

**10501 Abstracts Collection**  
**Advances and Applications of Automata on**  
**Words and Trees**  
— Dagstuhl Seminar —

Christian Glaßer<sup>1</sup>, Jean-Éric Pin<sup>2</sup>, Nicole Schweikardt<sup>3</sup>, Victor Selivanov<sup>4</sup>, and  
Wolfgang Thomas<sup>5</sup>

<sup>1</sup> Universität Würzburg, DE

Glasser@informatik.uni-wuerzburg.de

<sup>2</sup> LIAFA University Paris VII, FR

Jean-Eric.Pin@liafa.jussieu.fr

<sup>3</sup> Goethe-Universität Frankfurt, DE

schweika@informatik.uni-frankfurt.de

<sup>4</sup> Pedagogical University - Novosibirsk, RU

vseliv@iis.nsk.su

<sup>5</sup> RWTH Aachen, DE

thomas@informatik.rwth-aachen.de

**Abstract.** From 12.12.2010 to 17.12.2010, the Dagstuhl Seminar 10501 “Advances and Applications of Automata on Words and Trees” was held in Schloss Dagstuhl – Leibniz Center for Informatics. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

**Keywords.** Automata theory, logic, verification, data structures, algorithms, complexity, games, infinite games with perfect information, reactive systems, specification and verification, combinatorics, hierarchies and reducibilities

## 10501 Executive Summary – Advances and Applications of Automata on Words and Trees

The aim of the seminar was to discuss and systematize the recent fast progress in automata theory and to identify important directions for future research. For this, the seminar brought together more than 40 researchers from automata theory and related fields of applications. We had 19 talks of 30 minutes and 5 one-hour lectures leaving ample room for discussions. In the following we give a collection of abstracts.

*Keywords:* Infinite games with perfect information, reactive systems, specification and verification, combinatorics, hierarchies and reducibilities

*Joint work of:* Glasser, Christian; Pin, Jean-Eric; Schweikardt, Nicole; Selivanov, Victor; Thomas, Wolfgang

*Extended Abstract:* <http://drops.dagstuhl.de/opus/volltexte/2011/3147>

## On the decidability of the mu-calculus alternation hierarchy

*Bahareh Afshari (University of Edinburgh, GB)*

An interesting question regarding the modal mu-calculus alternation hierarchy is one of decidability, namely given a formula can one decide whether it is equivalent to a formula with a lower alternation depth. Martin Otto showed in 1999 that it is decidable whether a given formula of modal mu-calculus can be expressed without fixed point operators, by reducing the problem to the monadic second order theory of binary trees, S2S, which by Rabin's classical result is decidable. In 2002, Ralf Küsters and Thomas Wilke extended Otto's result to the first level of the hierarchy, proving it is decidable whether a formula of the modal mu-calculus is equivalent to one without greatest fixed point operators. This result was obtained by reducing the problem to the non-emptiness problem of a  $\Sigma_1$ -test automaton. However, these techniques fail to extend to the higher levels of the hierarchy and the general question has remained open. I will give partial results for the class of  $\Delta_2$  formulae and discuss generalising the techniques.

*Joint work of:* Afshari, Bahareh; Quickert, Sandra

## Equilibria in Quantitative Reachability Games

*Veronique Bruyere (Université de Mons, BE)*

We study turn-based quantitative multiplayer non zero-sum games played on finite graphs with reachability objectives. In this framework each player aims at reaching his own goal as soon as possible. We prove existence of finite-memory Nash (resp. secure) equilibria in multiplayer (resp. two-player) games.

## Distance Automata and Regular Cost Functions

*Thomas Colcombet (University Paris-Diderot, CNRS, FR)*

This survey will cover some important results concerning distance automata and their extensions, namely Krob's undecidability result, Hashiguchi's result of decidability of the limitedness problem (we use Leung's approach), and the decidability of the star-height problem due to Hashiguchi (we use Kirtsten's approach).

In the last part we will consider the extension of these results to the more general framework of regular cost functions.

*Keywords:* Distance automata, regular languages, limitedness, tropical semiring, regular cost functions

## Small Fragments of First-order Logic over Finite and Infinite Words (Part 1)

*Volker Diekert (Universität Stuttgart, DE)*

Over finite words, the algebraic approach is the method of choice for deciding the membership problem of fragments of first-order logic. We show how to combine algebraic and topological properties in order to generalize these results to infinite words. The first part of the talk considers decidable fragments with the linear order predicate  $<$  as the only binary relation. The results were published at STACS 2009 (joint work with Manfred Kufleitner). In Part 2 of the talk Manfred Kufleitner will report on a continuation of this work. The second part is based on material which will be presented at STACS 2011 (Jakub Kallas, Manfred Kufleitner, and Alexander Lauser).

## Weighted logics for unranked tree automata

*Manfred Droste (Universität Leipzig, DE)*

We define a weighted monadic second order logic for unranked trees and the concept of weighted unranked tree automata, and we investigate the expressive power of these two concepts. We show that weighted tree automata and a syntactically restricted weighted MSO-logic have the same expressive power in case the semiring is commutative or in case we deal only with ranked trees, but, surprisingly, not in general. This demonstrates a crucial difference between the theories of ranked trees and unranked trees in the weighted case.

Joint work with Heiko Vogler (TU Dresden).

*Keywords:* Weighted logics, unranked tree automata, weighted tree automata, formal power series

*See also:* M. Droste, H. Vogler: Weighted logics for unranked tree automata, Theory of Computing Systems 48 (2011), 23-47.

## Why the Wadge Hierarchy of $\omega$ -Regular Tree Languages is Huge

*Jacques Duparc (University of Lausanne, CH)*

I will provide a lower bound for the length of the Wadge hierarchy on languages recognized by non deterministic tree automata which shows that there is a huge gap between deterministic and non deterministic machines.

## Axiomatizing Regular Tree Languages

*Zoltan Esik (University of Szeged, HU)*

We present complete axiomatizations of regular languages of ranked trees.

## Weighted automata with pebbles and weighted FO logic with transitive closures

*Paul Gastin (ENS - Cachan, CNRS, FR)*

We introduce new classes of weighted automata on words. Equipped with pebbles and a two-way mechanism, they go beyond the class of recognizable formal power series, but capture a weighted version of first-order logic with bounded transitive closure. In contrast to previous work, this logic allows for unrestricted use of universal quantification. Our main result states that pebble weighted automata, nested weighted automata, and this weighted logic are expressively equivalent. We also give new logical characterizations of the recognizable series.

Joint work with Benedikt Bollig, Benjamin Monmege and Marc Zeitoun

## Local vs global regularity of words and automata theory

*Juhani Karhumaeki (University of Turku, FI)*

One of the fundamental topics in mathematics is search for relations between *local* and *global* regularities. We analyze this phenomena in connection with infinite words. Local regularity here means that the word possesses everywhere some local (finitely describable) regularity condition, such as some type of local periodicity, while the global regularity means that the word is periodic (or ultimately periodic).

More concretely, local periodicity could be that each long enough suffix of the word ends up with a square or with cube, respectively. In the case of squares the local regularity does not imply the ultimate periodicity, while the cubes do so. A striking exact border of order of repetitions in this setting was shown by

Mignosi, Restivo and Salemi, see [1], where it is also shown that the Fibonacci word proves the optimality.

We analyze this problem area in several concrete different settings, including some where abelian repetitions are considered instead of ordinary repetitions as models of local regularity. In particular, finite automata are very useful tools in solving and describing these phenomena. This is further emphasized by the following property: In many, if not all, natural cases if a local property need not imply the ultimate periodicity, then there are nondenumerably many words (or objects) satisfying this local property  $\tilde{U}$  which is directly visible from the structure of the corresponding automaton. The nondenumerability of such objects justifies our terminology stating that the local property allows a *chaotic* behaviour. In this terminology ultimately periodic words can be viewed as *predictable*. In this view we are looking for local properties which would separate predictable and chaotic behaviours in infinite words.

Finally, an in-between property of the equality and the abelian equality is defined, namely so-called *k-abelian* equality. Two words are *k-abelian* equal if they possess the same prefixes and suffixes of length  $k - 1$ , respectively, and moreover each factor of length  $k$  occurs in these words equally many times. The above problems are also considered with respect to these equivalence relations.

We also analyze problems of determining the smallest alphabet where 2-abelian squares and cubes can be avoided in infinite words.

[1] Mignosi, Filippo; Restivo, Antonio; Salemi, Sergio. Periodicity and the golden ratio. Theoret. Comput. Sci. 204 (1998), no. 1-2, 153-167.

## Small Fragments of First-order Logic over Finite and Infinite Words (Part 2)

*Manfred Kufleitner (Universität Stuttgart, DE)*

Over finite words, the algebraic approach is the method of choice for deciding the membership problem of fragments of first-order logic. We show how to combine algebraic and topological properties in order to generalize these results to infinite words. In the second part of the talk we consider fragments which allow the linear order predicate and, in addition, the successor predicate as binary predicates. The talk is the continuation of the lecture (Part 1) by Volker Diekert who reported on joint results being published at STACS 2009. The results in this talk (Part 2) will be presented at STACS 2011 (joint work with Jakub Kallas and Alexander Lauser).

## The nondeterministic parity index problem

*Christof Löding (RWTH Aachen, DE)*

The nondeterministic parity index problem consists in finding for a given regular language of infinite trees the minimal range of priorities needed by a nondeterministic parity automaton accepting the language. In this talk I present our recent approach (joint with Thomas Colcombet) to tackle this problem. The main idea is to translate the problem into a limitedness problem for distance parity automata. We have developed techniques to solve the latter problem but there are still some gaps and up to now we can only solve special cases.

*Joint work of:* Colcombet, Thomas; Löding, Christof

## Isomorphism Problems on Automatic Structures

*Markus Lohrey (Universität Leipzig, DE)*

Automatic structures are finitely presented structures where the universe and all relations can be recognized by finite automata. It is known that the isomorphism problem for automatic structures is complete for  $\Sigma_1^1$ , the first existential level of the analytical hierarchy. Positive results on ordinals and on Boolean algebras raised hope that the isomorphism problem is simpler for transitive relations. We prove that this is not the case. More precisely, the following results are shown:

(i) The isomorphism problem for automatic equivalence relations is complete for  $\Pi_1^0$  (first universal level of the arithmetical hierarchy).

(ii) The isomorphism problem for automatic trees of height  $n > 1$  is  $\Pi_{2n-3}^0$ -complete.

(iii) The isomorphism problem for automatic order trees with only countably many infinite paths is recursively equivalent to true first-order arithmetic.

(iv) The isomorphism problem for automatic order trees is  $\Sigma_1^1$ -complete.

(v) The isomorphism problem for automatic linear orders is  $\Sigma_1^1$ -complete.

*Keywords:* Automatic structures, isomorphism problems, arithmetical and analytical hierarchy

## DAG evaluation and the red-blue problem

*Pierre McKenzie (Université de Montréal, CA)*

We consider the generalisation of the tree evaluation problem considered by Braverman, Cook, McKenzie, Santhanam and Wehr (2009) to the case of directed acyclic graphs encoded in such a way as to render the resulting function mononone.

We define the *red-blue problem* for a red-blue-colored dag  $G$  with red-colored sources and a single blue-colored sink as that of identifying a blue-colored vertex having all red in-neighbours.

Appealing to the dart games of Raz and McKenzie (1999), we observe that the decision tree depth of the red-blue problem for  $G$  bounds the monotone Boolean circuit depth of the dag evaluation problem for  $G$  from below.

We conclude with bounds on the decision tree depth of the red-blue problem for various dags.

*Joint work of:* McKenzie, Pierre; work in progress with Yara Elias

## On separation problem in the index hierarchy

*Damian Niwinski (University of Warsaw, PL)*

This is a joint work with Henryk Michalewski.

In a previous work with Szczepan Hummel, we exhibited a pair of disjoint co-Buchi recognizable sets of trees inseparable by any Borel set. These sets are the game tree language  $W_{0,1}$  and its renamed copy  $W'_{0,1}$ .

Now we show that this pair is complete for all disjoint pairs of co-analytic sets.

This confirms that tree automata are indeed capable of recognizing really hard examples. The proof involves a construction of a “dichotomic” automaton, which is perhaps of some interest for its own.

*Keywords:* Tree automata, parity games, separation property, co-analytic sets, completeness

## Language equations: the story of computational completeness

*Alexander Okhotin (University of Turku, FI)*

Equations with formal languages as unknowns are best known for the case of systems  $X_i = \phi_i(X_1, \dots, X_n)$  using union and concatenation, which define the context-free grammars. Recent research has exposed strong connections between language equations  $\phi(X_1, \dots, X_n) = \psi(X_1, \dots, X_n)$  and computability; in particular, it was found that their unique (least, greatest) solutions can represent exactly the recursive (r.e., co-r.e.) languages. This talk describes the research path from encountering the first undecidable properties of language equations (Parikh et al., 1985; Charatonik, 1998) to establishing the computational completeness of their ultimately simple case: systems over a one-letter alphabet using concatenation only (Jez, Okhotin, 2008), and, in particular, systems of two equations  $XXK = XXL$ ,  $XM = N$  with regular constants  $K, L, M, N \subseteq a^*$  (Lehtinen, Okhotin, 2010).

## On Non-Forgetting Deterministic Restarting Automata That are Monotone

*Friedrich Otto (Universität Kassel, DE)*

The non-forgetting restarting automaton is an extension of the restarting automaton that is obtained by combining the restart operation with a change of the internal state just like the move-operations and the rewrite operation. Thus, when executing a restart operation the automaton is not forced to reset its internal state to the initial state. We present a hierarchy of language classes that are characterized by various types of non-forgetting, deterministic, and monotone restarting automata.

This hierarchy ranges from the deterministic context-free languages to the so-called left-to-right regular languages. In particular, we show that for non-forgetting, deterministic, monotone restarting automata, the RRWW-model is strictly more powerful than the RWW-model. This is the first time that for a particular type of restarting automata the expressive power of the RRWW-variant is separated from that of the corresponding RWW-variant.

*Keywords:* Restarting automaton, non-forgetting, separation result

*Joint work of:* Friedrich Otto and Hartmut Messerschmidt

*See also:* H. Messerschmidt, F. Otto; A Hierarchy of Monotone Deterministic Non-Forgetting Restarting Automata, Theory of Computing Systems, DOI 10.1007/s00224-009-9247-x

## Decidable Expansions of Labelled Linear Orderings

*Alexander Rabinovich (Tel Aviv University, IL)*

*Abstract:* Let  $M = (A, <, P_1, \dots, P_k)$  where  $(A, <)$  is a linear ordering and  $P_1, \dots, P_k$  are monadic predicates on  $A$ . We show that if the monadic second-order theory of  $M$  is decidable, then there exists a non-trivial expansion  $M'$  of  $M$  by a monadic predicate such that the monadic second-order theory of  $M'$  is still decidable.

*Keywords:* Monadic Logic, Maximal decidable structures

*Joint work of:* Alexander Rabinovich and Alexis Bes

## Parsing Unary Boolean Grammars Using Online Convolution

*Christian Reitwiessner (Universität Würzburg, DE)*

In contrast to context-free grammars, the extension of these grammars by explicit conjunction, the so-called conjunctive grammars can generate (quite complicated) non-regular languages over a single-letter alphabet (Jez, DLT 2007).



Given these expressibility results, we study the parsability of Boolean grammars, an extension of context-free grammars by conjunction and negation, over a unary alphabet and show that they can be parsed in time  $O(|G|M(n \log n) \log n)$  where  $M(n)$  is the time to multiply two  $n$ -bit integers. This multiplication algorithm is transformed into a convolution algorithm which in turn is converted to an online convolution algorithm which is used for the parsing.

*Joint work of:* Alexander Okhotin and Christian Reitwiessner

*Full Paper:* <http://drops.dagstuhl.de/opus/volltexte/2011/3146>

## The enumerating series brings a structuring vision to abstract numeration systems

*Jacques Sakarovitch (ENST - Paris, FR)*

We consider again the fact that any rational abstract numeration system is faithfully, and effectively, described by an  $IN$ -recognisable series. We give a simple proof of this result, which yields a matrix representation of this series. Then, this representation allows a simple computation of the value of words in this system and easy constructions for the recognition of recognisable sets of numbers.

This point of view also opens a number of new problems on  $IN$ -recognisable series. We show that it is decidable whether an  $IN$ -rational series corresponds to a rational abstract numeration system. Some further problems seem to be very difficult.

Joint work with Pierre-Yves Angrand presented at JM 2010.

*Keywords:* Recognisable series, abstract numeration system

## Advances and Applications of Automata on Data Words and Data Trees

*Thomas Schwentick (TU Dortmund, DE)*

Finite or infinite strings or trees with labels from a finite alphabet play an important role in computer science. They can be used to model many interesting objects including system runs in Automated Verification and XML documents in Database Theory. They allow the application of formal tools like logical formulas to specify properties and automata for their implementation. In this framework, many reasoning tasks that are undecidable for general computational models can be solved algorithmically, sometimes even efficiently.

Nevertheless, the use of finitely labelled structures usually requires an early abstraction from the *real* data. For example, theoretical research on XML processing very often concentrates on the document structure (including labels) but

ignores attribute or text values. While this abstraction has led to many interesting results, some aspects like key or other integrity constraints can not be adequately handled.

In Automated Verification of software systems or communication protocols, infinite domains occur even more naturally, e.g., induced by program data, recursion, time, communication or by unbounded numbers of concurrent processes. Usually one approximates infinite domains by finite ones in a very early abstraction step.

An alternative approach that has been investigated in recent years is to extend strings and trees by (a limited amount of) data and to use logical languages with a restricted expressive power concerning this data. As an example, in the most simple setting, formulas can only test equality of data values. The driving goal is to identify logical languages and corresponding automata models which are strong enough to describe interesting properties of data-enhanced structures while keeping decidability or even feasibility of automatic reasoning.

The talk gives a basic introduction into automata models that have been proposed for *data strings* and *data trees*, that is, strings and trees enhanced by data values. The emphasis is on expressiveness and complexity. Furthermore, some recent applications of these automata (inside theory) are discussed.

*Keywords:* Automata, Data strings, data words, data trees

## Fine hierarchy of omega-regular k-partitions

*Victor Selivanov (A. P. Ershov Institute - Novosibirsk, RU)*

We develop the theory of  $\omega$ -regular  $k$ -partitions (for arbitrary  $k \geq 2$ ) that extends the theory around the Wagner hierarchy of regular  $\omega$ -languages. In particular, we characterize the structure of Wadge degrees of  $\omega$ -regular  $k$ -partitions, prove the decidability of any level of the corresponding hierarchy, establish coincidence of the reducibilities by continuous functions and by functions computed by finite automata on the  $\omega$ -regular  $k$ -partitions, and show the undecidability of the first-order theory of the structure of Wadge degrees of regular  $k$ -partitions for each  $k \geq 3$ .

*Keywords:* Acceptor, transducer, omega-regular k-partition, reducibility, fine hierarchy

## Topologies refining the Cantor topology

*Ludwig Staiger (Martin-Luther-Universität Halle-Wittenberg, DE)*

The space of one-sided infinite words plays a crucial rôle in several parts of Theoretical Computer Science. Usually, it is convenient to regard this space as a metric space, the CANTOR space.

It turned out that for several purposes topologies other than the one of the CANTOR space are useful, e.g. for studying fragments of first-order logic over infinite words or for a topological characterisation of random infinite words.

It is shown that both of these topologies refine the topology of the CANTOR space. Moreover, from common features of these topologies we extract properties which characterise a large class of topologies. It turns out that, for this general class of topologies, the corresponding closure and interior operators respect the shift operations and also, to some respect, the definability of sets of infinite words by finite automata.

*Joint work of:* Sibylle Schwarz and Ludwig Staiger

*Full Paper:*

<http://www.informatik.uni-halle.de/arbeitsgruppen/theorie/publikationen/cdmtcs/>

*See also:* Schwarz, S. and Staiger, L., Topologies refining the Cantor topology on  $X^\omega$ , In (C. S. Calude et al. Eds.) Theoretical Computer Science, IFIP Advances in Information and Communication Technology, Vol. 323, Springer-Verlag, Berlin 2010, pp. 271-285.

## Degree Operations

*Bill Wadge (University of Victoria, CA)*

I will present the game/automata characterizations of the Wadge degrees in my dissertation (lub, successor, countable ordinal multiplication, sharp), of the degree multiplication operation of Steele and van Wesep, and report on the search for degree exponentiation.

*Keywords:* Wadge degrees operations

## Definability in the structures with Subword Order

*Ludmila Yartseva (EPFL - Lausanne, CH)*

We develop a theory of (first-order) definability in the subword partial order in parallel with similar theories for the  $h$ -quasiorder of finite  $k$ -labeled forests and for the infix order. In particular, any element is definable (provided that words of length 1 or 2 are taken as parameters), the first-order theory of the structure is atomic and computably isomorphic to the first-order arithmetic. We also characterize the automorphism group of the structure and show that any arithmetical predicate invariant under the automorphisms of the structure is definable in the structure.

Also we obtain some similar results on definability for the subword order over repetition-free words.

*Keywords:* Subword order, repetition-free words, infix order, definability, automorphism, least fixed point, first-order theory, biinterpretability

*Joint work of:* Ludmila Yartseva, Oleg Kudinov and Victor Selivanov