An Optimal Real-time Pricing Algorithm for the Smart Grid: A Bi-level Programming Approach*

Fan-Lin Meng and Xiao-Jun Zeng

School of Computer Science, University of Manchester
Manchester, United Kingdom
`mengf@cs.man.ac.uk, x.zeng@manchester.ac.uk`

Abstract

This paper proposes an improved approach to our previous work [11]. [11] uses Stackelberg game to model the interactions between electricity retailer and its customers and genetic algorithms are used to obtain the Stackelberg Equilibrium (SE). In this paper, we propose a bi-level programming model by considering benefits of the electricity retailer (utility company) and its customer. In the upper level model, the electricity retailer determines the real-time retail prices with the aim to maximize its profit. The customer reacts to the prices announced by the retailer aiming to minimize their electricity bills in the lower level model. In order to make it more tractable, we convert the hierarchical bi-level programming problem into one single level problem by replacing the lower lever's problem with his Karush–Kuhn–Tucker (KKT) conditions. A branch and bound algorithm is chosen to solve the resulting single level problem. Experimental results show that both the bi-level programming model and the solution method are feasible. Compared with the genetic algorithm approach proposed in work [11], the branch and bound algorithm in this paper is more efficient in finding the optimal solution.

1998 ACM Subject Classification G.1.6 Optimization

Keywords and phrases Real-time Pricing, Demand Response, Smart Gird, Bi-level Programming, Branch and Bound Algorithm

Digital Object Identifier 10.4230/OASIcs.ICCSW.2013.81

1 Introduction

The traditional electricity grid is facing many existing and potential problems with the increased demand from customers in recent years, and the reliability of the grid has been put in danger. In addition, the average household electricity load has the potential to double with the deployment of plug-in hybrid electric vehicles (PHEVs), which will further endanger the existing grid.

Instead of building more power plants to meet the peak demand of customers, demand response is a better choice for solving the above problems, especially with the development of the smart grid.

Real-time pricing (RTP) is one of the most important DR strategies, where the prices announced by retailers change typically hourly to reflect variations of the price in the wholesale market over time. Generally, customers are notified of RTP prices the day before or a few hours before the delivery time. One of the most typical types of RTP is day-ahead RTP, in which customers receive the prices for the next 24 hours [6].

* Parts of this paper appeared in the proceedings of UKCI 2012 [11].

2013 Imperial College Computing Student Workshop (ICCSW'13).
Editors: Andew V. Jones, Nicholas Ng; pp. 81–88
OpenAccess Series in Informatics
OASICS Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

There exists much literature on RTP. However, the results and analysis in this paper differ from the related work in several aspects:

In the work of [16], they analytically models the customers' preferences and customers' electricity consumption patterns in form of utility functions and it shows that the proposed algorithm can benefit both customers and energy providers. However, no explicit form of the customers' utility functions is given. [8] and [18] further develop the work of [16]. Both works use the same concept of utility functions to model the satisfaction of customers as [16], but similarly no explicit form of the utility functions are given. As a result, the approach is unable to help the customers to find the best scheme to minimize their bills. To overcome this weakness, the approach given in this paper aims to provide the best solution for customers to achieve the minimal bills.

Since the RTP design needs the participation of electricity retailer and its customers and the decision makings are sequential, i.e., the electricity retailer announces the prices first, then its customers react to the prices by shifting the energy use. The interactions between electricity retailer and its customers can be represented as a leader-follower Stackelberg game, and thus can be modelled using a bi-level programming model. In fact, the bi-level programming problem is a static Stackelberg game where two players try to maximize their individual objective functions [1].

Due to the hierarchical structure of the Stackelberg game or bi-level programming model, many real-world problems with two decision levels can be modelled using the bi-level programming approach. Price setting problems are two decision levels problems and have been studied for several years using bi-level programming approach [14, 15, 7]. [2] proposes a decision-making scheme for electricity retailers based on Stackelberg game. They model the customers' preference and satisfaction as utility functions. [4] presents an optimal demand response scheduling with Stackelberg game approach. Similar to [2], they model the customers' behaviour patterns as utility functions. However, no explicit form of utility functions are given. The difference between our work and [2] and [4] lies in that we model the follower level problem (lower level problem) with appliance-level details, which is more practical and thus more difficult to solve.

The main focus of this paper is to propose a decision making scheme based on bi-level programming model for the electricity retailer and its customer by considering the benefits of both participants and give efficient solutions to the proposed model.

The rest of this paper is organized as follows. The background of bi-level programming model is introduced in Section 2. In Section 3, the system model and the solution method are given. Experimental results are presented in Section 4. The paper is concluded in Section 5.

2 Background of Bi-level Programming Model

Decision making problems in decentralized organizations are often modelled as Stackelberg games, and they are formulated as bi-level mathematical programming problems. The major feature of a bi-level programming problem is that it includes two optimization problems within a single instance. The lower level executes its own optimal policy after decisions are made at the upper level [10].

The general formulation of a bi-level programming problem can be represented as follows:

$$
\begin{aligned}
(Upper \quad Level) \quad & \min_{x} F(x,y) \\
s.t. \quad & G(x,y) \leq 0
\end{aligned}
$$

where $y = y(x)$ is implicitly defined by:

$$(Lower \quad Level) \quad \min_{y} f(x,y)$$
$$s.t. \quad g(x,y) \leq 0$$

where F is the objective function of the upper level problem; f is the objective function of the lower level problem; G is the constraint set of the upper level decision vector; g is the constraint set of the lower level decision vector; x is the decision vector of the upper level; and y is the decision vector of the lower level. $y = y(x)$ is called reaction function. To solve the bi-level programming model, one needs to obtain the reaction function by solving the lower-level problem and replace the variable y in the upper level problem with the reaction function [17]. However, for many real applications, the reaction function $y = y(x)$ is not able to be explicitly represented. Thus, the problems can not be handled in the above way and become more difficult to solve.

Even though the simple linear bi-level programming problems are proven to be NP-hard, there are many methods arising in the last thirty years for solving the bi-level programming problems, such as the extreme-point approach for the linear bi-level programming, branch and bound method, descent methods, penalty function methods and trust region methods [5]. In this paper, the branch and bound method is chosen to solve our proposed bi-level programming problems as this algorithm was proved to be able to obtain the global optimal solutions [1].

3 System Model and Solution Method

In this section, we provide a mathematical representation of the considered decision making problem. Firstly, our focus is to formulate the electricity consumption scheduling problem in response to the real-time pricing in each household as an optimization problem that aims to minimize the payment bills. Secondly, we model the profit optimization problem for the retailer who will offer the 24 hours real-time prices to the customer.

We define $P = [p^1, p^2, ..., p^h, ...p^H]$ as the leader's strategy space, where p^h represents the electricity price at hour h and H represents the scheduling time window. We assume that $p^{min} \leq p^h \leq p^{max}$, where p^{min} represents the minimum price that the retailer (utility company) can offer to the customer and p^{max} represents the maximum price that the retailer can offer. It is also reasonable to assume that the price that the retailers can offer is greater than the wholesale price of each hour. The prices of p^{min} and p^{max} are usually designed based on history data and conditions of the wholesale price. However, since most of the retail markets now are regulated, there exists a price cap for the retail price and p^{max} should be less than the price cap. For the following part of this paper, we set $\mathcal{H} \triangleq \{1, 2, ..., H\}$. Usually, $H = 24$. We define the set of appliances in the customer's household S. In this paper, we only consider the one-leader, one-follower case, i.e., in our model, only one electricity retailer and one customer are considered.

3.1 Lower-level Model Problem

This model improves that of [13]. In their work, a upper limit for hourly electricity usage is set for each household, but we do not have such constraints for the optimization problem at customer's side as there is no such usage limits in practice. Instead, we consider the total upper limit of hourly usage in the optimization problem at retailers' side. This is to represent the maximum load capacity of power networks. Therefore, we can actually control the hourly use of electricity of each household by properly determining the retail price, which is more practical from an application point of view.

For each appliance $s \in S$, we define an electricity consumption scheduling vector:

$$\mathrm{e}_s = [e_s^1, ..., e_s^h, ..., e_s^H] \tag{1}$$

where H is the scheduling window. For each hour $h \in \mathcal{H} \triangleq \{1, 2, ..., H\}$, $e_s^h \geq 0$ represents the customer's electricity consumption of appliance s at time h.

It is reasonable to assume that the energy consumption of each appliance s during a typical day is maintained at the same level and the total electricity consumed by appliance s in a typical day is defined as E_s. Moreover, the customer needs to set a valid scheduling window $\mathcal{H}_s \triangleq \{\alpha_s, ..., \beta_s\}$ by specifying the beginning operation time $\alpha_s \in \mathcal{H}$ and the end operation time $\beta_s \in \mathcal{H}$ of appliance s. Based on the above analysis, we have

$$\sum_{h=\alpha_s}^{\beta_s} e_s^h = E_s \tag{2}$$

and

$$e_s^h = 0, \forall h \in \mathcal{H} \backslash \mathcal{H}_s \tag{3}$$

After defining the minimum power level γ_s^{min} and the maximum power level γ_s^{max} for each appliance $s \in S$, we have

$$\gamma_s^{min} \leq e_s^h \leq \gamma_s^{max}, \forall h \in \mathcal{H}_s. \tag{4}$$

Then, the payment bill optimization problem for the customer can be modelled as follows:

$$\begin{aligned} &\min_{\boldsymbol{e_s^h}} \textstyle\sum_{h=1}^{H} p^h \times (\sum_{s \in S} e_s^h) \\ &s.t. \\ &\textstyle\sum_{h=\alpha_s}^{\beta_s} e_s^h = E_s, \\ &e_s^h = 0, \forall h \in \mathcal{H} \backslash \mathcal{H}_s, \\ &\gamma_s^{min} \leq e_s^h \leq \gamma_s^{max}, \forall h \in \mathcal{H}_s. \end{aligned} \tag{5}$$

3.2 Upper-level Model Problem

In this section, we model the profit of the retailer by using the revenue subtracting the energy cost imposed on the retailer. We will discuss about the energy cost model first, and then a profit maximization model will be proposed.

In the practical application scenario, to determine the retail price, we need to consider many factors such as running cost of the retailers including the payments incurred in the wholesale market and so on. For simplicity, we define a cost function $C_h(L_h)$ indicating the cost of providing electricity by the retailers at each hour $h \in \mathcal{H}$, where L_h represents the amount of power provided to the customer at each hour of the day. We assume that the cost function $C_h(L_h)$ is increasing in L_h for each h [12, 8, 3]. In view of this, we design the cost function as follows [12].

$$C_h(L_h) = a_h L_h + b_h \tag{6}$$

where $a_h > 0$ and $b_h \geq 0$ at each hour $h \in \mathcal{H}$.

For each hour $h \in \mathcal{H}$, by defining the minimum price that the retailer (utility company) can offer p^{min} and the maximum price p^{max}, we have $p^{min} \leq p^h \leq p^{max}$. Note that there is usually a maximum load capacity, denoted as $E_h^{\max}$, of power networks at each hour. Thus, we have following constraints:

$$\sum_{s \in S} e_s^h \leq E_h^{\max}, \forall h \in \mathcal{H} \tag{7}$$

Then the profit maximization problem can be modelled as (8).

$$\begin{aligned} &\max_{\boldsymbol{p^h}} \{ \sum_{h \in \mathcal{H}} p^h \times \sum_{s \in S} e_s^h - \sum_{h \in \mathcal{H}} C_h (\sum_{s \in S} e_s^h) \} \\ &s.t. \\ &p^{min} \leq p^h \leq p^{max} \\ &\sum_{s \in S} e_s^h \leq E_h^{\max}, \forall h \in \mathcal{H} \end{aligned} \tag{8}$$

3.3 Solution Method

Instead to solve the bi-level problem in its hierarchical form (Eqs.(8) and (5)), we convert it into a standard mathematical program by replacing the follower's problem (lower level problem, Eq.(5)) with his Karush–Kuhn–Tucker (KKT) conditions. Then a branch and bound algorithm is chosen to solve the resulting non-linear programming problem [1]. We adopt the YALMIP solver, which is based on the above mentioned algorithm and implemented in Matlab, to solve our bi-level programming mode [9].

4 Experimental Results

We simulate a simple one energy retailer (utility company), one customer case. It is assumed that the customer has 4 appliances: dish washer, washing machine, clothes dryer and PHEV. Note that the scheduling horizon is from 8AM to 8AM (the next day).

For the cost of the energy provided to the customer by utility company, we model this as a cost function. We choose a simple linear cost function: $C_h(L_h) = a_h L_h + b_h$, where L_h represents the amount of power provided to the customer at each hour of the day. For simplicity we assume that $b_h = 0$ for all $h \in \mathcal{H}$. Also, we have $a_h = 5.5$ cents during the day, i.e., from 8AM to 12AM and $a_h = 4.0$ cents at night hours, i.e., from 13AM to 8AM (the next day). Finally, the parameter settings of these home appliances can be found in Table 1.

Getting ideas from the time-of-use pricing (ToU), we divide the 24 hours prices into three levels, i.e., peak hours(5PM-12AM), mid-peak hours(8AM-5PM) and off-peak hours(12AM-8AM). For peak hours, the prices range from 12 cents to 14 cents. Similarly, the prices range

Table 1 Home Appliances' Parameter Settings.

Appliance Name	E_s	H_s	γ_s^{min}	γ_s^{max}
Dish washer	1.8kwh	8PM-6AM	0.1kwh	1.0kwh
Washing machine	1.94kwh	8AM-8PM	0.1kwh	1.0kwh
Clothes dryer	3.4kwh	7PM-7AM	0.25kwh	3.0kwh
PHEV	9.9kwh	8PM-7AM	0.3kwh	2.0kwh

Table 2 24 Hours Optimal Prices Offered by the Retailer.

Time	8AM	9AM	10AM	11AM	12PM	1PM	2PM	3PM
Price(cents)	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00
Time	4PM	5PM	6PM	7PM	8PM	9PM	10PM	11PM
Price (cents)	12.00	12.00	12.00	14.00	14.00	14.00	14.00	14.00
Time	12AM	1AM	2AM	3AM	4AM	5AM	6AM	7AM
Price(cents)	14.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00

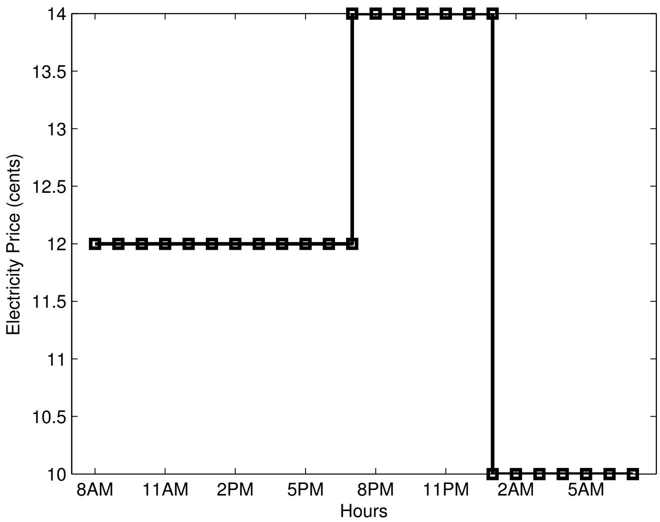

Figure 1 Optimal Real-time Prices Offered by the Retailer.

from 8 cents to 12 cents for mid-peak hours while the prices float between 6 cents to 10 cents for off-peak hours.

The aim of our proposed optimal RTP scheme is to find the optimal 24 hours prices by maximizing the retailer's profit (upper level problem). Besides this, with this identified price information, the customer can achieve his best benefit, i.e., minimize his payment bills (lower level problem).

Applying the open source solver YALMIP to our proposed bi-level programming problems, we can get the optimal 24 hours prices shown as Table 2 and Figure 1.

With the purpose to design a benchmark, we assume, without our proposed optimal appliances scheduling scheme, the appliances start working right at the beginning of the time interval H_s and at its typical power level. The energy consumption comparison of appliances with and without scheduling can be seen from Figure 2. We can easily find from Figure 2 that the customer shifts the energy use from high price periods to low price period. As a result, with our proposed scheduling scheme, the electricity bill of the customer for one day is reduced from 2.35 $ to 1.94 $.

Based on the above analysis, we can see that with this bi-level programming model not only the electricity retailer can maximize his benefits, but the customer can also benefit from the reduced electricity bills.

Last but not least, the proposed branch and bound algorithm is more efficient compared

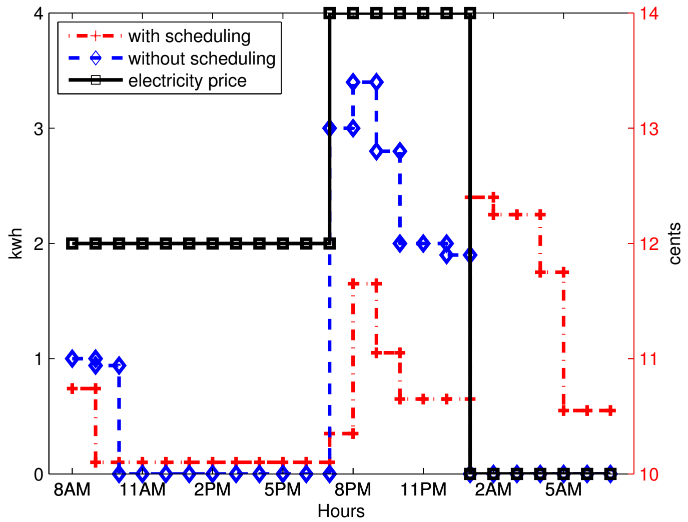

Figure 2 Energy Consumption Comparison with Scheduling and without Scheduling under Obtained Optimal Real-time Pricing.

with genetic algorithms used in [11]. Ten separate experiments for each approach have been done for the computation time comparison. The average time cost of the genetic algorithm approach in obtaining the optimal solution to our proposed bi-level programming model is around 120 seconds while the branch and bound algorithm takes only around 8 seconds.

5 Conclusion and Future Work

We propose a bi-level programming approach to model the interactions between the retailer and its customer. First, a electricity bill minimization model (lower level model) has been proposed for the customer to incentive him to change his electricity use pattern. Second, a profit maximization model (upper level model) for the retailer has been modelled. A branch and bound algorithm is chosen to solve this propose bi-level programming problem. As the simulation results show that both the retailer and the customer can benefit from the proposed framework, it has great potential to improve the implementation of current energy pricing programs, help customers to reduce the increasing energy bills, and change their energy usage patterns.

This work can be extended in several directions. First, we will enrich the lower level problem by considering the trade-off between minimizing bills and satisfying customer's comfort. Second, we will extend the current one-leader one follower case to one-leader multiple-followers bi-level programming model in our future work.

References

1 Jonathan F Bard and James T Moore. A branch and bound algorithm for the bilevel programming problem. *SIAM Journal on Scientific and Statistical Computing*, 11(2):281–292, 1990.

2 S. Bu, F. Richard Yu, and Peter X. Liu. A game-theoretical decision-making scheme for electricity retailers in the smart grid with demand-side management. In *2011 IEEE*

International Conference on Smart Grid Communications (SmartGridComm), pages 387–391, 2011.

3 C. Chen, S. Kishore, and L.V. Snyder. An innovative rtp-based residential power scheduling scheme for smart grids. In *Acoustics, Speech and Signal Processing (ICASSP), 2011 IEEE International Conference on*, pages 5956–5959. IEEE, 2011.

4 Jiang Chen, Bo Yang, and Xinping Guan. Optimal demand response scheduling with stackelberg game approach under load uncertainty for smart grid. In *Smart Grid Communications (SmartGridComm), 2012 IEEE Third International Conference on*, pages 546–551. IEEE, 2012.

5 Benoît Colson, Patrice Marcotte, and Gilles Savard. Bilevel programming: A survey. *4OR*, 3(2):87–107, 2005.

6 Seppo Kärkkäinen Corentin Evens. Pricing models and mechanisms for the promotion of demand side integration. Technical Report VTT-R-06388-09, VTT Technical Research Centre of Finland, 2009.

7 Martine Labbé and Alessia Violin. Bilevel programming and price setting problems. *4OR*, pages 1–30, 2013.

8 Na Li, Lijun Chen, and Steven H. Low. Optimal demand response based on utility maximization in power networks. In *2011 IEEE Power and Energy Society General Meeting*, pages 1–8, 2011.

9 Johan Lofberg. Yalmip: A toolbox for modeling and optimization in matlab. In *Computer Aided Control Systems Design, 2004 IEEE International Symposium on*, pages 284–289. IEEE, 2004.

10 Huina Mao, Xiao-Jun Zeng, Gang Leng, Yong-Jie Zhai, and John A Keane. Short-term and midterm load forecasting using a bilevel optimization model. *Power Systems, IEEE Transactions on*, 24(2):1080–1090, 2009.

11 Fan-Lin Meng and Xiao-Jun Zeng. A stackelberg game approach to maximise electricity retailer's profit and minimse customers' bills for future smart grid. In *Computational Intelligence (UKCI), 2012 12th UK Workshop on*, pages 1–7. IEEE, 2012.

12 A Mohsenian-Rad and VWS Wong. Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid. *Smart Grid, IEEE*, 1(3):320–331, 2010.

13 Amir-Hamed Mohsenian-Rad and Alberto Leon-garcia. Optimal Residential Load Control With Price Prediction in Real-Time Electricity Pricing Environments. *Smart Grid, IEEE*, 1(2):120–133, 2010.

14 Erling Pettersen, Andrew B Philpott, and Stein W Wallace. An electricity market game between consumers, retailers and network operators. *Decision support systems*, 40(3):427–438, 2005.

15 Vesna Radonjić and Vladanka Aćimović-Raspopović. Responsive pricing modeled with stackelberg game for next-generation networks. *annals of telecommunications-annales des télécommunications*, 65(7-8):461–476, 2010.

16 Pedram Samadi, Amir-Hamed Mohsenian-Rad, Robert Schober, Vincent W. S. Wong, and Juri Jatskevich. Optimal Real-Time Pricing Algorithm Based on Utility Maximization for Smart Grid. In *2010 First IEEE International Conference on Smart Grid Communications*, pages 415–420, 2010.

17 Huijun Sun, Ziyou Gao, and Jianjun Wu. A bi-level programming model and solution algorithm for the location of logistics distribution centers. *Applied Mathematical Modelling*, 32(4):610–616, 2008.

18 Peng Yang, Gongguo Tang, and Arye Nehorai. A game-theoretic approach for optimal time-of-use electricity pricing. *Power Systems, IEEE Transactions on*, 28(2):884–892, 2013.