A Case Study on Optimizing Toll Enforcements on Motorways*

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Abstract

In this paper we present the problem of computing optimal tours of toll inspectors on German motorways. This problem is a special type of vehicle routing problem and builds up an integrated model, consisting of a tour planning and a duty rostering part. The tours should guarantee a network-wide control whose intensity is proportional to given spatial and time dependent traffic distributions. We model this using a space-time network and formulate the associated optimization problem by an integer program (IP). Since sequential approaches fail, we integrated the assignment of crews to the tours in our model. In this process all duties of a crew member must fit in a feasible roster. It is modeled as a Multi-Commodity Flow Problem in a directed acyclic graph, where specific paths correspond to feasible rosters for one month. We present computational results in a case-study on a German subnetwork which documents the practicability of our approach.

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1 Introduction

The Vehicle Routing Problem (VRP) is an extensively studied optimization problem with a lot of variants and very different solution approaches, see [8, 4] for an overview. The core is always to determine a set of tours to execute given tasks. In this paper we will present a model to set up tours as well, but under some unusual settings and assumptions that lead to another variant of vehicle routing problems.

We address the problem of computing tours for toll control inspectors on motorways. In 2005 Germany introduced a distance-based toll on motorways for commercial trucks with a weight of at least 12 tonnes. The enforcement of the toll is the responsibility of the German Federal Office for Goods Transport (BAG). It is implemented by a combination of an automatic enforcement by stationary control gantries and by random tours of mobile control teams. There are about 300 control teams distributed over the entire network. The teams consist mostly of two inspectors, but in some cases of only one. Each team can only control highway sections in its associated *control area*, close to the depot. Germany is subdivided into 21 control areas. Our approach could also be applied to other countries and toll systems if they use mobile control tours. Furthermore there must be central databases that provide on-demand information on drivers, if they have paid tolls or not.

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Our challenge is to solve the VRP for the mobile teams. The tours should guarantee a network-wide control whose intensity is proportional to given spatial and time dependent traffic distributions. Similar to classical vehicle routing problems we have a *length restriction* for all tours according to daily working time limitations. We model this problem using a space-time network and formulate an associated optimization problem as an Integer Program. A typical problem instance is to compute a monthly plan for one control area of the German network.

The paper is structured as follows. In Section 2 the *Toll Enforcement Problem (TEP)* [2] is introduced and distinguished from classical vehicle routing approaches. In the following section the graph and IP model for the TEP is introduced and the integration of the rostering part is presented. In Section 4 we explain our settings for the case study and the computational experiments. Finally, in Section 5 the results are discussed and some directions for future research are provided.

2 Optimal Toll Enforcement

In contrast to most of the Vehicle Routing problems, where a set of given demands or tasks has to be met, in the TEP this is different. Since the number of teams is fixed, the goal here is to control as efficiently as possible with the available personnel. If we assign a profit value to each section that could be covered by a tour, then our problem is related to a *vehicle routing problem with profits* or a *prize-collecting vehicle routing problem*. In the case of only one vehicle this is known as the prize-collecting TSP (or TSP with profits). There are only a few applications for the case of several vehicles, see Feillet et al. [5] for a literature survey. A suitable approach to prize collecting in our setting is to set the profit to the number of trucks that pass through a motorway section during a predefined time interval. This has the effect to reward the controls on highly utilized sections. Furthermore, the profit values differ during different time intervals. For example, the section with the highest profit might not be the same during the rush hour and during the night. Hence, not only the sections of a tour must be determined, but also the starting time and the duration of a section control.

A second difference is with respect to driver assignments. Naturally vehicle routing problems result in a set of tours. Drivers are assigned to the tours in a subsequent step. The feasibility of crew assignments is not part of the algorithms to solve the classical models. But in the toll control setting it is not possible to ignore the availability of crews. There are only a few drivers that can perform a planned tour since each tour must start and end at the home depot of its associated team. Thus, sequential approaches to plan the tours independently of the crews will fail.

If we assign a crew to each tour, it must fit within a feasible crew roster, respecting all legal rules, over a time horizon of several weeks. Minimum rest times, maximal amounts of consecutive working days, and labor time regulations must be satisfied. Hence, a personalized duty roster planning must be used in our application. Therefore, we developed a novel integrated approach, that leads to a new type of vehicle routing problems. To the best knowledge of the authors there is no optimization approach to toll enforcement in the literature yet. Related publications deal with problems such as tax evasion or ticket evasion in public transport; they mainly discuss the expected behavior of evaders or payers from a theoretical point of view, e.g. [1], or optimal levels of inspection, see [3].

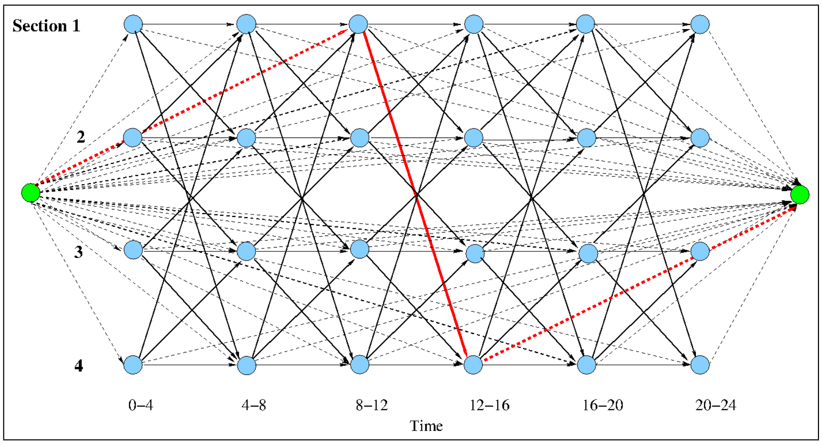

Figure 1 Construction of the tour planning graph.

3 A Graph Model for the Planning of Inspector Tours

The TEP can be described in terms of a *section graph* $G = (S, N)$. The nodes $s \in S$ correspond to so-called *control sections*, that is, sub-parts of the network with a length of approximately 50 km. The edges $n \in N$ of G connect two section nodes, if they have at least one motorway junction or motorway exit in common. The temporal dimension of our model involves a given planning horizon T, e.g., one month, and some time discretization Δ, e.g., two hours. According to the time discretization, we extend G to a space-time digraph $D = (V, A)$, the *tour planning graph.* Its nodes $v \in V$ are either defined as a pair of a section and a point in time, i.e., $v = (s,t) \in S \times [0, T]$, or they represent start and end nodes d_s and d_t for the vehicle paths (depot nodes). Directed arcs connect either adjacent time intervals for the same section, i.e., $a = ((s, t_1), (s, t_2))$ with $t_2 = t_1 + \Delta$, starting at $t_1 = 0$ until $t_2 = T$, or they connect adjacent sections, i.e., $(s_1, s_2) \in N$ implies $((s_1, t_i), (s_2, t_{i+1})) \in A \,\forall t_i \in \{0, \Delta, \ldots, T - \Delta\}$. In addition, arcs that connect the start depot node with all other non-depot nodes and all non-depot nodes with the end depot node model the beginning and the end of a tour. Figure 1 illustrates this construction by a network with four sections and a time discretization of $\Delta = 4$ hours. One drawback of this approach is that the size of D depends on Δ. With $\Delta = 4$ a feasible tour consists in controlling two sections, e.g., see the red path in Figure 1. In practice, there is always a break for the drivers after the first half of the sections in the tour is controlled. Hence, the time and location of the break does not need to be modeled explicitly in D.

A profit value and a length is associated with each arc $a \in \delta^+(v), v \in V$ to collect the profit for visiting v during a control tour. We consider the problem of finding a (d_s, d_t)-path in D for each vehicle f on each day that respects the restriction of a maximum tour length. This is called the *Tour Planning Problem (TPP).* We model the TPP as a classical 0/1 Multi-commodity flow problem [7] in D.

Let P be set of all paths in D, that represent feasible control tours and $P_{f,j} \subset P$ the set of all paths that are feasible for vehicle $f \in F$ and start at day $j \in J$. In addition for a section $s \in S$, the set of all paths $p \in P$ that visit a node $v = (s, t_i) \in V$ is denoted by P_s. By κ_s the minimum control quota, i.e., the minimum number of control visits on section s during the planning horizon, is indicated. We introduce binary variables z_p, $p \in P$, to decide that a tour is chosen or not. Then the following IP solves the TPP:

$$\max \sum_{p \in P} w_p z_p \tag{1}$$

$$\sum_{p \in P_{f,j}} z_p \leq 1, \qquad \forall (f,j) \in F \times J \tag{2}$$

$$\sum_{p \in P_s} z_p \geq \kappa_s, \qquad \forall s \in S \tag{3}$$

$$z_p \in \{0,1\}, \quad \forall p \in P. \tag{4}$$

In the objective function (1) the profit of the selected tours is maximized. Constraints (2) guarantee that each vehicle performs at most one tour per day. Constraints (3) requires that at least κ_s paths, that traverse section s, are chosen in a feasible solution. Finally constraints (4) demand the path variables being binary.

3.1 Integration of Duty Roster Planning

The second task in the TEP is the planning of the rosters, called the *Inspector Rostering Problem (IRP)*. There, the objective is to minimize the total costs. In contrast to many other duty scheduling and rostering approaches the goal in this setting is not to minimize crew costs. In the IRP the costs penalize some feasible but inappropriate sequences of duties, see Section 4 for examples. This is more related to the criterion of driver friendliness.

We formulate the IRP again as a Multi-Commodity flow problem in a directed graph $\tilde{D} = (\tilde{V} = (\hat{V} \cup \{s,t\}), \tilde{A})$ with two artificial start and end nodes s,t. The nodes $\hat{v} \in \hat{V}$ represent duties as a pair of day and time interval. The arcs $(u,\hat{v}) \in \tilde{A} \subseteq \tilde{V} \times \tilde{V}$ model a feasible sequence of two duties according to legal rules.

Let M be the set of all inspectors and b_m the start value of the time account of m. For each month there is a regular working time for each inspector. This needs not to be met exactly, but there is a feasible interval for the nominal value of the working time account. Hence, by l_m we denote the lower bound for the nominal value of inspector m and by u_m the upper bound, respectively. In addition, let t_v be the duration of duty $v \in \hat{V}$. The costs of a direct sequence of duties u and $v \in \hat{V}$ in a roster are indicated by $c_{u,v}$. A variable $x^m_{u,v}$ for each arc (u,v) and inspector m is introduced. According to that we propose the following integer programming formulation for the IRP:

$$\min \sum_{m \in M} \sum_{(u,v) \in \tilde{A}} c_{u,v} x^m_{u,v} \tag{5}$$

$$\sum_{v} x^m_{s,v} = 1, \qquad \forall m \in M \tag{6}$$

$$\sum_{k} x^m_{v,k} - \sum_{u} x^m_{u,v} = 0, \qquad \forall v \in \hat{V}, m \in M \tag{7}$$

$$b_m + \sum_{u \in \hat{V}} \sum_{v} t_u x^m_{u,v} \leq u_m, \qquad \forall m \in M \tag{8}$$

$$b_m + \sum_{u \in \hat{V}} \sum_{v} t_u x^m_{u,v} \geq l_m, \qquad \forall m \in M \tag{9}$$

$$x^m_{u,v} \in \{0,1\}, \quad \forall (u,v) \in \tilde{A}, m \in M. \tag{10}$$

As already mentioned, in the objective function (5) the cost is minimized. By Constraints (6) we assure that exactly one arc per inspector with a non-zero flow value is leaving depot s. The resulting path of all non-zero flow arcs for an inspector is called *Inspector Roster Path*. The flow conservation in the non-depot nodes is expressed by constraints (7). The inequalities (8) and (9) enforce for each inspector that the planned roster does not exceed the interval for the nominal value of the working time account. In the last constraint (10) the flow variables are restricted to be binary. In this model the use of arcs variables allows to handle small and medium size instances, i.e., instances that have up to 160000 flow variables in the rostering part.

Finally a formulation for the TEP is derived, by combining the TPP and the IRP by so-called *coupling constraints*. To this end, by $P_{f,v}$ we define the set of all control paths feasible for vehicle f and duty $v \in \hat{V}$. In addition, the parameter n_f gives the number of inspectors in vehicle f and $m \in f$ denotes, that inspector m uses vehicle f in a fixed assignment. This leads to the following equation:

$$\sum_{p \in P_{f,u}} n_f z_p - \sum_{m \in f} \sum_{v} x^m_{u,v} = 0 \quad \forall f \in F, u \in \hat{V} \tag{11}$$

Each control path p belongs to a predefined time interval. Hence, by (11) it is guaranteed that for each control path p in D all inspectors in the corresponding team have a feasible roster path, where a duty in the time horizon of p is scheduled.

The objective function of the TEP is therefore a combination of collecting the profit (1) and minimizing the cost (5). In practice, we maximize a linear combination of these two objectives. A coefficient is used to set the proportion of the rostering costs in the integrated model. We have observed that for several instances, the solution which maximizes the profit (1) contains no penalized duty sequence arcs (i.e., (5) is at its minimum). More details about the rostering costs will be discussed in the next sections.

4 Case Study – Instances and Settings

We have implemented the above described model in an optimization tool, called TC-OPT. We tested TC-OPT on some real world instances from one control area with about 20 inspectors. We used a set of standard (legal) rules, like minimum rest times, working time regulations and some other constraints mentioned in the sections above. In addition, manually generated reference plans from this control area are given. This allows a comparison of our novel approach with plans that are representative for the current manual planning of the control tours.

We selected six instances, three for August 2011 (aug1, aug2, aug3) and three for October 2011 (oct1, oct2, oct3). Table 1 distinguishes the instances according to several criteria. The column “mincontrol for all sections” indicates whether we used the minimum control quota constraint, see eq. (3), for all sections (case “yes”) or if some sections can be omitted during control (case “no”). The fourth column gives information about so called “rotation penalties” used in our model. This relates to the artificial costs we introduced in Section 3.1. A sequence of two duties d_1 and d_2 of an inspector from day t to day $t+1$ is called a *rotation*, if the starting-time of d_2 is different from d_1. If it starts later, e.g., from Mo 8-17 to Tu 10-19, we call this a *forward rotation*. If the second duty begins earlier, e.g., Mo 8-17 to Tu 6-15, it is a *backward rotation*.

Table 1 Overview on general settings for the test instances. All other parameter and data, e.g., inspectors, teams, sections or holidays, are the same for all instances. If the data depends on the selected month, it is the same in all instances belonging to the same month.

instance	Δ	mincontrol for all sections	rotation penalty	traffic data from
aug1	4	yes	moderate	last month
aug2	2	yes	moderate	last month
aug3	4	no	moderate	last month
oct1	4	yes	moderate	last month
oct2	4	yes	strong	last month
oct3	4	yes	moderate	last year

It is legal to use rotations in a duty roster, if the minimum rest time between the end of a duty and the beginning of the next is not less then 11h. Beside of that, is it an important goal to avoid or to minimize rotations in a roster. Because it is more employee-friendly if subsequent duties start always on the same time and if changing to another start time occurs only after some days off. According to that we integrated *rotation (penalty) factors* in our model which must be chosen in relation to the profit of the control tours. The value "strong" means that the penalty factors are higher then the profits of all tours while for "moderate" this holds only for a majority of the tours but not for all. In general the factor of the backward rotation should be higher, since this strongly infects the length of rest times between duties. The last column indicates the period, from where we took the profit values in the objective function of the TPP (1). All data depending on the selected month, like holidays, fixed duties or working time accounts, are same in all of the three instances belonging to the same month. All other data, e.g. team assignments or the selection of sections, are the same for all instances.

Another important aspect of the control planning is that a control may not start at any time. There are given time intervals when the tours can take place. We call them *working time windows*. For our test setting we used six different time windows, two starting in the morning, one mid-day interval and three that start in the afternoon or in the evening. A major constraint in our model is that a certain duty mix is maintained. Therefore, for each time window there is a minimal and maximal contingent of all duties, e. g., the duties from 6am to 3pm must be at least 20% of all duties and at most 50%. The main significance of these constraints is to define upper bounds on the number of late and night duties.

5 Case Study - Results and Discussion

We were able to solve all instances with a proven optimality. There is no optimality gap with more than 10%. Hence, for all instances we received a feasible control plan. Before the solution behavior is dicussed in detail in Section 5.1 the quality of the optimized plans is analysed first. Comparing the manual and the optimized plan we see several benefits in using TC-OPT. It is easier to handle the balance of the working hours, see eq. (8) and (9), especially in the case of different working hours in a team. The second is that we could comply with the duty mix constraints, which is more difficult in manual planning. Furthermore, it was possible to prove the benefit in introducing the rotation factors. Comparing instance oct2 to oct1 we were able to reduce the number of rotation when increasing the factor. For some instances, e.g., aug1, even a low factor suffices to avoid rotations between two scheduled duties.

In Figure 2 the distribution of the controls on the 15 sections of the control area is shown.

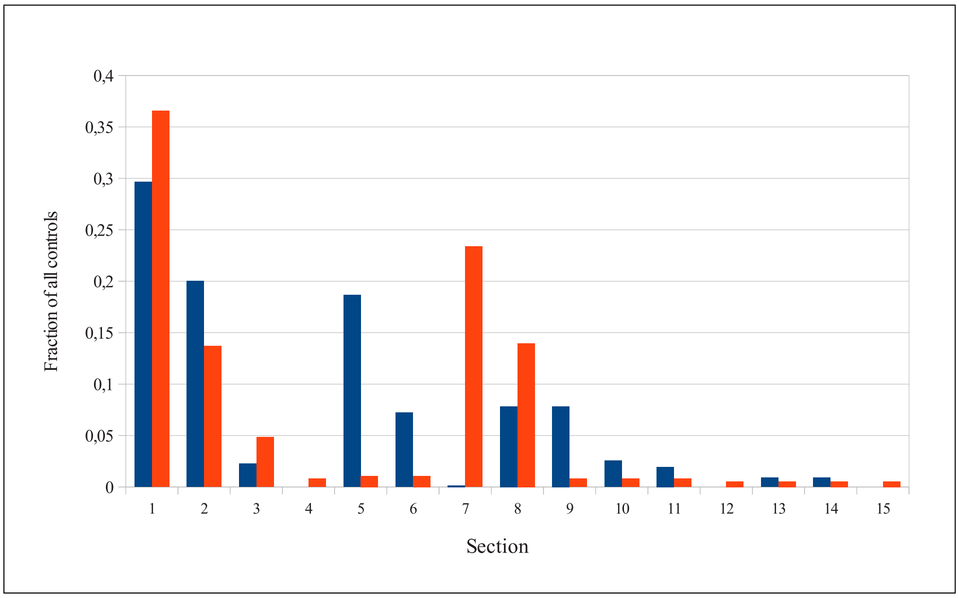

Figure 2 Control distribution over all sections in percent for instance aug1, the left blue columns show the reference plan, while the right columns in red represent the TC-OPT solution.

The main difference between the optimized and the reference plan is that the latter controls a lot on sections five and six, while the optimized plan has a focus on sections seven and eight. The difference originates from the observation, that there is significantly more traffic on sections seven and eight than on five and six. The same holds for sections one and two, where TC-OPT controls the first section twice as much as the second one. So the objective function clearly tends towards the sections with the most traffic. The sections with very low traffic, like 12 or 15, were only controlled by the required minimum quota. We can conclude, that the control is mostly planned according to the traffic distribution. The use of minimum control quota constraints (3) achieves a better control coverage of the whole network compared to the reference plans.

Choosing the time discretization to two hours, as in aug2, the control distribution along the sections is quite similar to the four-hour case. The only difference is a slightly higher part of the control on some low traffic sections. This originates from the possibility to control up to four sections during a tour in the two-hour case. As a result, one can control high traffic sections as well as low traffic sections in a common tour.

Another important aspect is the control distribution according to different days in a week. The main focus of the control is between Monday and Friday according to the fact that there is much less traffic at the weekend. One reason for that is the Sunday truck ban on German motorways that tolerates only small exceptions, e.g., for some urgent food transports.

Beside of the distribution over the weekdays it is interesting to study the distribution during a day, i.e., a daily control pattern. It is important to mention that those values heavily depend on the chosen duty mix constraints. Usually labour agreements restrict the number of late and night shifts. Those regulations have a major impact on the mix constraints and thereby also on the daily control pattern. According to that, our optimization tool allows the planners to predefine the intervals for the duty mix constraints. This may have the side effect that the distribution can differ a lot between different areas.

Figure 3 shows a comparison of the optimized plan (in red) and the reference plan (in

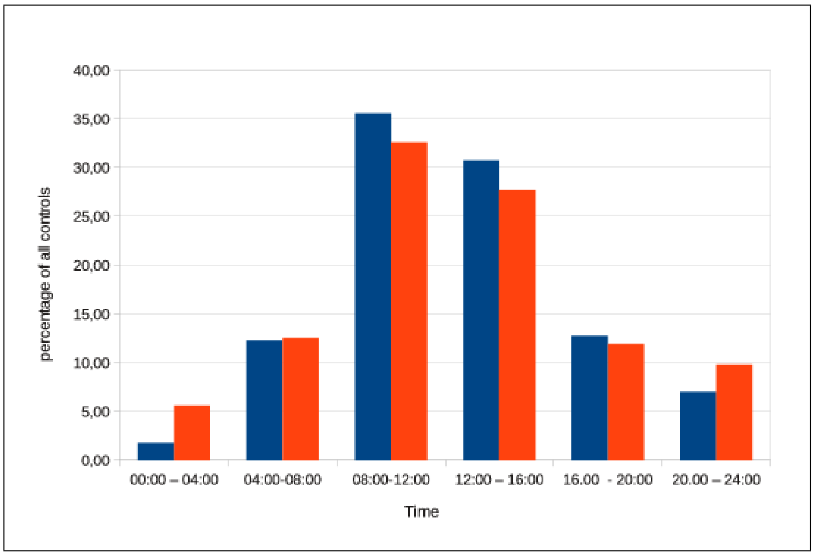

Figure 3 Control distribution of instance aug1 for different time intervals across a day in percent. The blue columns show the reference plan, while the red represent a solution by TC-OPT.

blue) according to different time intervals across a day. The values are summarized over all days of the planning period and depend on the exemplary chosen duty mix setting of our test. As one can see, the part of control during late evening and night is higher in the TC-OPT result. This follows from the observation that on some motorway sections, in particular on central network axis, the truck traffic during the night is not much less comparing to the traffic during the day. Consequently, the profit value for a control on some highly utilized sections at night is higher than the value for a control on a low traffic section during the day. Hence, TC-OPT tries to schedule more night duties.

5.1 IP Solution Analysis

After discussing the quality of the solution regarding to several important aspects in practice, we analyse the solution behavior of our algorithmic approach. All computations were done on a Blade Server with an 8-core Intel Xeon CPU with 3.2 GHz and SUSE Linux 11.4 as operating system. The memory limit for the solution tree was 40 GB. Furthermore, there was a time limit of 2 days (= 172800 seconds) for each instance. As an IP Solver CPLEX 12.3 [6] by IBM with the default parameter setting was applied by using up to eight threads. Table 2 presents all relevant data for the solution analysis. The second and third column display the number of rows and columns of the IPs. As mentioned before, the chosen time discretization has a huge influence of the size of the IP, especially on the number of columns. The next column shows the root LP value, v(lp), i.e., the value of the linear relaxation of the IPs. This value is the first dual bound value during the solution process. Since our problem is a maximization problem, the dual bound value is an upper bound on the optimal solution. The fifth column gives the dual bound value b^* at the end of the solution process. In the sixth column the best primal bound v^* is denoted, which is the best integer value or in other words the best solution found. The next columns display the gap between primal and dual bound, the time, when the first solution was found and the overall solution time,

Table 2 IP-Solution analysis for all test set instances. The value of the root LP is denoted by v(lp) and the best integer solution by v^*. By b^* we name the value of the dual bound at the end of the optimization run. The time limit for the optimization run equals $2days = 48h = 172800sec$. The column "time 1st sol." indicates when the first primal solution was found while "time(ip)" gives the overall solution time.

instance	#rows	#columns	v(lp)	b^*	v^*	gap(%)	time 1st sol.[sec.]	time(ip) [sec.]
aug1	11424	198179	569790.98	524510.24	523232.20	0.24	3900	172800.00
aug2	14574	771893	562176.71	519017.06	515664.10	0.65	14700	172800.00
aug3	11424	198179	570545.77	526971.61	525560.03	0.27	2400	172800.00
oct1	11799	211680	615269.50	570377.87	535365.49	6.54	6240	170474.0[1]
oct2	11799	211680	615269.50	570517.72	520215.84	9.67	6050	139514.0[1]
oct3	11799	211680	582188.86	542402.84	511003.29	6.14	7900	147333.3[1]

i.e., time(ip). The gap is computed by $(b^* - v^*)/v^*$.

The most important result is that we were able to compute feasible solutions for all instances with a gap of at least 10%. The gap for all August-instances was even less than 1% at all, even for the huge 2-hour-discretization instance. This encouraging result is enhanced by the fact that for each instance a first feasible solution could be found during the first two and a half hours of the solution time. In addition, the initial gap, i.e., the gap after the first solution, is less then 9% for all august-Instances and less than 21% for all october-instances. An interesting observation is that the instance with the biggest number of rows and columns, aug2, is not the one with the highest solution gap. All October-instances were much more difficult to solve than the August-instances. The reason for this lies in the duty mix constraints that were much more tighter in the October-instances. This makes it more difficult to find feasible solutions that satisfy the small feasibility intervals for the time windows.

Beside of that the October-instances lead to an huge Branch & Bound tree that even exceeds the very high amount of memory. On the one hand, this is a point, where our models should be improved in the future, but on the other hand the wasteful memory consumption relates partly to the before-mentioned duty mix requirement that is chosen too strict.

5.2 Conclusion and Future Research

We presented the first model of optimizing the tours of toll control inspectors on motorways. This problem was derived as a special Vehicle Routing Problem with profits where also the crew scheduling has to be taken into account. Therefore, our model consists of two parts. The first one, the Tour Planning Problem, is to plan tours of inspectors in the network. The second part, the Inspector Rostering Problem, builds up a feasible duty roster of each inspector. For both problems an appropriate graph model was presented. We formulated the two as Multi-Commodity Flow Problems in their planning graphs. Each of the problem is formulated by an IP, and by coupling constraints they are integrated into one common formulation. The model is implemented in a tool, named TC-OPT.

The main issue of this paper was the presentation of a case-study on the TEP. We optimized several duty plans of two different planning horizons for one exemplary chosen

[1] Memory-Limit reached

control area. A set of standard legal rules was integrated in our model. We were able to solve all six instances with only a small optimality gap within two days. Furthermore, a comparison with a reference plan, that represents different aspects of traditional planning approaches, showed that TC-OPT could support the planners. It was able to improve the quality of the optimized plan in many ways. Another important aspect of our case study is the duration of the solution process. According to the usual planning horizon a duty plan has to be computed only once in a month. Hence, our time limit of two days is reasonable from a practical point of view. In urgent cases the time limit can be significantly reduced since the first solution is found after a few hours in most cases.

Hence, it can be concluded that we are on the right way to get an optimization tool that satisfies all requirements of the toll control planning problem. An important aspect in our future research is to be able to compute feasible plans for all control areas, with different settings in a reasonable time by moderate hardware requirements. We will test some impacts in our model to get smaller computation times, like problem-dependent reduction techniques on the sizes of the planning graphs or variations on several parameters and strategies of the MIP solution process. Also additional rules, that are not legally defined, but very common in practice, will be integrated in our model. A typical example is a fairer planning of late and night shifts.

References

1 M.G. Allingham and A. Sandmo. Income Tax Evasion: a Theoretical Analysis. *Journal of Public Economics*, 1(3-4):323–338, 1972.

2 R. Borndörfer, G. Sagnol, and E. Swarat. An IP Approach to Toll Enforcement Optimization on German Motorways. Technical Report ZIB Report 11-42, Zuse Institute Berlin, Takustr. 7, 14195 Berlin, Germany, 2011.

3 C. Boyd, C. Martini, J. Rickard, and A. Russell. Fare Evasion and Non-Compliance: a Simple Model. *Journal of Transport Economics and Policy*, 23(2):189–197, 1989.

4 J.-F. Cordeau, G. Laporte, M. W.P. Savelsbergh, and D. Vigo. Vehicle Routing. In C. Barnhart and G. Laporte, editors, *Handbooks in Operations Research and Management Science, Transportation*, volume 14, chapter 6, pages 367–428. Elsevier, 2007.

5 D. Feillet, P. Dejax, and M. Gendreau. Traveling Salesman Problems with Profits. *Transportation Science*, 39(2):188–205, 2005.

6 IBM ILOG CPLEX, IBM Software Group. *User-Manual CPLEX 12.3*, 2011.

7 R. M. Karp. On the Computational Complexity of Combinatorial Problems. *Networks*, 5(1):45–68, 1975.

8 P. Toth and D. Vigo. *The Vehicle Routing Problem.* Society for Industrial and Applied Mathematics, Philadelphia, USA, 2002.