Semi-algebraic geometry in computational game theory – a consumer's perspective*

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Abstract

We survey recent applications of real algebraic and semi-algebraic geometry in (computational) game theory.

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Category Invited Talk

1 Introduction to the talk

Real algebraic geometry and semi-algebraic geometry are well-established tools in game theory (e.g., [2, 11, 9]). In this talk, we survey recent work applying these tools to (mostly) computational settings of game theory. Examples include:

- A bound on the discount factor for which the value of a discounted stochastic game is guaranteed to well-approximate the value of the corresponding undiscounted stochastic game [6, 7, 5, 10]. The bound is in terms of the combinatorial parameters of the game and is relatively tight. This refines work of Milman [9].
- An analysis of a recursive bisection algorithm for solving stochastic games [6, 7].
- A tight upper bound on the worst case complexity of the strategy iteration algorithm for concurrent reachability games [6, 7, 5].
- An existence proof of "monomial" near-optimal strategies for concurrent reachability games [4].
- Approximating the value of a concurrent reachability game can be done in the polynomial time hierachy [8, 3].
- Computational (polynomial-time) equivalence between approximating a Nash equilibrium and approximating a trembling hand equilibrium of a game in strategic form (joint work with Etessami, Hansen, and Sørensen, in preparation).

The applications rely on generic tools and off-the-shelf theorems of real algebraic and semi-algebric geometry [1]. The talk is therefore given from the perspective of a consumer of real and semi-algebraic geometry and should be accessible to an audience with little or no knowledge of this topic (which is a level of knowledge similar to that of the speaker). We do briefly discuss what kind of improvements of the results might hopefully be obtained in the future by looking under the hood into the beautiful machinery of semi-algebraic geometry.

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