Optimization of electricity trading using linear programming*

Minja R. Marinović[1], Dragana D. Makajić-Nikolić[1], Milan J. Stanojević[1], and Lena S. Đorđević[1]

1 **Faculty of organizational sciences, University of Belgrade**
Jove Ilića 154, Belgrade, Serbia
`marinovic75709@fon.bg.ac.rs, gis@fon.bg.ac.rs, milans@fon.bg.ac.rs, lena.djordjevic@fon.bg.ac.rs`

Abstract

In the last two decades, the liberalization of the electricity markets have been established in order to increase efficiency, harmonize and reduce electricity prices, make a better use of resources, give customers the right to choose their supplier and provide customers with a better service. This change made the electricity market competitive and introduced several new subjects. In this paper, we study one of these subjects: Electricity Trading Company (ETC) and its daily trading process. We present a linear mathematical model of total daily profit maximization subject to flow constraints. It is assumed that the demand and supply are known and some of them are arranged. Possible transmission capacities are known but also additional capacities can be purchased. All trading, transmission prices and amounts are subject of auctions. First, we present energy trading problem as directed multiple-source and multiple-sink network and then model it using linear programming. Also, we provide one realistic example which is slightly changed in order to save confidentiality of the given data.

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1 Introduction

Until the nineties, power plants power production and transmission were carried out between monopolistic public power companies. This situation changed since electricity markets have been deregulated allowing customers to choose their provider and producers. Today, we have a Single European electricity market which provides seamless competition within the electricity supply chain. All participants enjoy a wide choice between competing electricity retailers, who source their requirements in competitive wholesale markets. Market participants are trying to satisfy demand in their own countries and supply electricity across borders into neighborhood markets. Cross-border trading and supply is a part of this market. Energy Trading Companies (ETCs) are buying transmission capacity from Transmission System Operators (TSOs) [1]. All time, TSOs consistently release to the market a truly maximum amount of cross-border transmission capacity. We should mention that ETCs are operating in the middle stage of an energy supply chain [11] and they are trying to manage the risks associated with fluctuating prices through buying and selling electricity contracts. Both

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traders and end-users apply financial instruments such as futures, options and derivatives to protect their exposures to prices and to speculate on price fluctuations.

There are few ways of trading electricity but two main ways are via the telephone in bilateral transactions (so called "Over The Counter" or OTC, usually through the intermediation of a broker), or it is traded through futures markets such as Nordpool or EEX. Some key factors influencing energy prices include geopolitical factors, global economic growth, short term weather impacting demand, supply disruptions from maintenance or unexpected outages, fuel price movements and product swapping in response to relative prices [3].

The literature concerning different issues in energy trading and transmission is extensive. In [8] the authors analyze the impact of CO2 cost on power prices and test two different average cost pricing policies, regional and zonal, that have different effects on electricity market of Central and West Europe. In [12] multiple interrelated markets is considered for electricity and propose a multi-stage mixed-integer stochastic model for capacity allocation strategy in a multi-auction competitive market. A generalized network flow model of the national integrated energy system that incorporates production, transportation of coal, natural gas, and electricity storage with respect to the entire electric energy sector of the U.S. economy is proposed in [10]. The authors have formulated a multi period generalized flow problem in which the total cost is minimized subject to energy balance constraints. The problem of energy allocation between spot markets and bilateral contracts is formulated as a general portfolio optimization quadratic programming problem in [6]. The proposed methodology with risky assets can be applied to a market where pricing, either zonal or nodal, is adopted to mitigate transmission congestion. In [9] the authors propose a zonal network model, aggregating individual nodes within each zone into virtual equivalent nodes, and all cross-border lines into equivalent border links. Using flow-based modeling, the feasibility of the least granularity zonal model where the price zones are defined by the political borders, is analyzed. The authors in [4] consider network systems with multiple-source multiple-sink flow such as electric and power systems. They observe the problem in which resources are transmitted from resource-supplying nodes to resource-demanding nodes through unreliable flow networks. In [7] they analyze the simultaneous optimization of power production and day-ahead power trading and formulated it as a stochastic integer programming model.

In this paper, we consider day-ahead planning from the perspective of a decision maker in ETC. Decisions that should be made are: where and how much electricity ETC should buy and sell and on which ways that energy should be transferred in order to maximize total daily profit. We represent this problem by a directed multiple-source and multiple-sink network and model using linear programming. We illustrate this approach on an example of one ETC trading on Central and South-East Europe (CSEE).

The paper is organized as follows. Section 2 is devoted to a description of the main assumptions of the observed problem. In Section 3, we present a LP model for day-ahead planning. Then, in Section 4 we report and discuss numerical results to illustrate the model application. Also, we present the solution of the LP model with fixed parameters and then we investigate the impact of prices on trading capacities and amounts. Conclusions along with perspectives regarding further work are given in Section 5.

2 Problem description

In this paper we are focusing on electricity trading from the perspective of ETC. The main task of trading section of ETC is to ensure each client's demands are met whatever the circumstances. Trading section also enables ETC to respond to the ever-changing state of the

Figure 1 Simplified CSEE electricity market.

region's transmission grid and production capacities. Besides that, this section takes care of different problems such as: spot and long term arrangements, making schedules, cross border capacities allocations, optimization of whole portfolio, managing different energy sources, customers in different countries and cross border energy flows and costs.

Efficiency of trading section of ETC can be improved by dealing with at least two optimization problems: long term and short term (day-ahead) planning. Long term planning include determination of electricity market e.g. buyers and suppliers interested for cooperation and transmission capacity that will be purchased for the next period. Day-ahead planning represents finding the optimal plan of selling and buying electricity that will maximize daily profit considering the available transmission capacities.

Day-ahead planning starts from an established network of potential buyers and suppliers and purchased transmission capacities based on long term decisions. One example of simplified electricity network where all buyers and suppliers from one country are represented by one node is shown in Figure 1. Similar network will be used later in the numerical example.

Daily demand and daily supply of each country (node) are known. Some electricity trades are already arranged and they must be fulfilled. All the electricity that has been bought during one day has to be sold the same day. Therefore, if there is a surplus or shortage of arranged supply, it will be traded through future markets.

In order that transmission capacities purchased for a long period to be disposed by ETC, it is necessary to announce the amount that will be used. Only the amounts that are announced one day before day-ahead planning are considered available. The transmission capacity that is not announced is subject to the "use it or lose it" principle and will be reoffered in the daily auction [5]. If daily trading exceeds the amount of announced capacity, it is possible to buy additional daily transmission capacities at the auction price.

The goal is to create a useful tool which will help decision makers in trading section of ETC to simulate market and network situations for different amounts and prices of electricity and transmission capacities in order to help them to maximize ETC's total profit.

3 Model formulation

The described problem can be modeled as a directed multiple-source and multiple-sink network. In this section, we will present notation for this problem which will be used for the LP mathematical model for day-ahead planning. Sets N, S, B and A represent: set of all nodes, set of all nodes representing sellers ($S \subseteq N$), set of all nodes representing buyers ($B \subseteq N$) and set of arcs representing transmission capacity between nodes ($A \subset N \times N$), respectively. Parameter su_j is the upper bound of electricity that can be bought from supplier j, for every $j \in S$ while sl_j represents the lower bound (arranged buying) of electricity that must be bought from supplier $j \in s$. Also, the upper bound bu_i and the lower bound bl_i (arranged sale) of electricity that can be sold, are given for each buyer i, where $i \in B$. Parameter f_{ij} represents announced daily transmission capacity of every arc $(i, j) \in A$. The maximal additional daily transmission capacity which is possible to buy on an arc $(i, j) \in A$ is denoted by hu_{ij}.

Parameters c_j, d_i, a_{ij} and t_{ij} represent: purchasing prices for supplier $j \in S$, selling price for buyer $i \in B$, price for additional transmission capacities for every arc $(i, j) \in A$ and taxes for additional transmission capacities on arc $(i, j) \in A$, respectively. In this model we have three variables:

- x_j – amount of electricity that should be bought from supplier for every $j \in S$;
- y_i – amount of electricity that should be sold to buyer for every $i \in B$;
- h_{ij} – amount of additional transmission capacities on arc for every $(i, j) \in A$.

We should mention that all electricity amounts and transmission capacities are expressed in MWh. The unit for all prices and taxes is Euro per MWh.

Using given notation, the LP mathematical model for day-ahead planning can be stated as:

(MMDAP)

$$\max Z(x, y, h) = \sum_{i \in B} d_i y_i - \sum_{j \in S} c_j x_j - \sum_{(i,j) \in A} (t_{ij} + a_{ij}) h_{ij} \tag{1}$$

s.t.

$$\sum_{j \in S} x_j - \sum_{i \in B} y_i = 0 \tag{2}$$

$$\sum_{(i,j) \in A} (f_{ij} + h_{ij}) - \sum_{(j,i) \in A} (f_{ji} + h_{ji}) = \begin{cases} 0 & j \in N \backslash (S \cup B) \\ -x_j & j \in S \backslash B \\ y_j & j \in B \backslash S \\ y_j - x_j & j \in B \cap S \end{cases} \tag{3}$$

$$sl_j \leq x_j \leq su_j, \; j \in S \tag{4}$$

$$bl_i \leq y_i \leq bu_i, \; i \in B \tag{5}$$

$$0 \leq h_{ij} \leq hu_{ij}, \; (i, j) \in A \tag{6}$$

The objective function (1) represents the difference between total income and expenses of buying electricity and additional transmission capacities. The constraint (2) provides that

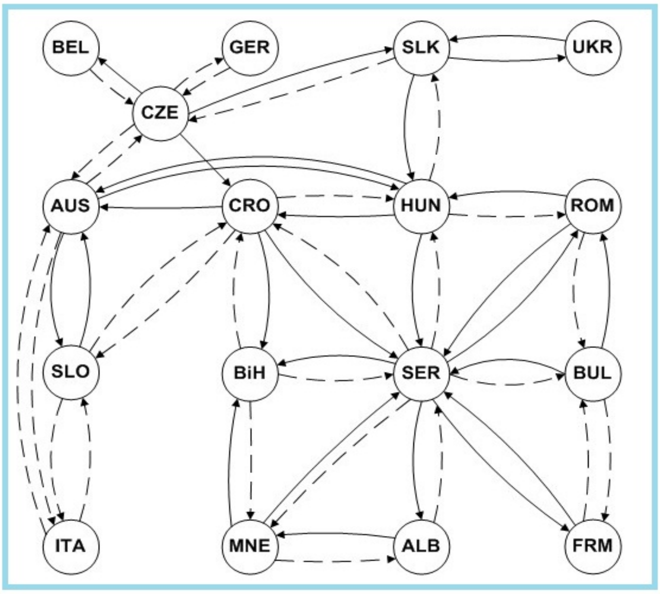

Figure 2 Graph representation of CSEE electricity market.

the sums of electricity that has been bought and sold during one day are equal. Since all nodes are transition nodes, the flow conservation constraint (3) must hold. This constraint has four different interpretations depending on the node type. $N\backslash(S \cup B)$ is the set of nodes without demand and supply. For each node from $N\backslash(S \cup B)$ constraints (3) ensure that the amount of electricity entering node must be equal to the amount of electricity leaving node. $S\backslash B$ is the set of source nodes in which the amount of electricity entering the node and electricity bought in this node must be equal to the amount of electricity leaving this node while $B\backslash S$ represents the set of sink nodes in which the amount of electricity entering a node must equal the amount of electricity leaving a node and electricity sold in this node. $B \cap S$ is the set of source-sink nodes. For each node from $B \cap S$ constraints (3) ensure that the amount of electricity entering the node and electricity bought in this node must equal the amount of electricity leaving this node and electricity sold in that node. Optimal amounts of electricity that should be bought and sold lies between their upper and lower boundaries given by constraints (4) and (5). Constraint (6) refers to maximal additional daily transmission capacity which is possible to buy on an arc.

4 Numerical examples

In order to evaluate the proposed model we consider two different ways of its application. First one is related to obtaining the optimal day-ahead plan when all parameters have fixed values. The second application is based on a more realistic situation when some parameters may vary within certain boundaries. Both applications are demonstrated on the CSEE network consisting of 17 nodes and 54 arcs (Figure 2).

Each country is presented by one node, which is characterized by its lower and upper bounds of electricity that can be bought and/or sold in/from that country as well as purchasing and selling prices. All arcs in Figure 2 (solid and dashed lines) represent cross-border connections on which it is possible to buy additional transmission capacity.

Table 1 Optimal solution.

	Buying in MWh			Selling in MWh		
node	min	Optimal	max	min	Optimal	max
BEL	18	40	40	25	31	67
GER	2	70	70	0	70	80
AUS	1	31	31	0	0	60
ITA	0	36	50	31	41	41
SLO	–	–	–	5	5	35
CZE	25	35	35	0	0	38
HUN	0	0	10	–	–	–
CRO	–	–	–	22	22	44
BiH	36	52	52	42	49	49
MNE	–	–	–	5	5	60
SLK	–	–	–	12	12	33
SER	0	46	46	0	80	80
ALB	5	10	35	–	–	–
FRM	0	40	40	3	3	57
ROM	–	–	–	4	38	70
BUL	–	–	–	14	14	25
UKR	10	10	20	–	–	–

Arcs represented by solid lines indicate the existence of announced transmission capacities purchased earlier. Maximal additional daily transmission capacities, amounts of announced transmission capacities, prices and taxes of each arc are also given. Due to confidentiality issues, we present in this paper slightly modified data which are within boundaries of common real-life situations.

The model has been implemented and solved using the GNU Linear Programming Kit [2]. The optimal solution for one scenario is given in Table 1. Marks "–" mean that there were no suppliers or buyers in the corresponding country.

In addition to the optimal trade amounts, the solution determines additional transmission capacities that should be purchased. As well, the solution provides information about total trade amounts in MWh and profit which will be made with this scenario.

Beside information about total daily trade and total daily income, the application has the possibility to show expenses of buying electricity and expenses of additional transmission capacities in Euros. In order to obtain a total profit certain corrections in the objective have to be made:

- Taxes for announced transmission capacities as well as costs of long term purchased transmission capacities evaluated on daily basis should be subtracted.
- Unit prices for previously arranged buying and selling (bl_j and sl_j) can differ from actual prices. These differences should be taken into account.

However, any of these corrections will not affect the optimal solution obtained by the model. In the further analysis we will use the term "profit" to refer to the value in (1), although it is just an approximation.

In order to demonstrate the second way of using MMDAP we perform a sensitivity analysis for some parameters. In this case we present a scenario where the selling price for node SER varies from 80 to 88 Euro/MWh. We analyze the influence of the selling price on the optimal

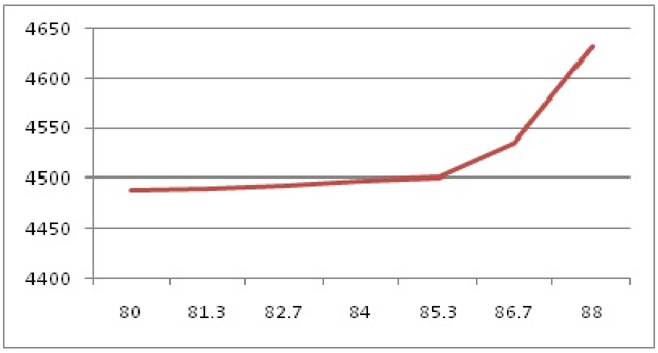

Figure 3 The impact of prices on the optimal profit.

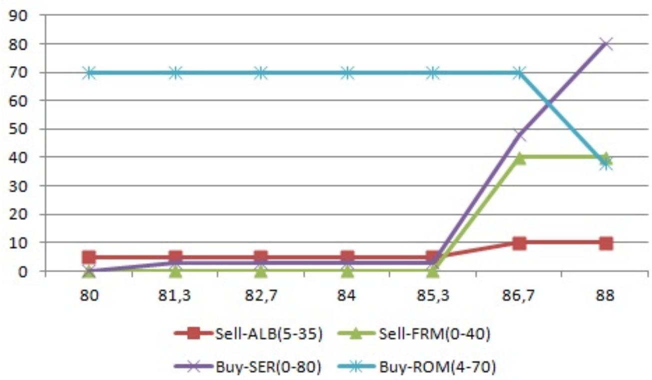

Figure 4 The impact of prices on the optimal amounts.

amount of electricity that should be bought and the optimal solution as a whole. A series of optimizations were made for all integer values of the price in the given interval. Figure 3 and 4 show the optimal amounts of electricity that should be bought and sold for some nodes and the corresponding profits for some characteristic prices of electricity, assuming that all other parameters remain the same.

The increases of the optimal amount of electricity as well as the profit were expected because the selling price increased, but all the solutions are optimal for those values. In other words, for different prices different amounts of electricity that should be bought (and corresponding profits) will be optimal.

Optimization results should give the decision maker information how to make bids on auctions in order to maximize ETC's total profit. On CSEE electricity market uniform price auction type is used. That assumes that each bidder in the auction bids a price and an amount. The price bid is considered the maximum price they are willing to pay per item, and the amount is the number of units they wish to purchase at that price. Typically these bids are sealed – not revealed to the other buyers until the auction closes. The auctioneer then serves the highest bidder first, giving them the number of units requested, then the second highest bidder and so forth until the supply of the commodity is exhausted. All bidders then pay a per unit price equal to the lowest winning bid (the lowest bid out of the buyers who actually received one or more units of the commodity) – regardless of their actual bid [13].

In practice, for each offer bidders make several bids with different amounts and prices in order to provide both: needed amount to be bought and the price to be as low as possible.

Table 2 List of bids derived from optimal solutions.

Bids	Amounts of el.	Price	Accumulated amount	Guaranteed profit
B1	3	481.3-85.3	3	4489-4501
B2	45	85.4-86.7	48	4535
B3	35	86.8-88	80	4631

On the basis of the presented optimal solutions we can suggest the ETC decision maker to make bids as presented in Table 2.
Depending on the price equal to the lowest winning bid (which will not be known till the end of the auction), ETC will buy 0, 3, 48 or 80 MWh from node SER. For prices that are presented as intervals in Table 2, ETC decision maker can give any price from that interval for which it estimates that can win. Anyhow, the corresponding amount will be optimal. The profits given in the rightmost column represent the optimal profit in case that corresponding bid becomes the lowest winning bid. If any lower bid wins, the profit will be bigger.

5 Conclusions

In this paper, problem of day-ahead energy planning in the Energy Trading Companies have been considered. It has been formulated as a linear programming problem. Day-ahead energy planning implies finding the optimal amounts of electricity that should be bought from each supplier and sold to each buyer and the optimal routes which can satisfy the daily demands using the purchased and additional energy transmission capacities.

A real numerical example was considered and demonstrate the usage of the model which has been presented in this paper. Since the developed model is linear, it can be used to solve real life problems of large dimensions.

As a topic of further research, the trading through futures markets can be taken into consideration. Another topic of future research is modeling long term planning strategy of Energy Trading Companies.

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