the even numbered scenarios, producing unbalanced scenarios with a balancing of 3 to 1. As the anchors are placed in the corners of the simulation area and at least three anchors are taken for the centroid algorithm, at least one path to an anchor has a quite different node degree than the other two paths.

The main evaluation metric is the localization error  $LEr$  [1] as distance between the exact real position and the estimated position of a node.

We computed and compared for all non anchors in a scenario the localization error for centroid, weighted centroid and node degree based improved weighted centroid algorithm. As expected the weighted centroid algorithm outperformed the simple centroid algorithm tremendously. Therefore we only compare the two weighted centroid algorithms here.

The localization error was averaged for all non anchors in each randomly generated scenario. The minimum, maximum and mean improvement in percent over 1000 scenarios using the node degree approach is shown in Table 1. Negative values show that some randomly generated scenarios had a better average performance for the normal weighted centroid algorithm. Nevertheless in all scenarios the mean localization error is reduced using node degree based improved weighted centroid. It performs better with longer routes in scenarios V to X with shorter communication range and more nodes. It also performs better with less anchors to choose, resulting also in longer and more different long paths to anchors. The most increase in performance was obtained with the unbalanced scenarios leading to paths with rather different node degrees.

The overall best performance of the node degree based improved weighted centroid localization algorithm was obtained in scenario X with the longest paths to the four anchors and an unbalanced scenario.

■ **Table 1** Simulation Scenarios and Localization Error Improvement

Scenario	No. of Nodes N	Range R	No. of Anchor Nodes	Balancing	LEr Improvement in %		
					Min	Max	Mean
I	100	10m	4	none	-3.92	6.39	1.38
II	100	10m	4	3:1	-8.37	15.04	4.45
III	100	10m	9	none	-2.43	3.16	0.25
IV	100	10m	9	3:1	-5,15	7,45	1,06
V	400	5m	4	none	-1,49	5,20	2,55
VI	400	5m	4	3:1	-0,11	8,78	3,05
VII	400	5m	9	none	0,19	5,11	2,76
VIII	400	5m	9	3:1	1,51	8,57	4,07
IX	1000	3m	4	none	0.97	3.92	2.91
X	1000	3m	4	3:1	5,43	14,87	8,64

## 6 Conclusions

In this paper we improved the weighted centroid algorithm for hop count based localization. With the small functional addition of counting the node degree on the paths to the referenced anchors we could improve the performance of hop count based localization. Simulations showed that for longer hop counts to the anchors and areas with different node degrees the proposed algorithm outperforms the known hop count based weighted localization algorithm.

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