Resource lambda-calculus: the differential viewpoint*

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Abstract

Milner's π calculus features a clear dichotomy between replicable and non-replicable resources, very much in the spirit of Linear Logic (LL). Analyzing Milner's encoding of the lazy λ-calculus in the π-calculus, Boudol introduced the λ-*calculus with resources* [1, 2] where functions can be applied to bags made of replicable and non-replicable arguments. This refinement of the syntax required to stick to a lazy reduction strategy implemented with explicit substitutions, used to postpone linear substitutions of non replicable resources.

Motivated by the discovery of denotational models of LL such as [3] where all morphisms of the associated cartesian closed category can be differentiated, we introduced in [4] the *differential* λ-*calculus* which features two ways of applying a function to an argument: the standard one and a linear one, implementing differentiation. This approach allows to generalize Boudol's idea: linear β-reduction – understood now as differentiation – can be performed everywhere in terms, and explicit substitutions are not needed anymore. Moreover, these differential operations fit very well in the LL framework: differentiation appears as a logical rule dual to dereliction, and the nice par/tensor symmetry of multiplicative LL extends to the exponentials, see [5]. In the conclusion of [6], Girard contemplated the possibility of introducing differential ideas in LL. There were probably very good reasons for not doing so at this early stage, since differential constructs are incompatible with both determinism and totality.

We present differential linear logic and its models, the associated resource and differential λ-calculi, and the Taylor expansion of promotion boxes. We also describe an *antiderivative* which seems to be available in many models of differential LL, and we present a very simple categorical axiom for this operation.

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