A Survey of Interface Tracking Methods in Multi-phase Fluid Visualization

Fang Chen[1] **and Hans Hagen**[2]

1 **University of Kaiserslautern**
AG Computer Graphics and HCI Group, TU Kaiserslautern, Germany
chen@cs.uni-kl.de

2 **University of Kaiserslautern**
AG Computer Graphics and HCI Group, TU Kaiserslautern, Germany
hagen@cs.uni-kl.de

Abstract

A central feature that scientists are interested in is the dynamics of fluid interfaces or the so called material boundaries in multi-fluid simulation . Visualization techniques for capturing fluid interface are based on one of about three basic algorithms. In this paper, we give a survey of the existing interface tracking algorithms, including backgrounds, terms, procedures as well as pointers to details and further reading. We also provide a glance at the mathematical fundamentals of multi-fluid dynamics for scientists who are interested in understanding the underlying math and physics of multi-phase fluid simulation.

Keywords and phrases Multi-phase fluid, interface tracking, topology methods

Digital Object Identifier 10.4230/OASIcs.VLUDS.2010.11

1 Introduction

Implicit surface, dynamic and free boundaries are interesting topics in multi-phase fluid visualization. Aside from mass and momentum transportation of the flow, scientists are particularly interested in locating the sharp interfaces between the components of fluids. An interface or boundary is an implicit surface in the fluid mixture where two different materials are separated. To track the dynamic behavior of these interfaces, special algorithms are needed.

Interface tracking is a visualization technique that allows the scientist to identify and follow dynamics of fluid. It is of great interest in several application fields as diverse as chemical engineering, combustion and astrophysics.

There are three main approaches in fluids interface tracking, front tracking method, volume of fluid method, and level set method. In the next few sections, we will cover the central procedure and main idea of these algorithms, and provide also possible numerical solutions for solving the equations of computational fluid dynamics.

This paper is organized as follows: Section 2 gives a brief introduction to the mathematical simulation of multi-fluids. In section 3, we will discuss the main three types of algorithms in tracking fluids interface with two of them covered in more details. Section 4 covers the numerical schemes that can be applied to solving the corresponding hyperbolic partial differential equations when conducting those tracking algorithms. We especially expand our discussion in the direction of point-based/mesh-free methods, as they serve as a better and more efficient solution to transient multi-fluid problems.

Visualization of Large and Unstructured Data Sets– IRTG Workshop, 2010.
Editors: Ariane Middel, Inga Scheler, Hans Hagen; pp. 11–19

OpenAccess Series in Informatics
Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

2 Simulation of multi-phase fluid

Interface tracking is a visualization technique that visualizes scientific data coming from multi-fluid simulations. To better understand the simulation data, it is also important to get an insight into the mathematical principles simulation process. In this section , we give a brief introduction to the mathematical formulation of multi-fluid simulation.

2.1 Navier-Stokes equations of multi-phase fluids

Numerical simulation of multiphase fluids are complex and challenging tasks. Current numerical algorithms mostly target a specific fluid model, and 3D simulations of multiphase fluids are still ongoing. What makes the problem so challenging is the large variety of physical phenomena in which the phases interact [9]. Here we provide a basic formulation of two-phase fluid problem, namely the conservation laws for mass and momentum.

Conservation laws for multiphase fluids problem are given follows: The first two equations describe the conservation of mass for each phase of flow, where α denotes the volume fraction of the dispersed phase. Here we consider the formulation proposed by Kuhnert and Tiwari [3, 17, 21]

$$\frac{d\alpha}{dt} = -\alpha(\nabla \cdot \mathbf{v_1}) \tag{1}$$

$$\frac{d(1-\alpha)}{dt} = -(1-\alpha)(\nabla \cdot \mathbf{v_2}) \tag{2}$$

Momentum conservation of the two phases are given as

$$\frac{d\mathbf{v_1}}{dt} = \frac{\nabla p}{\rho_1} + \frac{1}{\alpha_1 \rho_1}\nabla \cdot S_1 + \mathbf{g} + \frac{\alpha_1}{\rho_1}\mathbf{F_1} \tag{3}$$

$$\frac{d\mathbf{v_2}}{dt} = \frac{\nabla p}{\rho_2} + (1 - \frac{\rho_1}{\rho_2})\mathbf{g} - \frac{\mathbf{1}}{(\mathbf{1}-\alpha_1)\rho_2}\mathbf{F_1} \tag{4}$$

where the interfacial dragging force $\mathbf{F_1}$ is given by [24]

$$\mathbf{F_1} = -\frac{\mathbf{3}}{\mathbf{4}}(\mathbf{1}-\alpha)\rho_1 \frac{\mathbf{C}}{\mathbf{d_2}}|\mathbf{v_2} - \mathbf{v_1}|(\mathbf{v_2} - \mathbf{v_1})$$

and the interfacial stress tensor [3] is given as

$$S_1 = \alpha\mu_1[\nabla\mathbf{v_1} + (\mathbf{v_1})^{\mathbf{T}} - \frac{\mathbf{1}}{\mathbf{3}}(\nabla \cdot \mathbf{v_1})\mathbf{I}]$$

where μ is the dynamic viscosity of the liquid. In this formulation, the conservation law for different phase of fluid are coupled by volume fraction α of the dispersed phase. For instance $\alpha = 0.47$ means that the dispersed phase of liquid takes 47% of the total volume.

3 Main approaches in fluid interface tracking

This section gives a short overview of the current interface tracking methods. As two main streams of interface tracking algorithms, volume of fluid method and level sets methods are elaborated in the following subsections. A list of pros and cons are provided in the overview as an comparison of each algorithm. Then we introduce the general concepts of marker functions and one-phase formulation of the multi-phase fluid problem, followed by more detailed discussion of each algorithm. As a supplementary material, we will cover the mathematical fundamental concepts of multi-fluid simulation in Appendix.

3.1 Overview

Generally, there are three interface tracking methods : Front Tracking Method (FT) [23], Level Set Method [15], and Volume of Fluid method(VOF) [8].

Front Tracking Method [23, 22, 6] advects the marked interface from an initial configuration and keeps the topology of the interface during the simulation. Therefore, this method is limited to topological changes in multiphase-fluid, such as merging or breaking of droplets. Thus we will not include it in our discussion.

Level Set Method was first introduced by Osher [15] in 1988. The material boundary or interface is defined as the zero set of [13, 14, 18] isocontour or isosurface of the given scalar field. Sethian [20] and Lakehal [9] applied the idea level set method into fluid simulation. In 2002, Enright, Fedkiw et. al. [4] have combined Lagrangian marker particles with LSM to obtain a maintain a smooth geometrical description of the fluid interface. However, it has been pointed out by Müller [10] and Garimella et. al. [5] that material volume is not well preserved in level set method, which is a main draw back of this method.

Volume of Fluid Methods [8] is one of best established interface volume tracking method currently in use [11, 19]. As name indicated, it keeps tracking of the volume of each fluid phase with a sub-volume. This method is therefore based on subcells or sub-volumes, and one tracks the volume percentage that one type of fluid takes up a sub volume cell.

Aside from the above three interface tracking algorithms, there also exist research directions for reconstruction of material interfaces. Reconstruction methods are mainly working on rebuilding a continuously interfaces out of discrete pieces or piecewise functions, while interface tracking algorithms focuses on tracking the dynamic behavior of the interface. Material reconstruction researches include *simple line interface* (SLIC) [12] and piecewise linear interface construction (PLIC) [16]. Recent development of discrete reconstruction algorithms can be found in [2, 1], and it is beyond the scope of our discussion in the survey.

3.2 One-phase formulation of the multi-fluid problem

A significant idea in dealing with multi-phase fluids is to use a one-phase formulation for the coupled systems.

Rather than looking at the coupled equation systems, one can reduce the complexity of computation by introducing a marker/indicator function χ.

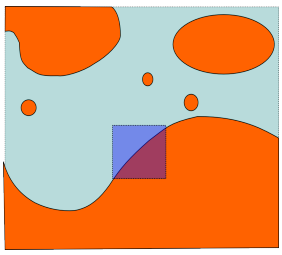
$$\chi = \begin{cases} 1 & \text{when it is in fluid 1} \\ 0 & \text{when it is in fluid 2} \end{cases} \tag{5}$$

As the name indicates, a marker function indicates the phase change by attaching a marker on one of the phases. Sometimes, markers can also be assigned to the exact interface between two materials, such as in level sets method. We will discuss that in the next subsection.

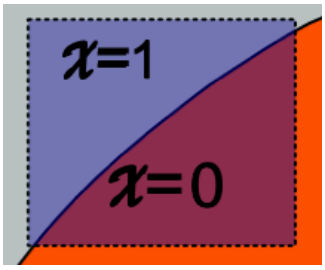
Therefore, instead of having two sets of variables for each phase of the fluid mixture (two phase formulation, see Appendix), a mark function marks the integration volume (i.e. when one look at a small volume element, 1b) by a characteristic function and conservation law for the two phase [9] results in only one :

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{6}$$

where $\rho = \rho_1 + (\rho_1 - \rho_2)\chi$.



(a) Two phase.



(b) One fluid formulation. Zoom in at an volume element

Figure 1 One and two phase formulation

3.3 Volume of Fluid

VOF method, originated by Hirt and Nichols [8] is a Eulerian method interface tracking method which fulfills the conservation of mass/volume. More importantly, topology change of the moving surfaces, such as merging and breakup of bubbles can be captured by VOF method.

The idea of VOF method is based on the one-fluid formulation. A fraction variable C is defined as the integral of the marker function χ in the controlled volume V:

$$C = \frac{1}{V}\int_V \chi(\vec{x}, t)dV \tag{7}$$

Typically the controlled volume V is the computational cell volume.

A zero value of C indicates the cell is fully occupied by fluid one, and when $C = 1$, then the cell is fully filled with the other fluid. While $0 < C < 1$, the line/surface separating the two fluids goes across this cell.

Given a velocity field of V, the volume fraction function should fulfills the transportation theory

$$\frac{dC}{dt} + \vec{v} \cdot \nabla C = 0 \tag{8}$$

Volume tracking of VOF method do not only include a computation for volume frace at each time step, but also needs to approximate and reconstruct the interface. This method is accurate in the sense of volume preserving, but reconstruction of interface process makes it difficult to keep the topology of the interface.

3.4 Level Set Method

Osher and Fredkiw [14] have provided an overview of level set method and the most recent results. Enright et al. [4] use a fast first order accurate semi- Lagrangian method to improve the mass conservation. Similar to VOF method, LSM employs the idea of one-fluid simulation. The implicit material boundary/interface is given by the zero set of the scalar field ϕ:

$$\Gamma : \{(x, y, z) | \phi(x, y, z) = 0\}$$

$$\phi = \begin{cases} > 0 & \text{in fluid 1} \\ < 0 & \text{in fluid 2} \\ = 0 & \text{at boundary } \Gamma \end{cases} \tag{9}$$

Here ϕ is a similar marker function which is given in equation 5, the only difference is that $\phi = 0$ is exactly where the interface lies. The evolution of the zero sets satisfied the topology equation (8). Finding the initial zero sets normally involves extracting isocontour or isosurface at starting time. For a dataset with volume fraction information, one normally defines the zero set to be the isosurface of $\alpha = 0.5$.

Moreover, normal and curvature of the level set is given as [9, 13, 4]:

$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} \tag{10}$$

$$\kappa = \nabla \cdot \frac{\nabla\phi}{|\nabla\phi|} \tag{11}$$

Constraints on curvature should be applied when doing physically correct simulation. A general concern for curvature is the minimization of surface energy, which is also related to surface tension. However, this approach is more directed to free surface flows. In bubbly flow, e.g., mixture of two immiscible fluids, the topology change of interfaces contains more information.

Level set method is the most widely used method currently. It has an simple mathematical formulation and easy to solve. However, level set method has the draw back that volume is not always preserved while advecting the interface. Such disadvantages can be corrected by applying a volume correction after each numerical advection.

4 Numerical Methods for solving the governing equations

In computational fluid dynamics, there are two ways looking at a fluid property:

- Euler specification: In this way, a fixed time independent grid is used to compute the propagation of flow property. Metaphorically, the observer is sitting on the river bank, staring at a fix point in the river, and measuring the flow quantity at this fixed point as water flows by. This type of method is normally computationally expensive because to advect flow quantity, one has to update every grid point based on the previous quantity at this point and its four neighboring points.
- Lagrangian specification: A Lagrangian specification of flow dynamics is also known as the particle based specification. Compared to the previous one, now the observer rides on a single fluid particle, moves with the flow, and measures the change of flow quantities. Change of the flow properties are related to the so-called *material derivative*:

$$\frac{\mathrm{D}f}{\mathrm{D}t} = \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f$$

4.1 Grid-based method

One classical way of handling hyperbolic partial differential equations is to solve them numerically on a fixed grid points. These types of methods solves a partial differential equation in an Euler way. To solve the governing equations for both VOF and LS methods, one can apply the standard advection schemes, such as upwind scheme, central difference, Lax-Friedrich and Lax-Wendroff schemes. To avoid dissipation, a second order scheme with flux limiters can be considered. A large body of literature is available for Euler-type solvers, such as [14, 7, 8, 14, 13].

4.2 Particle-based method

In many simulation cases, volume fraction of second phase of fluid is given at a point cloud at each time frame. At each time slice, the point cloud was reinitialized, and there is correspondence between the points from one time slice to the next ones, see figure 2a. Therefore, conventional interpolation between two time slices would involve a resampling on a regular grid or project the points from one times step to the other. An intuitive way of interpolating the volume fraction in between two time steps is doing a linear combination between the two, such as:

$$f(t_0 + \Delta t) = \frac{t_0 - t_1 + \Delta t}{t_0 - t_1} f(t_0) + \frac{-\Delta t}{t_0 - t_1} f(t_1)$$

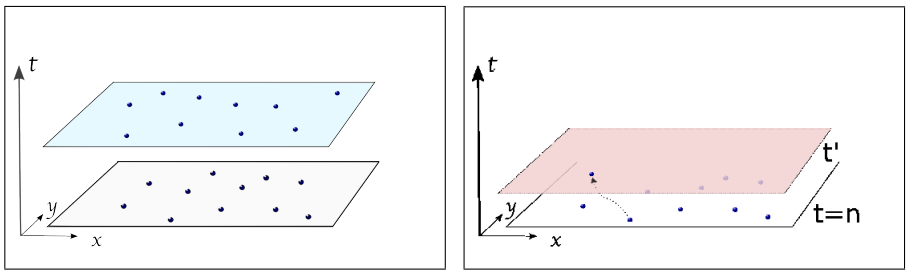
However, a linear blending gives us a physically incorrect interpolation resulting from the ignorance of flow direction. Displayed in figure 3a, the middle layer is a linear interpolation of the two slices. For a good interpolation, one would expect the high density region (marked as blue) moves a long the flow to the bottom time slice and then to the upper one. However, in the linear case, we can observe a non-smooth advection of these regions. What went wrong? We didn't consider flow directions. The solution to this problem is to use given velocity information and advect the flow property from one time step to another. To advect the flow property, we need to go back to the Navier-Stokes governing equation which described the motion of fluid:

$$\frac{\partial f}{\partial t} + \nabla \cdot (f\vec{v}) = 0 \tag{12}$$

where f is a fluid property such as mass or density. And Lagrangian formulation of equation (12) yields

$$\frac{\mathrm{D}f}{\mathrm{D}t} = -f\nabla \cdot \vec{v}. \tag{13}$$

where $-f\nabla \cdot \vec{v}$ is the advected term.



(a) Input point cloud **(b)** Lagrangian advection on a point

Figure 2 Original point cloud and Lagrangian methomaked

To solve the governing equation in a Lagrangian way, the following two steps are carried out:

- Advect a point along the velocity field (see Fig. 2b):

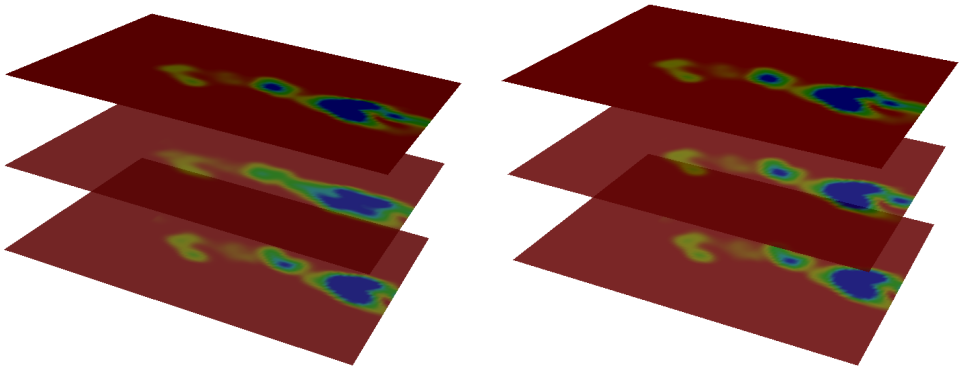
  $$\mathbf{x}^{n+1} = \mathbf{x}^n + \vec{v}\Delta t.$$

  For a higher order approximation or when then Δt is too large, one has to consider higher order scheme to advect the point, and adjust time step length when flow derivation is too large.

- Update the fluid property from equation (13)

$$f(\mathbf{x})^{t+1} = \frac{f(\mathbf{x})^t}{1 + \Delta t(\nabla \cdot \vec{v}^t)}$$

If convergence is desired, one can simply do a forward interpolation from t_n to t_{n+1}, and then perform a backward interpolation the other way around. A continuous result can be obtained by averaging the two. An example of Lagrangian interpolation is shown in figure 3b. Compare to Fig. 3a, one can observe that Lagrangian method gives us a smooth transition between time steps.



(a) Linear interpolation. Middle slice is a linear blending of the other two time slices.

(b) Linear interpolation. The side in the middle is interpolated using Lagrangian scheme.

4.3 Remarks

In both grid-based and particle based method, stability or convergence is an unneglegible issue. First order upwind scheme is unstable thus can cause unwanted numerical solution. In the case of particle based method, advection term of computational particles can be approximated by finite difference schemes. Moving particles around must be handle with care when velocity deviation is high. A common way of treating deviation problem is to apply an adaptive integration length.

5 Conlusion

In this paper, we presented a survey of the current interface tracking methods. The goal of interface tracking is to locate the interface separating fluid materials over time. Depending on the requirements of the interface tracking, such as continuity of volume preserving, some methods may be better suited than others. In this survey, we focus on a few current interface tracking methods that track the evolution of interface directly. Unlike the interface reconstruction methods, interface tracking methods track the interface propagation over time and require the knowledge of an initial stage.

We explore some key interface methods such as level set method which covers a wide range of interface-tracking problem. Depending on the mesh type of the given data, we also provide several corresponding numerical solvers to the underlying tracking equations. Thorough concepts and discussion can be found in further literatures such as [14, 15, 20].

Acknowledgements This work is supported by International Research and Training Group at University of Kaiserslautern (IRTG) and Deutsche Forschungsgemeinschaft (DFG , German research foundation).

References

1 John C. Anderson, Christoph Garth, Mark A. Duchaineau, and Kenneth I. Joy. Discrete Multi-Material Interface Reconstruction for Volume Fraction Data. *Comput. Graph. Forum*, 27(3):1015–1022, 2008.

2 John C. Anderson, Christoph Garth, Mark A. Duchaineau, and Ken Joy. Smooth, Volume-Accurate Material Interface Reconstruction. *IEEE Transactions on Visualization and Computer Graphics*, 16(5):802–812, 2010.

3 C. Drumm, S. Tiwari, J. Kuhnert, and H-J. Bart. Finite pointset method for simulation of the liquid-liquid flow field in an extractor. *Computers and Chemical Engineering*, 32(12):2946–2957, 2008.

4 D. Enright, R. P. Fedkiw, and Ferziger J. H. et al. A hybrid particle level set method for improved interface capturing. *J. Comput. Phys.*, 183:83–116, 2002.

5 R. V. Garimella and V. et. al. Dyadechko. *Interface Reconstruction in Multi-fluid, Multi-phase Flow Simulations.* pages 19–32, Springer, 2005.

6 O. Gloth, D. Hänel, L. Tran, and R. Vilsmeier. A front tracking method on unstructured grids. *Computers & Fluids*, 32(4):547–570, 2003.

7 Edwige Godlewski and Pierre-Arnaud Raviart. *Numerical approximation of hyperbolic systems of conservation laws.* Springer-Verlag New York, 1996.

8 C. W. Hirt and B. D. Nichols. Volume of fluid (VOF) method for the dynamics of free boundaries. *Journal of Computational Physics*, 39(1):201–225, 1981.

9 D. Lakehal, M. Meier, and M. Fulgosi. Interface tracking towards the direct simulation of heat and mass transfer in multiphase flows. *International Journal of Heat and Fluid Flow*, 23:242–257, 2002.

10 Matthias Müller. Fast and robust tracking of fluid surfaces. In *SCA '09: Proceedings of the 2009 ACM SIGGRAPH/Eurographics Symposium on Computer Animation*, pages 237–245, New York, NY, USA, 2009. ACM.

11 M. Marek, W. Aniszewski, and A. Boguslawski. Simplified volume of fluid method for two-phase flows. *Task Quarterly*, 3:255–265, 2008.

12 W. Noh and P. Woodward. Simple line interface method. In *A. van de Vooren and P. Zandbergen, editors, Proceedings of the 5th international conference on fluid dynamics, Lecture notes in Physics*, volume 59, 1976.

13 S. Osher and R. P. Fedkiw. Level Set Methods: An Overview and Some Recent Results. *Journal of Computational Physics*, 169(2):463–502, May 2001.

14 S. J. Osher and R. P. Fedkiw. *Level Set Methods and Dynamic Implicit surfaces.* Springer, 2003.

15 S. Osher and J. A. Sethian. Fronts propagating with curvature-dependent speed: Algorithms based on Hamilton-Jacobi formulations. *Journal of Computational Physics*, 79:12–49, 1988.

16 W. J. Rider and D. B. Kothe. Reconstructing volume tracking. *Journal of computational physics*, 141(2), 1998.

17 R. Rieger, C. Weiss, G. Wigley, H. J. Bart, and R. Marr. Investigating the process of liquid-liquid extraction by means of computational fluid dynamics. *Computers and Chemical Engineering*, 20(12):1467–1475, 1996.

18 J. A. Sethian. Curvature and the Evolution of Fronts. *Commun. Math. Phys.*, 101:487–499, 1985.

19 M. Sussman and E. G. Puckett. A coupled level set and volume-of-fluid method for computing 3D and Axisymmetric incompressible two-phase flows. *Journal of Computational Physics*, 162:301–337, 2000.

20 J. A. Sethian and P. Smereka. Level set methods for fluid interfaces. *Annu. Rev. Fluid Mech.*, 35:341–372, 2003.

21 S. Tiwari and J. Kuhnert. Modeling of two-phase flows with surface tension by finite pointset method (FPM). *J. Comput. Appl. Math.*, 203(2):376–386, 2007.

22 Hiroshi Terashima and Gretar Tryggvason. A front-tracking/ghost-fluid method for fluid interfaces in compressible flows. *Journal of Computational Physics*, 228(11):4012–4037, 2009.

23 S. O. Unverdi and G. Tryggvason. A front tracking method for viscous Incompressible multi-fluid flows. *Journal of Computational Physics*, 100(1):25–37, 1992.

24 F. Wang and Z. S. Mao. Numerical and experimental investigation of liquid-liquid two-phase flow in stirred tanks. *Industrial and Engineering chemistry research*, 44(15):5776–5787, 2005.