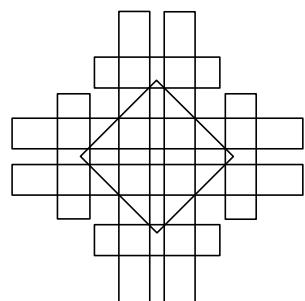


# **31st International Symposium on Computational Geometry**

**SoCG'15, June 22–25, 2015, Eindhoven, The Netherlands**

Edited by

**Lars Arge  
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Dedicated to the memories of Ferran Hurtado and Jiří Matoušek.



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## ■ Foreword

The research papers, the abstracts of invited talks, and the descriptions of videos and multimedia presentations in this volume constitute the proceedings of the 31st International Symposium on Computational Geometry (SoCG'15), which was held as part of CG Week 2015 at TU Eindhoven, the Netherlands, June 22-25, 2015.

There were 154 papers submitted to SoCG'15 of which the program committee selected 59 for presentation after a substantial review process involving 241 external reviewers. The online submission and the review were conducted using EasyChair. Several papers have been selected for the special issues of Discrete & Computational Geometry and the Journal of Computational Geometry dedicated to SoCG'15.

In addition to the technical papers, four submissions—three videos and one applet—were received in response to the Call for Video and Multimedia. All four were reviewed and accepted for presentation. The extended abstracts that describe the accepted submissions are included in this proceedings. The final versions of the videos for the accepted submissions will be archived at <http://www.computational-geometry.org>.

The Best Paper Award went to the paper “Combinatorial Discrepancy for Boxes via the  $\gamma_2$  Norm” by Jiří Matoušek and Aleksandar Nikolov. The Best Student Presentation Award will be determined at the symposium based on input of the attendees. We thank all authors of submitted papers, videos and multimedia presentations. We also thank all people who gave their time for the quality and success of this conference, especially the local organizers, the external reviewers, and the program committee members.

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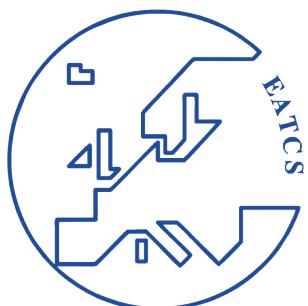


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