Report from Dagstuhl Seminar 14451

Optimality and Tight Results in Parameterized Complexity

Edited by

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 14451 "Optimality and tight results in parameterized complexity". Over the last two decades parameterized complexity has become one of the main tools for handling intractable problems. Recently, tools have been developed not only to classify problems, but also to make statements about how close an algorithm is to being optimal with respect to running time. The focus of this seminar is to highlight and discuss recent, relevant results within this optimality framework and discover fruitful research directions. The report contains the abstracts of the results presented at the seminar, as well as a collection of open problems stated at the seminar.

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1 Executive Summary

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While many seemingly hard computational problems can be solved satisfactorily in practice, classical complexity dictates that they are in fact intractable (NP-hard) in general. This is an unsatisfactory situation since one would desire a more productive interplay between more heuristic practical results and theoretically proven theorems.

Parameterized complexity analyzes the complexity in finer detail by considering additional problem parameters beyond the input size and expresses the efficiency of the algorithms in terms of these parameters. In this framework, many NP-hard problems have been shown to be (fixed-parameter) tractable when certain structural parameters of the inputs are bounded. In the past two decades, there has been tremendous progress in understanding which problems are fixed-parameter tractable and which problems are not (under standard complexity assumptions).

In recent years, the field of parameterized complexity seems to have evolved beyond merely classifying problems as fixed-parameter tractable or not. The focus shifted to understanding how close the algorithmic results are to the "best possible" algorithm for the problem. Thanks to significant recent advances on both algorithms (upper bounds) and complexity (lower bounds), we have now a tight understanding of many problems and many algorithmic results can be now proven optimal under reasonable assumptions. Moreover, it turns out that the search for optimality can be formulated with respect to different aspects of parameterized complexity and each such aspect gives a separate challenging but doable research direction. One can consider the optimality of algorithms for parameterized problems (either fixed-parameter tractable or not), the optimality of preprocessing algorithms, and the optimality of algorithms with respect to the generality of the problem being solved. The goal of the seminar was to bring together experts in the area of parameterized complexity and algorithms, highlight these research directions and the relevant recent results, and discuss future research topics.

The scientific program of the seminar consisted of 25 talks. Among these there were five 60 minute tutorials on the core topics of the seminar: Marek Cygan and Michał Pilipczuk ("Exponential Time Hypothesis, Part 1+2") covered the Exponential Time Hypothesis (ETH), focussing on techniques for proving tight runtime lower bounds under ETH. Daniel Lokshtanov ("The Strong Exponential Time Hypothesis") introduced Strong ETH as well as related lower bound techniques, and Virginia Vassilevska Williams ("Implications of SETH for polynomial time problems") gave an overview of tight lower bounds for efficiently solvable problems under Strong ETH. Finally, Dániel Marx ("Every Graph is easy or hard") covered the topic of dichotomy theorems for graph problems. Throughout, the tutorials were well-received both as a means of introduction to the topics but also as a convenient way of catching up on very recent results pertaining to the seminar. Furthermore, with most tutorials being held on Monday and Tuesday morning this set a productive atmosphere for tackling open problems regarding tight parameterized complexity results. A further 60 minute contributed talk by Saket Saurabh discussed the recent breakthrough result of fixed-parameter tractability of Graph Isomorphism with respect to treewidth. The rest of the talks were 25-minute presentations on recent research of the participants.

The time between lunch and afternoon coffee was left for self-organized collaborations and discussions. An open problem session was organized on Monday evening. Notes on the presented problems can be found in this report.

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3 Overview of Talks

3.1 On Courcelle's Conjecture

Hans L. Bodlaender (Utrecht University, NL)

Joint work of Bodlaender, Hans L.; Heggernes, Pinar; Telle, Jan Arne

Courcelle's theorem tells that every problem that is formulatable in Counting Monadic Second Order Logic can be solved in linear time on graphs of bounded treewidth, with a specific type of algorithm – the form can be captured by the notion of regularity, i.e., the algorithm can be seen as a finite state tree automaton. Courcelle's conjecture states that the reverse also holds, i.e., each regular graph problem can be formulated in CMSOL. In the talk, the status of this conjecture is discussed. Recently, we found a proof for the case of k-chordal graphs.

3.2 The Complexity of Counting k-Matchings Revisited

Radu Curticapean (Universität des Saarlandes, DE)

Joint work of Curticapean, Radu; Marx, Dániel
Main reference R. Curticapean, D. Marx, "Complexity of Counting Subgraphs: Only the Boundedness of the Vertex-Cover Number Counts," in Proc. of the 55th IEEE Annual Symposium on Foundations of Computer Science (FOCS'14), pp. 130–139, IEEE, 2014; pre-print available as arXiv:1407.2929v1 [cs.CC], 2014.
URL http://dx.doi.org/10.1109/FOCS.2014.22
URL http://arxiv.org/abs/1407.2929v1

In this talk, we present a novel #W[1]-hardness proof for the problem of counting matchings of size k. A hardness proof for this problem was posed as an open problem in the book on parameterized complexity theory by Flum and Grohe, and remained open until recently (C., 2013). However, this first proof relies upon rather complicated algebraic arguments. Furthermore, some steps in this proof can only be verified with the help of a computer, and lastly, the proof can not provide a lower bound under the Exponential Time Hypothesis. In fact, its parameter blowup cannot be easily bounded by any explicit function, similar to a hardness proof for counting k-cycles by Flum and Grohe that was known before. We present a new proof, published as part of a recent paper (C., Marx, 2014) on the parameterized complexity of counting subgraphs. This new proof is significantly simpler and, in particular, uses only first principles from linear algebra. The reduction is from the problem of counting vertex-colorful copies of a 3-regular graph H in a vertex-colored graph G. Given such an instance (H, G) as input, with $|V(H)| = k$, we show how to construct 3^k edge-colored graphs on $O(k)$ colors such that the following holds: The number of vertex-colorful H-copies in G is equal to a linear combination of the numbers of edge-colorful matchings in the constructed graphs. The coefficients in this linear combination are easily computed. Then, the problem of counting edge-colorful matchings of size k can be reduced to the uncolored version via simple inclusion-exclusion. Since this reduction incurs only linear blowup, it can be used to transfer a lower bound of $n^{O(k/logk)}$ for the vertex-colorful subgraph problem under ETH (Marx, 2007) to the problem of counting k-matchings. Using simple self-contained reductions from counting k-matchings, we then obtain the same lower bounds for counting directed/undirected paths/cycles of length k.

3.3 Tutorial: Lower Bounds Based on the Exponential Time Hypothesis – Part 1

Marek Cygan (University of Warsaw, PL)

Joint work of Cygan, Marek; Fomin, Fedor; Kowalik, Łukasz; Lokshtanov, Daniel; Marx, Dániel; Pilipczuk, Marcin; Pilipczuk; Saurabh, Saket

This talk will be an introduction to lower bounds based on the Exponential Time Hypothesis. We will discuss the Sparsification Lemma and different types of lower bounds that one can obtain based on ETH.

3.4 Parameterized Complexity of Bandwidth on Trees

Markus S. Dregi (University of Bergen, NO)

Joint work of Dregi, Markus S.; Lokshtanov, Daniel
Main reference M. S. Dregi, D. Lokshtanov, "Parameterized Complexity of Bandwidth on Trees," in Proc. of the 41st Int'l Colloquium on Automata, Languages, and Programming (ICALP'14), LNCS, Vol. 8572, pp. 405–416, Springer, 2014.
URL http://dx.doi.org/10.1007/978-3-662-43948-7_34

The bandwidth of a n-vertex graph G is the smallest integer b such that there exists a bijective function $f : V(G) \to \{1, \dots, n\}$, called a layout of G, such that for every edge $uv \in E(G)$, $|f(u) - -f(v)| \leq b$. In the Bandwidth problem we are given as input a graph G and integer b, and asked whether the bandwidth of G is at most b. We present two results concerning the parameterized complexity of the Bandwidth problem on trees. First we show that an algorithm for Bandwidth with running time $f(b)n^{o(b)}$ would violate the Exponential Time Hypothesis, even if the input graphs are restricted to be trees of pathwidth at most two. Our lower bound shows that the classical $2^{O(b)}n^{b+1}$ time algorithm by Saxe [SIAM Journal on Algebraic and Discrete Methods, 1980] is essentially optimal. Our second result is a polynomial time algorithm that given a tree T and integer b, either correctly concludes that the bandwidth of T is more than b or finds a layout of T of bandwidth at most $b^{O(b)}$. This is the first parameterized approximation algorithm for the bandwidth of trees.

3.5 Kernelization Lower Bounds from Weaker Hardness Assumptions

Andrew Drucker (University of Edinburgh, GB)

Main reference A. Drucker, "New Limits to Classical and Quantum Instance Compression," ECCC TR12-112, Revision #3, December 24, 2014.
URL http://eccc.hpi-web.de/report/2012/112/

The OR-SAT and AND-SAT problems have played an important role in giving evidence of kernelization hardness for parametrized problems. It is known that these problems don't have polynomial kernels, unless NP is in co-NP/poly. More recently I have shown this even holds unless NP is in the uniform class co-AM. The newer result's proof involves some quite different ideas, and might stimulate further research. I will provide a high-level introduction to this work.

3.6 FPT Algorithms for the Workflow Satisfiability Problem with User-Independent Constraints: Optimality and Empirical Evaluation

Gregory Z. Gutin (Royal Holloway University of London, GB)

The Workflow Satisfiability Problem (WSP) asks whether there exists an assignment of authorised users to the steps in a workflow specification, subject to certain constraints on the assignment. The problem is NP-hard and usually parameterized by the number k of steps (as k is usually relatively small in practice). The parameterized WSP is W[1]-hard and researchers consider a special family of constraints, user-independent constraints, which are of interest in practice and for which WSP is in FPT. Researchers also consider a subfamily of user-independent constraints, regular constraints. We discuss optimal complexity of FPT algorithms for both families of constraints (they are $O^*(2^k)$ and $O^*(2^{k \log k})$, respectively) and recent experimental results with implementations of fixed- parameter tractable algorithms for WSP with user-independent constraints. For more information, see [1, 2, 3, 4].

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3.7 Shortest Two Disjoint Paths in Polynomial Time

Thore Husfeldt (IT University of Copenhagen, DK)

Joint work of Björklund, Andreas; Husfeldt, Thore
Main reference A. Björklund, T. Husfeldt, "Shortest Two Disjoint Paths in Polynomial Time," in Proc. of the 41st Int'l Colloquium on Automata, Languages, and Programming (ICALP'14), LNCS, Vol. 8572, pp. 211–222, Springer, 2014.
URL http://dx.doi.org/10.1007/978-3-662-43948-7_18

Given an undirected graph and two pairs of vertices (s_i, t_i) for $i \in \{1, 2\}$ we show that there is a polynomial time Monte Carlo algorithm that finds disjoint paths of smallest total length joining s_i and t_i for $i \in \{1, 2\}$ respectively, or concludes that there most likely are no such paths at all. Our algorithm applies to both the vertex- and edge-disjoint versions of the problem. Our algorithm is algebraic and uses permanents over the quotient ring $\mathbf{Z}_4[X]/(X^m)$ in combination with Mulmuley, Vazirani and Vazirani's isolation lemma to detect a solution. We develop a fast algorithm for permanents over said ring by modifying Valiant's 1979 algorithm for the permanent over $\mathbf{Z}_{2^l}$.

3.8 Uniform Kernelization Complexity of Hitting Forbidden Minors

Bart Jansen (University of Bergen, NO)

Joint work of Giannopoulou, Archontia C.; Jansen, Bart M. P.; Lokshtanov, Daniel; Saurabh, Saket
Main reference A. C. Giannopoulou, B. M. P. Jansen, D. Lokshtanov, S. Saurabh, "Uniform Kernelization Complexity of Hitting Forbidden Minors," arXiv:1502.03965v1 [cs.DS], 2015.
URL http://arxiv.org/abs/1502.03965v1

Several results are known concerning the optimality of the sizes of problem kernels. For example, a result of Dell and van Melkebeek shows that k-Vertex Cover does not have a kernel with $O(k^{2-\epsilon})$ bits, for any positive ϵ, unless NP is in coNP/poly. For other problems, we are much further from establishing the optimal kernel size. A large family of vertex deletion problems can be captured in the framework of $\mathcal{F}$-Minor-free Deletion problems. For a fixed, finite family $\mathcal{F}$, the $\mathcal{F}$-Minor-free Deletion problem takes as input a graph G and an integer k, and asks whether k vertices can be removed from G such that the resulting graph does not contain any graph in $\mathcal{F}$ as a minor. This generalizes k-Vertex Cover, k-Feedback Vertex Set, k-Vertex Planarization, and other problems. A breakthrough result by Fomin et al. (FOCS 2012) shows that if $\mathcal{F}$ contains a planar graph (implying that $\mathcal{F}$-minor-free graphs have constant treewidth), $\mathcal{F}$-Minor-Free Deletion has a polynomial kernel. Concretely, there is a function g (which is not known to be computable) such that $\mathcal{F}$-Minor-Free Deletion has a kernel with $O(k^{g(\mathcal{F})})$ vertices. As $\mathcal{F}$-Minor-Free Deletion captures a large number of classical problems, it would be desirable to find kernels of optimal size. A first question towards finding the optimal kernel size for such problems is whether $\mathcal{F}$-Minor-Free Deletion has kernels of uniformly polynomial size, i.e., of size $g(\mathcal{F}) \cdot k^c$ for some constant c that does not depend on the family $\mathcal{F}$. We show that this is not the case: assuming NP is not in coNP/poly, $\mathcal{F}$-Minor-Free Deletion does not have uniformly polynomial kernels. We can also prove the following contrasting, positive result: there is a function g such that for every t, the Treedepth-t Deletion problem has a kernel with $g(k) \cdot k^c$ vertices for a small, absolute constant c. Since for every fixed t, Treedepth-t Deletion is an instance of $\mathcal{F}$-Minor-Free Deletion, this shows that when the family of forbidden minors enforce sufficient structure on the solution graphs, uniformly polynomial kernels can be obtained. Our results therefore form the first step into analyzing exactly which aspects of the family of forbidden minors determine the degree of the polynomial in the optimal kernel size.

3.9 Parameterized Complexity of Mixed Chinese Postman Problem

Mark Jones (Royal Holloway University of London, GB)

Joint work of Gutin, Gregory; Jones, Mark; Wahlstrom, Magnus
Main reference G. Gutin, M. Jones, M. Wahlstrom, " Structural Parameterizations of the Mixed Chinese Postman Problem," arXiv:1410.5191v3 [cs.CC], 2015.
URL http://arxiv.org/abs/1410.5191v3

Given a mixed graph G, the Mixed Chinese Postman Problem (MCPP) asks us to find a minimum weight closed walk traversing each edge and arc at least once. The MCPP parameterized by the number of edges in G or the number of arcs in G is fixed-parameter tractable as proved by van Bevern et al. (in press) and Gutin, Jones and Sheng (ESA 2014), respectively. The parameterized complexity of MCPP with respect to treewidth was an open question of van Bevern et al. Answering this question, we show that somewhat unexpectedly, MCPP is W[1]-hard with respect to not only treewidth but also pathwidth. On the positive side, we show that MCPP is fixed-parameter tractable (FPT) with respect to treedepth. We are unaware of any natural graph parameters between pathwidth and treedepth and so our results provide a dichotomy of the complexity of MCPP. Furthermore, to the best of our knowledge MCPP is the first natural problem known to be W[1]-hard with respect to treewidth but FPT with respect to treedepth.

3.10 Flip Distance is in FPT time $O(n + k \cdot c^k)$

Iyad A. Kanj (DePaul Uniersity – Chicago, US)

Joint work of Kanj,Iyad; Xia, Ge
Main reference I. Kanj, G. Xia, "Flip Distance is in FPT time $O(n + k \cdot c^k)$," arXiv:1407.1525v1 [cs.DS], 2014.
URL http://arxiv.org/abs/1407.1525v1

Let $\mathcal{T}$ be a triangulation of a set $\mathcal{P}$ of n points in the plane, and let e be an edge shared by two triangles in $\mathcal{T}$ such that the quadrilateral Q formed by these two triangles is convex. A *flip* of e is the operation of replacing e by the other diagonal of Q to obtain a new triangulation of $\mathcal{P}$ from $\mathcal{T}$. The *flip distance* between two triangulations of $\mathcal{P}$ is the minimum number of flips needed to transform one triangulation into the other. The Flip Distance problem asks if the flip distance between two given triangulations of $\mathcal{P}$ is k, for some given $k \in \mathbb{N}$. It is a fundamental and a challenging problem whose complexity for the case of triangulations of a convex polygon remains open for over 25 years. In this talk we present an algorithm for Flip Distance that runs in time $O(n + k \cdot c^k)$, for a constant $c \leq 2 \cdot 14^{11}$, which implies that the problem is fixed-parameter tractable. The NP-hardness reduction for Flip Distance given by Lubiw and Pathak can be used to show that, unless the Exponential Time Hypothesis fails, Flip Distance cannot be solved in time $O^*(2^{o(k)})$. Therefore, one cannot expect an asymptotic improvement in the exponent of the running time of our algorithm.

3.11 Fast Witness Extraction Using a Decision Oracle

Łukasz Kowalik (University of Warsaw, PL)

Joint work of Björklund, Andreas; Kaski Petteri; Kowalik, Łukasz
Main reference A. Björklund, P. Kaski, Ł. Kowalik "Fast Witness Extraction Using a Decision Oracle," in Proc. of the 22th Annual European Symposium on Algorithms (ESA'14), LNCS, Vol. 8737, pp. 149–160, Springer, 2014.
URL http://dx.doi.org/10.1007/978-3-662-44777-2_13

The gist of many (NP-)hard combinatorial problems is to decide whether a universe of n elements contains a witness consisting of k elements that match some prescribed pattern. For some of these problems there are known advanced algebra-based FPT algorithms which solve the decision problem but do not return the witness. We investigate techniques for turning such a YES/NO-decision oracle into an algorithm for extracting a single witness, with an objective to obtain practical scalability for large values of n. By relying on techniques from combinatorial group testing, we demonstrate that a witness may be extracted with $O(k \log n)$ queries to either a deterministic or a randomized set inclusion oracle with one-sided probability of error. Furthermore, we demonstrate through implementation and experiments that the algebra-based FPT algorithms are practical, in particular in the setting of the k-Path problem. Also discussed are engineering issues such as optimizing finite field arithmetic.

3.12 Tutorial: Every Graph is Easy or Hard: Dichotomy Theorems for Graph Problems

Dániel Marx (Hungarian Academy of Sciences – Budapest, HU)

Given a family of algorithmic problems, a dichotomy theorem characterizes each member of the family either as "easy" or as "hard." A classical example is the result of Hell and Nešetřil classifying the complexity of H-Coloring for every fixed H: it is polynomial-time solvable if H is bipartite and NP-hard for *every* nonbipartite graph. Some dichotomy theorems characterize the complexity of a family of problems in a more general setting, where a problem in the family is defined not just by fixing a single graph H, but by fixing a (potentially infinite) *class* of graphs. For example, a result of Grohe characterizes the complexity of graph homomorphisms when the left-hand side graph is restricted to be a member of a fixed class of graphs. In the talk, we survey classical and recent dichotomy theorems arising in the context of graph problems.

3.13 PointLine Cover: The Easy Kernel is Essentially the Best

Geevarghese Philip (MPI für Informatik – Saarbrücken, DE)

Joint work of Kratsch, Stefan; Philip, Geevarghese; Ray, Saurabh
Main reference S. Kratsch, G. Philip, S. Ray, "Point Line Cover: The Easy Kernel is Essentially Tight," in Proc. of the 25th Annual ACM-SIAM Symp. on Discrete Algorithms (SODA'14), pp. 1596–1606, SIAM, 2014.
URL http://dx.doi.org/10.1137/1.9781611973402.116

The input to the NP-hard Point Line Cover problem (PLC) consists of a set P of n points on the plane and a positive integer k, and the question is whether there exists a set of at most k lines which pass through all points in P. By straightforward reduction rules one can efficiently reduce any input to one with at most k^2 points. We show that this easy reduction is essentially tight under standard assumptions. More precisely, we show that unless the polynomial hierarchy collapses to its third level, for any *epsilon* > 0, there is no polynomial-time algorithm that reduces every instance (P, k) of PLC to an equivalent instance with $O(k^{2-\epsilon})$ points. This answers, in the negative, an open problem posed by Lokshtanov (PhD Thesis, 2009). Our proof uses the machinery for deriving lower bounds on the *size* of kernels developed by Dell and van Melkebeek (STOC 2010, JACM 2014). It has two main ingredients: We first show, by a reduction from Vertex Cover, that unless the polynomial hierarchy collapses, PLC has no kernel of *total size* $O(k^{2-\epsilon})$ bits. This does not directly imply the claimed lower bound on the *number of points*, since the best known polynomial-time encoding of a PLC instance with n points requires $\omega(n^2)$ bits. To get around this hurdle we build on work of Alon (Mathematika, 1986) and devise an *oracle communication protocol* of cost $O(n \log n)$ for PLC. This protocol, together with the lower bound on the total size (which also holds for such protocols), yields the stated lower bound on the number of points. While a number of essentially tight polynomial lower bounds on total sizes of kernels are known, our result is to the best of our knowledge, the first to show a nontrivial lower bound for structural/secondary parameters.

3.14 Hitting Forbidden Subgraphs in Graphs of Bounded Treewidth

Marcin Pilipczuk (University of Warwick, GB)

Joint work of Cygan, Marek; Marx, Dániel; Pilipczuk, Marcin; Pilipczuk, Michał
Main reference M. Cygan, D. Marx, M. Pilipczuk, M. Pilipczuk, "Hitting Forbidden Subgraphs in Graphs of Bounded Treewidth," in Proc. of the 39th Int'l Symp. on Mathematical Foundations of Computer Science 2014 (MFCS'14), LNCS, Vol. 8635, pp. 189–200, Springer, 2014.
URL http://dx.doi.org/10.1007/978-3-662-44465-8_17

We study the complexity of a generic hitting problem H-Subgraph Hitting where given a fixed pattern graph H and an input graph G, the task is to find a set $X \subseteq V(G)$ of minimum size that hits all subgraphs of G isomorphic to H. In the colorful variant of the problem, each vertex of G is precolored with some color from $V(H)$ and we require to hit only H-subgraphs with matching colors. Standard techniques shows that for every fixed H, the problem is fixed-parameter tractable parameterized by the treewidth of G; however, it is not clear how exactly the running time should depend on treewidth. For the colorful variant, we demonstrate matching upper and lower bounds showing that the dependence of the running time on treewidth of G is tightly governed by $\mu(H)$, the maximum size of a minimal vertex separator in H. That is, we show for every fixed H that, on a graph of treewidth t, the colorful problem can be solved in time $2^{\mathcal{O}(t^{\mu(H)})} \cdot |V(G)|$, but cannot be solved in time $2^{o(t^{\mu(H)})} \cdot |V(G)|^{O(1)}$, assuming the Exponential Time Hypothesis (ETH). Furthermore, we give some preliminary results showing that, in the absence of colors, the parameterized complexity landscape of H-Subgraph Hitting is much richer. A preliminary version of this work appeared at MFCS 2014.

3.15 Tutorial: Lower Bounds Based on the Exponential Time Hypothesis – Part 2

Michał Pilipczuk (University of Warsaw, PL)

Joint work of Cygan, Marek; Fomin, Fedor; Kowalik, Łukasz; Lokshtanov, Daniel; Marx, Dániel; Pilipczuk, Marcin; Pilipczuk; Saurabh, Saket

During the second part of the tutorial on lower bounds based on ETH, we shall see two examples of specific methodologies for proving negative results about the existence of parameterized algorithms with a certain form of the running time. Firstly, we look into problems that are solvable in slightly-superexponential running time, i.e., $O^*(2^{O(k \log k)})$, and then we examine geometric and planar problems for which XP algorithms with running time $n^{O(\sqrt{k})}$ exist. For both of these families, ETH can be used to pinpoint the optimum form of the running time of a parameterized algorithm solving the problem.

3.16 Tree Deletion Set Has a Polynomial Kernel (but no $OPT^{O(1)}$ Approximation)

Ondrej Suchý (Czech Technical University – Prague, CZ)

Joint work of Giannopoulou, Archontia C.; Lokshtanov, Daniel; Saurabh, Saket; Suchý, Ondrej
Main reference A. C. Giannopoulou, D. Lokshtanov, S. Saurabh, O. Suchý, "Tree Deletion Set Has a Polynomial Kernel (but no $OPT^O(1)$ Approximation)," in Proc. of the 34th Int'l Conf. on Foundations of Software Technology and Theoretical Computer Science (FSTTCS'14), LIPIcs, Vol. 29, pp. 85-96, 2014.
URL http://dx.doi.org/10.4230/LIPIcs.FSTTCS.2014.85

In the Tree Deletion Set problem the input is a graph G together with an integer k. The objective is to determine whether there exists a set S of at most k vertices such that $G \setminus S$ is a tree. The problem is NP-complete and even NP-hard to approximate within any factor of $\mathtt{OPT}^c$ for any constant c. In this talk we give an $O(k^5)$ size kernel for Tree Deletion Set. An appealing feature of our kernelization algorithm is a new reduction rule, based on system of linear equations, that we use to handle the instances on which Tree Deletion Set is hard to approximate.

3.17 Backdoors, Satisfiability, and Problems Beyond NP

Stefan Szeider (TU Wien, AT)

In this talk I will survey parameterized complexity results for problems related to finding and using backdoors, mainly focusing on the propositional satisfiability problem (SAT). In addition I will discuss recent results on using backdoors to break through complexity barriers between higher levels of the Polynomial Hierarchy.

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2 Ronald de Haan and Stefan Szeider. *Compendium of Parameterized Problems at Higher Levels of the Polynomial Hierarchy.* Electronic Colloquium on Computational Complexity (ECCC) 21:143 (2014). http://eccc.hpi-web.de/report/2014/143/

3.18 Tutorial: Implications of Strong ETH for Polynomial Time Solvable Problems

Virginia Vassilevska Williams (Stanford University, US)

Joint work of Vassilevska Williams, Virginia; Roditty, Liam; Abboud, Amir; Weimann, Oren

The Strong Exponential Time Hypothesis (SETH) asserts that there is no $2^{(1-\epsilon)n}\mathtt{poly}(m)$ time algorithm (for constant $\epsilon > 0$) that solves k-SAT (for arbitrary k) formulas on n

variables and m clauses. Although unproven, SETH has been a popular conjecture. Recent research has resulted in a variety of conditional lower bounds based on SETH for wide-studied problems within polynomial time. In this talk I will survey some of these. Some examples of results that would refute SETH include:

- a truly subquadratic time 1.499-approximation algorithm for the diameter problem in sparse graphs,
- any fully dynamic algorithm for maintaining the number strongly connected components of a directed graph with nontrivial update time and
- a truly subquadratic time algorithm for local sequence alignment.

3.19 Parameterized Complexity of the k-Chinese Postman Problem

Anders Yeo (Singapore University of Technology and Design, SG)

Joint work of Gutin,Gregory; Muciaccia, Gabriele; Yeo, Anders
Main reference G. Gutin, G. Mucciaccia, A.Yeo, "Parameterized complexity of k-Chinese Postman Problem," Theoretical Computer Science, 513(2013):124–128; pre-print available as arXiv:1308.0482v2 [cs.DS], 2013.
URL http://dx.doi.org/10.1016/j.tcs.2013.10.012
URL http://arxiv.org/abs/1308.0482v2

We consider the following problem called the k-Chinese Postman Problem (k-CPP): given a connected edge-weighted graph G and integers p and k, decide whether there are at least k closed walks such that every edge of G is contained in at least one of them and the total weight of the edges in the walks is at most p? The problem k-CPP is NP-complete, and van Bevern et al. and Sorge asked whether the k-CPP is fixed-parameter tractable when parameterized by k. We will prove that the k-CPP is indeed fixed-parameter tractable. In fact, we prove a stronger result: the problem admits a kernel with $O(k^2 \log k)$ vertices.

3.20 On Graph Motif Problems Parameterized by Dual

Christian Komusiewicz (TU Berlin, DE)

Joint work of Komusiewicz, Christian; Fertin, Guillaume
Main reference Work in progress

The Graph Motif problem has as input a vertex-colored graph $G = (V, E)$ with k different vertex colors and asks whether there is a connected subgraph on k vertices containing each color exactly once. We study Graph Motif parameterized by $\ell = |V| - k$. For general graphs we show that, assuming the strong exponential time hypothesis, a previous $O(2^\ell \cdot |E|)$ time algorithm is optimal. We then provide a faster algorithm for trees. We also consider the List-Colored Graph Motif problem. In this extension of Graph Motif each vertex may choose its color from a list of colors. For this variant, we strengthen previous hardness results by showing for example that the problem remains W[1]-hard when the color lists have length at most two.

3.21 The Complexity of Geometric Problems in High Dimension

Christian Knauer (Universität Bayreuth, DE)

Many important NP-hard geometric problems in $\mathbb{R}^d$ are trivially solvable in time $n^{O(d)}$ (where n is the size of the input), but such a time dependency quickly becomes intractable for higher-dimensional data, and thus it is interesting to ask whether the dependency on d can be mildened. We try to adress this question by applying techniques from parameterized complexity theory.

More precisely, we describe two different approaches to show parameterized intractability of such problems: A framework that gives fpt-reductions from the k-clique problem to a large class of geometric problems in $\mathbb{R}^d$, and a different approach that gives fpt-reductions from the k-Sum problem.

While the second approach seems conceptually simpler, the first approach often yields stronger results, in that it further implies that the d-dimensional problems reduced to cannot be solved in time $n^{o(d)}$, unless the Exponential Time Hypothesis (ETH) is false.

3.22 Tutorial: Lower Bounds Based on the Strong Exponential Time Hypothesis

Daniel Lokshtanov (University of Bergen, NO)

Joint work of Cygan, Marek; Fomin, Fedor; Kowalik, Łukasz; Lokshtanov, Daniel; Marx, Dániel; Pilipczuk, Marcin; Pilipczuk; Saurabh, Saket

This talk will be an introduction to lower bounds based on the Strong Exponential Time Hypothesis (SETH). We will discuss the hypothesis itself, as well as the different kinds of lower bounds that can be obtained assuming the SETH. We also discuss related research directions and open problems.

3.23 Fixed-Parameter Tractable Canonization and Isomorphism Test for Graphs of Bounded Treewidth

Saket Saurabh (The Institute of Mathematical Sciences, IN)

Joint work of Lokshtanov, Daniel; Pilipczuk, Marcin; Pilipczuk, Michał; Saurabh, Saket
Main reference D. Lokshtanov, M. Pilipczuk, M. Pilipczuk, S. Saurabh, "Fixed-parameter tractable canonization and isomorphism test for graphs of bounded treewidth," in Proc. of the 55th IEEE Annual Symposium on Foundations of Computer Science (FOCS'14), pp. 186–195, IEEE, 2014.
URL http://dx.doi.org/10.1109/FOCS.2014.28

We give a fixed-parameter tractable algorithm that, given a parameter k and two graphs G_1, G_2, either concludes that one of these graphs has treewidth at least k, or determines whether G_1 and G_2 are isomorphic. The running time of the algorithm on an n-vertex graph is $2^{O(k^5 \log k)} \cdot n^5$, and this is the first fixed-parameter algorithm for Graph Isomorphism parameterized by treewidth.

Our algorithm in fact solves the more general CANONIZATION problem. We design a procedure working in $2^{O(k^5 \log k)} \cdot n^5$ time that, for a given graph G on n vertices, either concludes that the treewidth of G is at least k, or:

- finds in an isomorphic-invariant way a graph $\tau(G)$ that is isomorphic to G;
- finds an isomorphism-invariant *construction term* – an algebraic expression that encodes G together with a tree decomposition of G of width $O(k^4)$.

Hence, the isomorphism test reduces to verifying whether the computed isomorphic copies or the construction terms for G_1 and G_2 are equal.

3.24 Fast Modular Permanents

Andreas Björklund, (Lund University, SE)

Joint work of Björklund, Andreas; Husfeldt, Thore; Lyckberg, Isak

We show that the permanent of an integer $n \times n$ matrix M can be computed in $2^{n-\Omega(n/(dlogd))}$ time in expectation, when you know a bound on the permanent $|per(M)| <= d^n$. This complements results for 0/1-matrices with d ones per row by Izumi and Wadayama (FOCS, 2012), and by Cygan and Pilipczuk (ICALP, 2013), in a surprisingly straight-forward way. Instead of explicitly using that many entries are zero in the matrix, we use that when the permanent is relatively small (as it is in sparse 0/1 matrices), we can employ Chinese remaindering and modular permanents to get almost the same asymptotic result for every sparse matrix with small non-zero elements, but for many matrices it gives better bounds. In particular, a random $\{-1, 0, 1\}$-matrix with d non-zero elements per row on average, with the non-zero elements sampled uniformly and independently from $\{-1, 1\}$, can be solved in $2^{n-\Omega(n/(\sqrt(d)logd))}$ expected time.

We use several ideas from Bax and Franklin's result on 0/1 permanent computation to get a fast algorithm for computing a permanent modulo p^m for prime p. In particular, we show that the permanent modulo 2^k for $k > 1$ can be computed in kn^{k+1} time, improving on Valiant's n^{4k-3} time algorithm from 1980.

4 Open Problems

4.1 Solving ILPF in Single Exponential Time in the Number of Variables

Stefan Kratsch (TU Berlin, DE)

Given a matrix $A \in \mathbb{Z}^{m \times p}$ and a vector $b \in \mathbb{Z}^m$, INTEGER LINEAR PROGRAMMING FEASIBILITY (ILPF) asks you to find a vector $x \in \mathbb{Z}^p$ such that $Ax \leq b$.

Question

Is there a $c^p m^{O(1)}$ time algorithm for some constant c, that solves ILPF?

Remark

The best known algorithm has running time $p^{O(p)} m^{O(1)}$ [1].

References

1 Ravi Kannan. Minkowski's convex body theorem and integer programming. *Math. Oper. Res.*, 12(3):415–440, August 1987.

4.2 Parameterized Cutwidth

Saket Saurabh (The Institute of Mathematical Sciences, IN)

Given a graph G and an integer w CUTWIDTH asks if there is a linear ordering of the vertices of G such that any vertical line intersects at most w edges.

Question

- Is there an XP-algorithm for CUTWIDTH parameterized by treewidth?
- Is there an XP-algorithm for CUTWIDTH parameterized by the size of the minimal feedback vertex set?

4.3 Batched k-CNF

Virginia Vassilevska Williams (Stanford University, US)

Question

Is there an $\epsilon > 0$ and an integer $l >= 1$ such that for all integers k and for all n, for any k-CNF formula F over n variables $x_1, \ldots, x_n$, the question of whether:

$$\forall x_1, \ldots, x_{n/l} \exists x_{n/l+1}, \ldots, x_{2n/l} \forall x_{2n/l+1}, \ldots, x_{3n/l} \exists \forall \cdots \exists x_{n-n/l+1}, \ldots, x_n F(x_1, \ldots, x_n)$$

is true can be decided in time $O^*(2^{(1-\epsilon)n})$?

4.4 Does Bounded Cliquewidth Imply Bounded Linear Cliquewidth?

Daniel Lokshtanov (University of Bergen, NO)

The cliquewidth of a graph G is the minimum number of labels needed to construct G by applying the following four operations:

- creating of a new vertex v with label i,
- taking the disjoint union of two labeled graphs G and H,
- making every vertex labeled i adjacent to every vertex labeled j and
- renaming label i to label j.

The linear cliquewidth of a graph G is defined as a restricted version of cliquewidth where one is only allowed to take the disjoint union of G and H if at least one of G and H is a graph on a single vertex.

Question

Let $\mathcal{F}_k$ be the class of minimal forbidden induced subgraphs in the class of graphs with cliquewidth at most k. Is there a function $f : \mathbb{N} \to \mathbb{N}$ such that the linear cliquewidth of every graph in $\mathcal{F}_k$ is bounded by $f(k)$?

Remark

This question is supposed to help on the road to proving that CLIQUEWIDTH is in FPT.

4.5 Quadratic Kernel for Planar Steiner Tree

Michał Pilipczuk (University of Warsaw, PL)

Given a planar graph G, a set of terminals $T \subseteq V(G)$ and an integer k, PLANAR STEINER TREE asks for a set of vertices $X \subseteq V(G)$ of size at most k such that $G[V \cup T]$ is connected.

Question

Does PLANAR STEINER TREE admit a kernel with $O(k^2)$ many vertices?

Remark

PLANER STEINER TREE has a kernel of size $O(k^{142})$ [1].

References

1 M. Pilipczuk, M. Pilipczuk, P. Sankowski, and E.J. Van Leeuwen. Network sparsification for steiner problems on planar and bounded-genus graphs. In *Foundations of Computer Science (FOCS), 2014 IEEE 55th Annual Symposium on*, pages 276–285, Oct 2014.

4.6 From Parity Set Cover to Parity SAT

Holger Dell (Universität des Saarlandes, DE)

Given a universe U of size n, a set family $\mathcal{F} \subseteq 2^U$ of size m and an integer t, $\oplus$Set Cover asks if the number of subsets $\mathcal{X} \subseteq \mathcal{F}$ of size at most t such that $\bigcup \mathcal{X} = U$ is odd. Similarly, $\oplus$SAT asks if the number of satisfying assignments of a SAT-instance is odd.

A serf-reduction is a reduction that preserves sub-exponential running times. Meaning that the resulting instance should be linear in the original instance and not require more than sub-exponential time to compute [2].

Question

Cygan et al. [1] gave a serf-reduction from $\oplus$SAT to $\oplus$Set Cover. Is there a serf-reduction from $\oplus$Set Cover to $\oplus$SAT?

References

1 Marek Cygan, Holger Dell, Daniel Lokshtanov, Dániel Marx, Jesper Nederlof, Yoshio Okamoto, Ramamohan Paturi, Saket Saurabh, and Magnus Wahlström. On problems as hard as CNF-SAT. In *Proceedings of the 27th Conference on Computational Complexity, CCC 2012, Porto, Portugal, June 26–29, 2012*, pages 74–84.

2 Russell Impagliazzo, Ramamohan Paturi, and Francis Zane. Which problems have strongly exponential complexity? In *Foundations of Computer Science, 1998. Proceedings. 39th Annual Symposium on*, pages 653–662. IEEE, 1998.

4.7 Lower Bounds for Computing Treewidth

Hans L. Bodlaender (Utrecht University, NL)

Question

Is there a lower bound on the time complexity for computing the treewidth of a graph? For example, is an $2^{O(k)}n^{O(1)}$ algorithm impossible under ETH?

4.8 TSP Below Average on Digraphs

Gregory Z. Gutin (Royal Holloway University of London, GB)

Let G be a complete digraph with integral weights on the arcs, h the average weight of a Hamiltonian cycle in G and k an integer. Then TSP Below Average asks whether there is a Hamiltonian cycle in G of weight at most $h - k$.

Question

Is TSP Below Average in FPT when parameterized by k?

Remark

It has been proven that TSP Below Average is in FPT when parameterized by k if G is a complete, undirected graph [1].

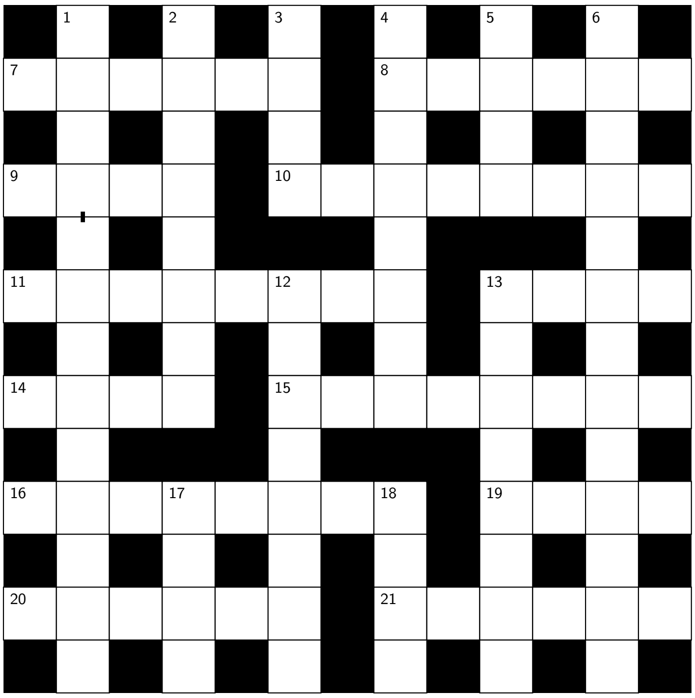
References

1 Gregory Gutin and Viresh Patel. Parameterized TSP: beating the average. *CoRR*, abs/1408.0531, 2014.

5 Puzzles

5.1 Cryptic Crossword

Thore Husfeldt (IT University of Copenhagen, DK)



Across

7 Freethinker Nelson hides a smaller problem. (6)
8 Mr No is crazy for graphs like K_5 and $K_{3,3}$. (6)
9 Remote application for algorithm rejected initially. (4)
10 Uppermost parts of perennial Unix command for displaying acyclic graph in PostScript? (8)
11 Peer pressured? (8)
13 Gadget for viewing booty in the mirror. (4)
14 Leaders of Japanese universities lament yearly when ICALP is. (4)
15 The German left room before Erik Demaine briefly blushed. (8)
16 Holger Dell, perhaps, to date teen in disguise. (8)
19 At the start of reduction, apply a trick. (4)
20 Rod is covered in soft hair, reportedly. (6)
21 Importance of mean and variance, for instance. (6)

Down

1 Important concept in exponential time complexity is belittling to slave. (4-9)
2 A connected, bridgeless cubic graph with chromatic index equal to 4 expresses "I love you" in a sharply critical fasion. (8)
3 Leaf lattice contains a set whose closure equals itself. (4)
4 Aunt had intercourse, as represented on a surface. (8)
5 Sounds like you mend one. (4)
6 Unix filesystem in important kernelization. (13)
12 Ask wordy drunk when Dagstuhl seminars take place. (8)
13 Robertson and Seymour's results maybe confused me and others. (8)
17 Blow nose for very long running times. (4)
18 At first, Eppstein liked minimum spanning trees. (4)

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