The design of transportation networks: a multi objective model combining equity, efficiency and efficacy

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Abstract

A network design problem consists in locating facilities (nodes and arcs) that enable the transfer of flows (passengers and/or goods) from given origin-destination pairs. The topic can have several applications within transportation and logistics contexts. In this work we propose a multi-objective model in which balancing or equity aspects, i.e. measures of the distribution of distances of users from the path, are considered. These kinds of models can be used when there is the need to balance risks or benefits among all the potential users deriving from the location of the path to be designed. The application of the proposed model to a benchmark problem used in the literature to test these kinds of models, shows that it is able to find solutions characterized by significant level of equity but also of efficiency and efficacy.

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1 Introduction

A location problem consists in the positioning of a set of facilities within a given space. The decision is made on the basis of an objective function, which can concern the minimization of costs or the maximization of benefits.
If the facilities have an extension such that the representation through points is ineffective we deal with a network design problem. For example communication systems, public transport and energy distribution require appropriate networks for their representation.
A network $G(N, A)$ consists of a set of nodes, $N = 1, ..., n$, and a set of arcs $A = ((i, j) : i, j \in N)$. At each arc there is usually associated a cost while at each node can be associated a demand service.
A network design problem requires the definition of a subset of arcs to be inserted in a solution in order to optimize an objective function subject to a set of constraints. In particular topological constraints which are often included in the problem formulation require that the solution presents given topological characteristics (path, tree, cycle, network).
The objective function can be related to an efficiency measure (i.e. total network cost) or to an efficacy measure concerning demand satisfaction aspects (i.e. cost, accessibility)[6], [8]. Many models consider the simultaneous presence of more objectives (multicriteria network design problem [1], [2], [3]).
In addition to the mentioned criteria, in many applications the need to obtain solutions related to the concept of equity occurs. This means that in the network design one could

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search for solutions in which some parameters (cost and/or benefits) are distributed as evenly as possible among the potential users.
The importance of taking into account such aspects derives from various considerations [7]. In general if users perceive a substantial equity in the treatment of the fruition of a service, they are more satisfied. In addition when facilities are considered "undesirable", an equitable distribution of the risk and/or disadvantage due to their locations can reduce the conflicts among users and can help in accepting possible solutions.
In this paper we propose a path location problem in which balancing aspects are explicitly considered. The remainder of the paper is organized as follows. In the following section we illustrate the main path location models proposed in the literature. Then we propose a formulation of a new model. Computational experiments are then performed, in order to analyze solutions provided by the proposed model; finally, some conclusions and directions for further researches are drawn.

2 Path Location Problem

Let $G = (N, A)$ a network of potential arcs which can be included in a solution and $(O, D) \in N$ a pair of nodes. A path location problem consists of selecting a subset of arcs, according to some criteria and defining a path from the origin O to the destination D.
In the literature different path location models have been proposed. The Maximum Coverage Shortest Path Problem (MCSP) [4] was formulated as a problem with two objectives. It was assumed that there is a demand for each node and that this demand is covered by a path if the path passes to some node located within a given distance (threshold).
The first objective is to identify the shortest (or minimum cost) path while the second objective is to maximize the total demand covered by the path. These two objectives are usually conflicting as in general when we increase the length of the path, the covered demand also increases.
The Maximum Population Shortest Path Problem (MPSP) [5] is a special case of the MCSP problem, where the threshold is zero (i.e. the demand is satisfied at a node if that node belongs to the path). Variants of MCSP can include constraints on the distances between any node not belonging to the path and the path itself (mandatory closeness constraints).
The Median Shortest Path Problem (MSPP) [5] is another bi-criteria path location model where the second objective is oriented to maximize the "accessibility" to the path. This can be measured as the sum of the distances from any node to the closest node of the path. In practice the objective aims at minimizing the average cost to reach the path.
We introduce the following notation:
N_i = set of nodes j such that the arc (i, j) exists
M_j =set of nodes i such that the arc (i, j) exists
P_i = set of nodes j such that the path from i to j exists
w_i = demand associated to the node i
d_{ij}= distance between node i and j
T_{ij} = the length of the shortest path connecting node i to node j
Q = a non empty subset of N
$|Q|$ = the cardinality of subset Q

$$X_{ij} = \begin{cases} 1 & \text{if a direct arc between the node } i \text{ and the node } j \text{ exists} \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{ij} = \begin{cases} 1 & \text{if node } i \text{ is assigned to node } j \\ 0 & \text{otherwise} \end{cases}$$

The MSPP can be formulated as:

$$\min Z = (Z_1, Z_2) = (\sum_i \sum_j d_{ij} X_{ij}, \sum_i \sum_j w_i T_{ij} Y_{ij}) \tag{1}$$

$$\text{s.t.} \quad \sum_{j \in N_O} X_{Oj} = 1 \tag{2}$$

$$\sum_{i \in N_D} X_{iD} = 1 \tag{3}$$

$$\sum_{i \in M_j} X_{ij} - \sum_{k \in N_j} X_{jk} = 0 \qquad \forall j \in N, j \neq O, j \neq D \tag{4}$$

$$\sum_{j \in N_i} X_{ij} + \sum_{j \in P_i} Y_{ij} = 1 \qquad \forall i \in N, i \neq O, i \neq D \tag{5}$$

$$Y_{ij} - \sum_{i \in M_j} X_{ij} \leq 0 \qquad \forall (i,j) \in A \tag{6}$$

$$\sum_{i \in Q} \sum_{j \in Q} X_{ij} \leq |Q| - 1 \qquad \forall Q \subseteq N, |Q| \geq 2 \tag{7}$$

$$X_{ij} \in \{0,1\} \qquad \forall (i,j) \in A \tag{8}$$

$$Y_{ij} \in \{0,1\} \qquad \forall (i,j) \in A \tag{9}$$

The objective function (1) is composed by two objectives. The first one (Z_1) measures the length (cost) of the path, while the function Z_2 measures the accessibility for each node. The constraints (2) and (3) assure, respectively, that the origin node and the destination node are on the median shortest path. The constraints set (4) states that demand of arcs entering and leaving a node is equal. The set of constraints (5) requires that each node is either in the path or is assigned to a node that is on that path. The set of constraints (6) prevents a node i, which is not on the path, to be assigned to a node j that does not belong to the path. The set of constraints (7) avoids the possible presence of cycles. The last constraints sets (8) and (9) ensure that the variables X_{ij} and Y_{ij} are binary.
Moreover in the Equity Constrained Shortest Path Problem, introduced in [9], [10] a path is considered feasible if the sum of the differences in cost between all pairs of nodes is less than a certain threshold value, called equity parameter. The introduction of equity for network design is also analyzed in [11].

3 The proposed model

We propose a version of the MSPP described above in order to include balancing aspects in the solution to be found. To this aim we introduce the parameter $r_k(i,j)$ defined as the risk or benefit perceived at node k due to the presence of the arc (i,j). We can then consider the total perceived cost or benefit at node k as the sum of the cost $r_k(i,j)$ due to any arc (i,j) belonging to a path P. We can assume that $r_k(i,j)$ depends on the distance from k to the closest node between i and j.
This way a balancing constraint can be defined as:

$$w_k \cdot (\sum_{(i,j) \in P} r_k(i,j)) - w_h \cdot (\sum_{(i,j) \in P} r_h(i,j)) \leq \mu \qquad \forall k, h \in N \tag{10}$$

where μ is an equity parameter representing the maximum cost or benefit difference between any pair of nodes k and h. Adding this constraints to the model (1) – (9) we could obtain a formulation that combines efficiency (the minimization of path length), efficacy

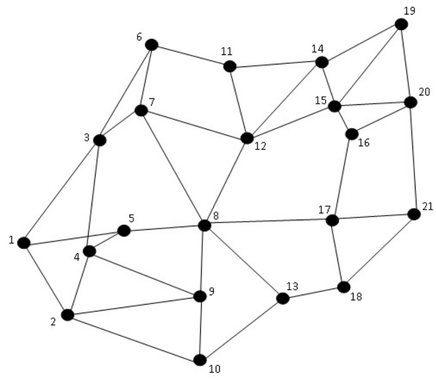

Figure 1 The used test problem.

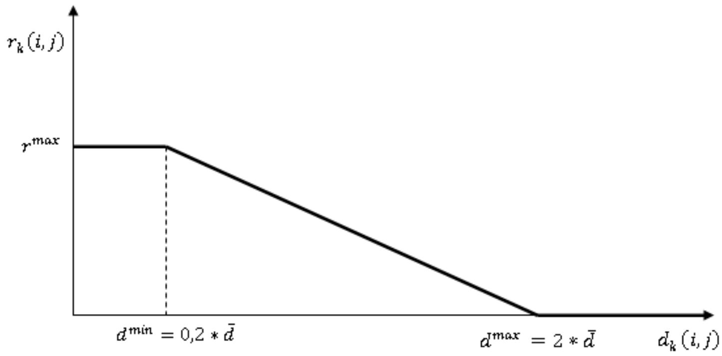

Figure 2 Expression of $r_k(i, j)$.

(the maximization of the accessibility), and the minimization of the disequity, i.e. better distribution of the cost or the benefit among the users.

4 Computational experiments

For testing our model we used a benchmark network introduced in [5] and represented by the graph in Figure 1 with 21 nodes and 39 arcs. Each node is associated a demand value and each arc is associated a length; finally, the distance $d(i, j)$ is associated to each pair of nodes (i, j).

Indicating with $d_k(i, j) = \min(d(i, k), d(j, k))$, in order to set the values of $r_k(i, j)$, we assumed that $r_k(i, j)$ is equal to a maximum value r^{max} if $d_k(i, j) < 0.2 * \overline{d}$ where $\overline{d}$ is the average of the distances; if $0.2 * \overline{d} < d_k(i, j) < 0.2 * d$ then $r_k(i, j)$ decreases linearly from r^{max} reaching 0 for $d_k(i, j) > 2 * \overline{d}$ (see Figure 2).

The arcs are divided into three categories corresponding to the values:

- $r^{max} = 3$;
- $r^{max} = 2$;
- $r^{max} = 1$.

A higher value of r^{max} indicates a bigger risk associated with that arc. In Figure 3 we report a representation of the three categories of arcs, identifying the different categories with different dotted lines.

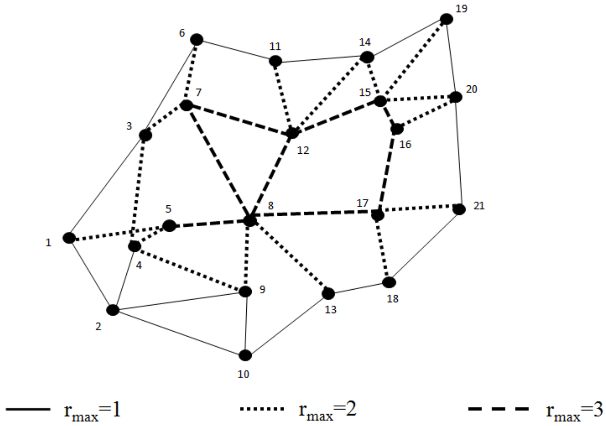

Figure 3 Arc categories.

In order to solve the model we consider the single-objective version of the MSPP model obtained by introducing a linear combination of the objective function $Z = (Z_1, Z_2)$ equal to:

$$Z' = \lambda \cdot Z_1 + (1 - \lambda) \cdot Z_2 \tag{11}$$

with λ included between 0 and 1. This way the optimal solution of the model (1) – (9), assuming Z is simple objective function with a fixed value of λ, is a Pareto solution for the model (1) – (9).

We solved the MSPP model and the proposed variant with the addition of constraints (10), using the software CPLEX 12.0. and throughout all the testing, we used a Pentium IV with 2.40 GHz and 4.00 GB of RAM running. The values of the equity parameter μ were fixed in order to assure that equity constraint permits to obtain different solutions from those provided by the model MSPP. For the considered instance we found the appropriate value of μ by iteratively solving the model in order to find the minimum value of μ $(1.8 * 10^{-6})$ in such a way that the model provides at least one feasible solution. Starting from this minimum value, μ is then increased with a step of $0.1 * 10^{-6}$ until constraints (10) are not active.

The computational times are very low; for all the analyzed instances we found the optimal solution in less than one minute.

In Figure 4 we report the path obtained for each value of the weight λ, from 0 to 1 with a step equal to 0.1. When $\lambda = 0.0$ as the objective function aims at minimizing the accessibility cost, the path visits each node of the graph (solution (a)). As λ increases, the number of visited nodes decreases until $\lambda \geq 0.5$ and the shortest path is obtained (solution (e)).

In Figure 5 we show the solutions provided by the proposed model with different values of λ assuming $\mu = 2.2 * 10^{-6}$. The obtained solutions do not correspond to those shown in Figure 4. In particular with $\lambda = 0.0$ the feasible solution which minimizes the accessibility cost (solution (f)) visits 10 of 21 nodes, with most of the arcs characterized by lower risk values. With values $\lambda = 0.1, 0.2, 0.3$ the solution is quite similar to those provided by the MSPP but with a selection of arcs with lower risk values. Finally with $\lambda \geq 0.4$ the solution is similar to the one founded by the MSPP with $\lambda \geq 0.5$.

In Figure 6 we report the value of Z' calculated through (11) for the two models varying both the parameters μ and λ.

We can highlight that the optimal solution of the MSPP for a given value of λ represents a

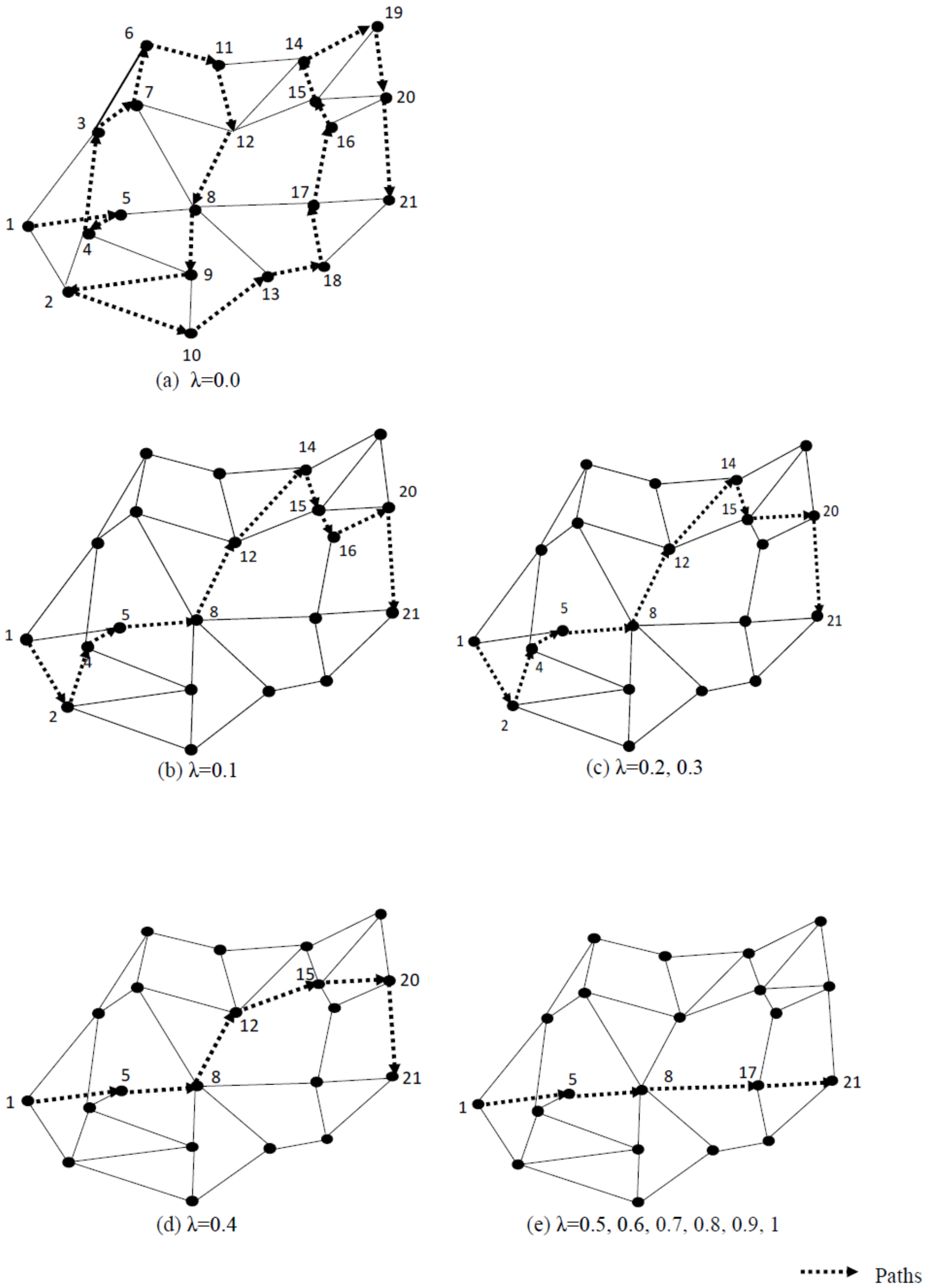

Figure 4 Solution for the MSPP by varying λ ($0 \leq \lambda \leq 1$) with step 0.1.

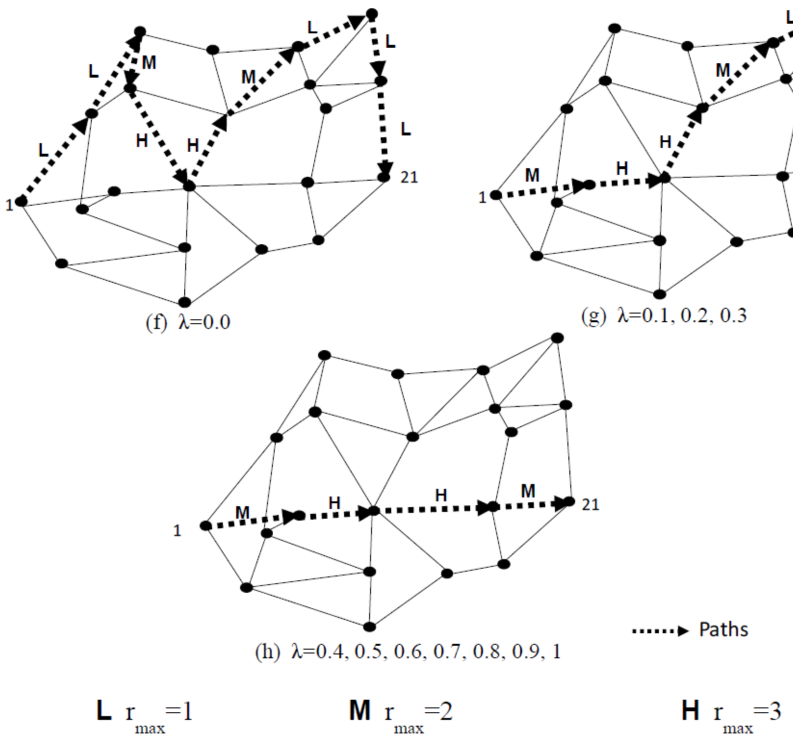

Figure 5 Solutions for the MSPP adding equity constraints with $\mu = 2.2 * 10^{-6}$, by varying λ ($0 \leq \lambda \leq 1$) with step 0.1.

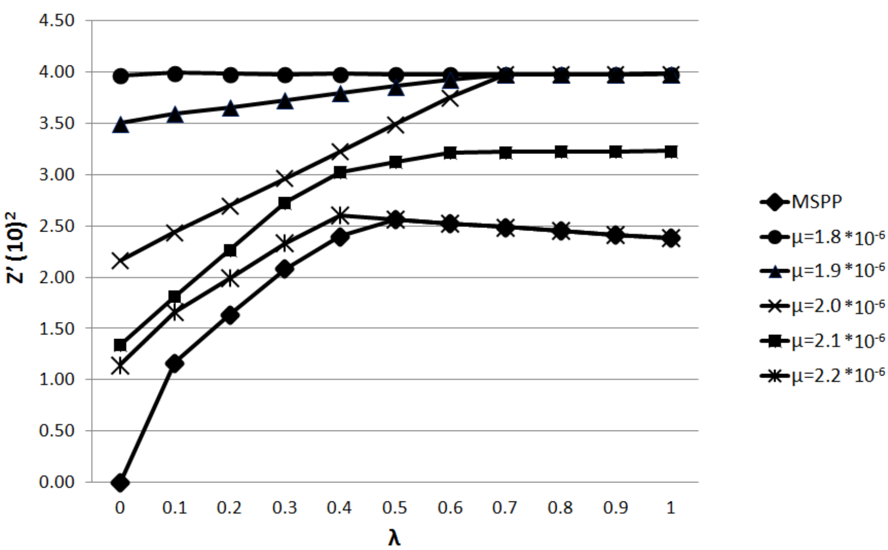

Figure 6 Values of Z' by varying μ and λ.

lower bound for Z when the equity constraint is introduced. When μ decreases we obtain solutions with significant difference from the lower bound due to the fact that the set of equity constraints is more active and then its presence reduces the set of feasible solutions. For example analyzing the obtained solution for $\lambda = 0.5$ for the lowest value of μ, and so for the highest level of equity, the solution is characterized by a higher value of $Z^{'}$. Increasing μ, and so decreasing the level of the equity, also the level of $Z^{'}$ decrease; so, the solution for intermediate values of μ are characterized by both level of equity and $Z^{'}$ (representing of the other two objectives), as wished. In addition, decrementing the level of equity the solution becomes more similar to those obtained by the MSPP model, until when $\mu \geq 2.3$ for which the solutions of the proposed model correspond with ones provided by the MSPP model for all the values of λ; in practice the constraint is not more effective.

5 Conclusion

In this work we analyzed the path location problems, taking into account the three main models in the literature: the Median Shortest Path Problem (MSPP), the Maximum Covering Shortest Path Problem (MCSPP) and the Equity Constrained Shortest Path problem. From the analysis of the literature we found the opportunity to include equity aspects in this context. For this reason we proposed a variant of the MSPP introducing balancing constraints. We formulated the model and we tested its effectiveness through experiments on a test problem. Comparing the solutions, it became evident the effect of the insertion of the equity constraints on the resulting paths; indeed we found paths with a higher level of equity and also with good values of the other two objectives. As further research we want to seek alternative methods of calculating the perceived risk from each node, in order to adapt the model to describe different applications.

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