

Dynamic Traffic Models in Transportation Science

Edited by

José R. Correa¹, Tobias Harks², Kai Nagel³, Britta Peis⁴, and
Martin Skutella⁵

1 University of Chile – Santiago de Chile, CL, correa@uchile.cl

2 Maastricht University, NL, t.harks@maastrichtuniversity.nl

3 TU Berlin, DE, nagel@vsp.tu-berlin.de

4 RWTH Aachen, DE, britta.peis@oms.rwth-aachen.de

5 TU Berlin, DE, martin.skutella@tu-berlin.de

Abstract

Traffic assignment models are crucial for traffic planners to be able to predict traffic distributions, especially, in light of possible changes of the infrastructure, e.g., road constructions, traffic light controls, etc. The starting point of the seminar was the observation that there is a trend in the transportation community (science as well as industry) to base such predictions on complex computer-based simulations that are capable of resolving many elements of a real transportation system. On the other hand, within the past few years, the theory of dynamic traffic assignments in terms of equilibrium existence and equilibrium computation has not matured to the point matching the model complexity inherent in simulations. In view of the above, this interdisciplinary seminar brought together leading scientists in the areas *traffic simulations*, *algorithmic game theory* and *dynamic traffic assignment* as well as people from industry with strong scientific background who identified possible ways to bridge the described gap.

Seminar October 4–9, 2015 – <http://www.dagstuhl.de/15412>

1998 ACM Subject Classification F.2 Analysis of Algorithms and Problem Complexity, I.6 Simulation and Modelling, E.1 Data Structures

Keywords and phrases Dynamic traffic equilibria, Complexity of equilibrium computation, Simulation, Dynamic network flow theory, Network optimization

Digital Object Identifier 10.4230/DagRep.5.10.19

Edited in cooperation with Miriam Schlöter

1 Executive Summary

José Correa

Tobias Harks

Kai Nagel

Britta Peis

Martin Skutella

License © Creative Commons BY 3.0 Unported license
© José Correa, Tobias Harks, Kai Nagel, Britta Peis, and Martin Skutella

Traffic assignment models play an important role for traffic planners to predict traffic distributions, especially, in light of possible changes of the infrastructure, e.g., road constructions, traffic light controls, etc. The prevailing *mathematical* approaches used in the transportation science literature to predict such distributions can be roughly classified into static traffic



Except where otherwise noted, content of this report is licensed under a Creative Commons BY 3.0 Unported license

Dynamic Traffic Models in Transportation Science, *Dagstuhl Reports*, Vol. 5, Issue 10, pp. 19–34

Editors: José R. Correa, Tobias Harks, Kai Nagel, Britta Peis, and Martin Skutella



DAGSTUHL
REPORTS

Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

assignment models based on aggregated static multi-commodity flow formulations and dynamic traffic assignment (DTA) models based on the methodology of flows over time. While static models have seen several decades of development and practical use, they abstract away too many important details and, thus, become less attractive. On the other hand, dynamic models are known to be notoriously hard to analyze in terms of existence, uniqueness and computability of dynamic equilibria.

In light of the prevailing computational difficulties for realistic-sized networks, the systematic optimization of such networks (e.g., by designing the network infrastructure, link tolls, or traffic light controls) becomes even more challenging as the resulting mathematical programs with equilibrium constraints contain already in the lower level presumably “hard” optimization-, complementarity- or variational inequality problems; not to speak of the resulting optimization problem for the first level.

On the other hand, there is a trend in the transportation science community to use *large-scale computer-based microsimulations* for predicting traffic distributions. The striking advantage of microscopic simulations over DTA models is that the latter usually ignore the feedback of changing network conditions on user behavior dimensions such as flexible departure time choice, mode choice, activity schedule choice, and such. Current simulation tools integrate all these dimensions and many more. The increasing model complexity, however, is by far not matched by the existing theory of dynamic traffic assignments. Against this background, the seminar provided (partial) answers to questions of the following type:

- Under which conditions do microscopic simulation models and dynamic traffic assignment models admit an equilibrium?
- Is an equilibrium efficiently (polynomial time) computable?
- Which models lead to multiple equilibria and how do the parameters of a learning process influence the resulting equilibrium outcome?
- What are the implications of possible intractability results (PPAD-hardness) on the plausibility of existing models?
- how do we compute optimal (or approximatively) network designs or traffic light controls subject to dynamic equilibrium constraints in polynomial time?

The seminar brought together leading researchers from three different communities – Simulations (SIM), Dynamic Traffic Assignment (DTA) and Algorithmic Game Theory (AGT) – and identified ways to narrow the existing gap between complex simulation based models and the existing theory. Among other points, the seminar initiated a systematic study of the complexity of equilibrium computations for DTA models – which is the core task when resolving dynamic traffic assignment problems. Equilibrium computation and its complexity status is a core topic in AGT. The seminar provided an excellent forum for a discourse of these questions between the DTA, SIM and AGT community which initiated several novel research questions and directions. The seminar also stimulated a conceptual discourse regarding the validity of DTA and microscopic simulation models in terms of their predictive power and use for optimization based approaches.

Overall, the seminar was a big success both in terms of stimulating new and very fruitful collaborations between so far separate communities and also with respect to novel insights and results on traffic equilibria and related concepts. We got enthusiastic feedback from many participants which is also reflected in the survey conducted by Dagstuhl.

2 Table of Contents

Executive Summary

José Correa, Tobias Harks, Kai Nagel, Britta Peis, and Martin Skutella 19

Overview of Talks

A Stackelberg Strategy for Routing Flow over Time <i>Umang Bhaskar</i>	23
Dynamic Equilibrium in Network Flows (a limited survey) <i>Roberto Cominetti</i>	23
Tight Bounds for Cost-Sharing in Weighted Congestion Games <i>Martin Gairing</i>	23
Cascading to Equilibrium: Hydraulic Computation of Equilibria in Resource Selection Games <i>Yannai A Gonczarowski</i>	24
Coordination Mechanisms and Routing over Time <i>Martin Hoefer</i>	24
Complexity and Approximation of the Continuous Network Design Problem <i>Max Klimm</i>	25
Present and Future of Dynamic Traffic at TomTom <i>Felix Koenig</i>	25
Detecting Braess's paradox in MATSim <i>Kai Nagel</i>	26
Congestion games with strategic departures <i>Marco Scarsini</i>	26
Computing Earliest Arrival Flows in Polynomial Space <i>Miriam Schlöter</i>	26
Atomic Selfish Routing over Time <i>Daniel Schmand</i>	27
Dynamic Atomic Congestion Games with Seasonal Flows <i>Marc Schroeder</i>	27
Inefficiency caused by Risk Aversion in Selfish Routing <i>Nicolás E. Stier-Moses</i>	28
Traffic signal optimization and user equilibria <i>Martin Strehler</i>	29
New iterative algorithm for the Link Transmission Model: DTA designed in support of optimization procedures <i>Chris M. J. Tampère</i>	29
Dynamic network loading for congested networks in DTA: survey of theory, models and properties <i>Chris M. J. Tampère</i>	30
Uniqueness of Nash Equilibria in Atomic Splittable Congestion Games <i>Veerle Timmermans</i>	30

Transportation Networks: Some Useful Tools for Analysing “Flows over time”?	
<i>Dave P. Watling</i>	31
Equilibrium Computation for Linear Complementarity Problems	
<i>Bernhard von Stengel</i>	31
Open Problems	
The Complexity of Computing a Maximum Robust Flow	
<i>Jannik Matuschke</i>	32
Strategic or non-strategic blockages	
<i>Kai Nagel</i>	32
A polyhedral approach to thin flows	
<i>Neil Olver</i>	32
Selfish Routing with Uncertainty – The Role of Information	
<i>Alexander Skopalik</i>	33
Open Question Session: “Flows over time” but not as we know it?	
<i>Dave P. Watling</i>	33
Participants	34

3 Overview of Talks

3.1 A Stackelberg Strategy for Routing Flow over Time

Umang Bhaskar (TIFR Mumbai, IN)

License © Creative Commons BY 3.0 Unported license
© Umang Bhaskar

Joint work of Bhaskar, Umang; Fleischer, Lisa; Anshelevich, Elliot

We study the efficiency of routing games in a dynamic queuing model introduced by Koch and Skutella. Prior work on routing games with static flows assumes that users care about either their maximum delay or their total delay. Both these measures are surrogates for measuring how long it takes to get all of a user's traffic through the network, and the use of the dynamic queuing model allows us to directly address this objective. We show that in this model, by reducing network capacity judiciously, the network owner can ensure that the equilibrium is no worse than a small constant times the optimal in the original network, for two natural measures of optimality. These are the first upper bounds on the price of anarchy in this model for general networks.

3.2 Dynamic Equilibrium in Network Flows (a limited survey)

Roberto Cominetti (University of Chile – Santiago de Chile, CL)

License © Creative Commons BY 3.0 Unported license
© Roberto Cominetti

We present an overview of the alternative equilibrium models that have been proposed for dynamic flows: volume-delay functions, LWR link dynamics, fluid queues, abstract models. We discuss both path-based and link-based models and revise the known existence results, pointing out some limitations as well as some open issues. In the second part of the talk we will focus on the computation of equilibria for the fluid queue model, raising a number of basic questions that remain unanswered.

References

- 1 Roberto Cominetti, José Correa, Omar Larré. *Dynamic equilibria in fluid queuing networks*. *Operations Research* 63(1):21–34, 2015.

3.3 Tight Bounds for Cost-Sharing in Weighted Congestion Games

Martin Gairing (University of Liverpool, GB)

License © Creative Commons BY 3.0 Unported license
© Martin Gairing

Joint work of Gairing, Martin; Kollias, Konstantinos ; Kotsialou, Grammateia

Main reference M. Gairing, K. Kollias, G. Kotsialou, "Tight Bounds for Cost-Sharing in Weighted Congestion Games," in Proc. of the 42nd Int'l Colloquium on Automata, Languages, and Programming (ICALP'15), LNCS, Vol. 9135, pp. 626–637, Springer, 2015.

URL http://dx.doi.org/10.1007/978-3-662-47666-6_50

This work studies the price of anarchy and the price of stability of cost-sharing methods in weighted congestion games. We require that our cost-sharing method and our set of cost functions satisfy certain natural conditions and we present general tight price of anarchy

bounds, which are robust and apply to general equilibrium concepts. We then turn to the price of stability and prove an upper bound for the Shapley value cost-sharing method, which holds for general sets of cost functions and which is tight in special cases of interest, such as bounded degree polynomials. Also for bounded degree polynomials, we close this talk with a somehow surprising result, showing that a slight deviation from the Shapley value has a huge impact on the price of stability. In fact, for this case, the price of stability becomes as bad as the price of anarchy.

3.4 Cascading to Equilibrium: Hydraulic Computation of Equilibria in Resource Selection Games

Yannai A Gonczarowski (The Hebrew University of Jerusalem, IL)

License © Creative Commons BY 3.0 Unported license

© Yannai A Gonczarowski

Joint work of Gonczarowski, Yannai A.; Tennenholtz, Moshe

Main reference Y. A. Gonczarowski, M. Tennenholtz, “Cascading to Equilibrium: Hydraulic Computation of Equilibria in Resource Selection Games,” arXiv:1403.7605v6 [cs.GT], 2015.

URL <http://arxiv.org/abs/1403.7605v6>

Drawing intuition from a (physical) hydraulic system, we present a novel framework, constructively showing the existence of a strong Nash equilibrium in resource selection games (i.e., asymmetric singleton congestion games) with nonatomic players, the coincidence of strong equilibria and Nash equilibria in such games, and the uniqueness of the cost of each given resource across all Nash equilibria.

Our proofs allow for explicit calculation of Nash equilibria and for explicit and direct calculation of the resulting (unique) costs of resources, and do not hinge on any fixed-point theorem, on the Minimax theorem or any equivalent result, on linear programming, or on the existence of a potential (though our analysis does provide powerful insights into the potential, via a natural concrete physical interpretation).

A generalization of resource selection games, called resource selection games with I.D.-dependent weighting, is defined, and the results are extended to this family, showing the existence of strong equilibria, and showing that while resource costs are no longer unique across Nash equilibria in games of this family, they are nonetheless unique across all strong Nash equilibria, drawing a novel fundamental connection between group deviation and I.D.-congestion. A natural application of the resulting machinery to a large class of constraint-satisfaction problems is also described.

3.5 Coordination Mechanisms and Routing over Time

Martin Hoefer (Universität des Saarlandes, DE)

License © Creative Commons BY 3.0 Unported license

© Martin Hoefer

Joint work of Hoefer, Martin; Mirrokni, Vahab; Röglin, Heiko; Teng, Shang-Hua

Main reference M. Hoefer, V. Mirrokni, H. Röglin, S.-H. Teng, “Competitive Routing over Time,” *Theoretical Computer Science*, 412(39):5420–5432, 2011.

URL <http://dx.doi.org/10.1016/j.tcs.2011.05.055>

We consider a framework that enhances network congestion games with a notion of time. Our temporal network congestion games are based on coordination mechanisms – local policies

that allow to sequentialize traffic on the edges. We study existence and complexity properties of pure Nash equilibria and best-response strategies for linear latency functions. In some cases our results can be used to characterize convergence properties of various improvement dynamics, by which the population of players can reach equilibrium in a distributed fashion.

3.6 Complexity and Approximation of the Continuous Network Design Problem

Max Klimm (TU Berlin, DE)

License © Creative Commons BY 3.0 Unported license
© Max Klimm

Joint work of Gairing, Martin; Harks, Tobias; Klimm, Max

Main reference M. Gairing, T. Harks, M. Klimm, “Complexity and Approximation of the Continuous Network Design Problem,” in *Approximation, Randomization, and Combinatorial Optimization – Algorithms and Techniques (APPROX/RANDOM’14) – Proc. of the 17th Int’l Workshop on Approximation Algorithms for Combinatorial Optimization Problems and 18th Int’l Workshop on Randomization and Computation, LIPIcs, Vol. 28, pp. 226–241, Schloss Dagstuhl, 2014.*

URL <http://dx.doi.org/10.4230/LIPIcs.APPROX-RANDOM.2014.226>

We revisit the classical (bilevel) continuous network design problem. Given a graph for which the latency of each edge depends on the ratio of the edge flow and the capacity installed, the goal is to find an optimal investment in edge capacities so as to minimize the sum of the routing costs of the induced Wardrop equilibrium and the investment costs for installing the edge’s capacities. We show that continuous network design is APX-hard. As for the approximation of the problem, we provide a detailed analysis for a heuristic studied by Marcotte [1]. Then, we propose a different algorithm and prove that using the better of the two algorithms results in improved approximation guarantees.

References

- 1 P. Marcotte. *Network design problem with congestion effects: A case of bilevel programming*. *Math. Program.*, 34:142–162, 1986.

3.7 Present and Future of Dynamic Traffic at TomTom

Felix Koenig (TomTom – Berlin, DE)

License © Creative Commons BY 3.0 Unported license
© Felix Koenig

TomTom is a world market leader in both dynamic routing and real-time traffic information. Independent research proves that in the present, drivers with different quality of traffic information, and different levels of sophistication in dynamic routing, experience different travel times between common origin and destination. This seems to contradict the common assumption that drivers in dense traffic tend to form a Nash equilibrium.

In future scenarios for transport in Smart Cities, mobility providers might control the routes taken by a significant share of vehicles on the road. This highlights the importance of coalition games in the future of dynamic traffic and can open up new research opportunities.

3.8 Detecting Braess's paradox in MATSim

Kai Nagel (TU Berlin, DE)

License  Creative Commons BY 3.0 Unported license
© Kai Nagel

Joint work of Thunig, Theresa; Nagel, Kai

MATSim (Multi-agent transport simulation) has, as network loading model, a relatively simple queue model, against which agents learn until they are roughly in a Nash Equilibrium. Moving the well-known Braess example into that simulation leads to some maybe unexpected consequences including the fact that the transient is rather different from the steady state, and the Braess paradoxon (that the additional road REDUCES overall system capacity) may not show up or not show up fully. Clearly, all of this can be explained and understood, but it points to the fact that seemingly small details in the representation of the traffic dynamics may lead to rather different solutions.

3.9 Congestion games with strategic departures

Marco Scarsini (LUISS Guido Carli – Rome, IT)

License  Creative Commons BY 3.0 Unported license
© Marco Scarsini

We consider a discrete-time atomic congestion game with a single source-destination pair and a finite number of players who have to reach their destination by an exogenously fixed time t_0 and can decide at what time to enter the system in order to achieve their goal. Players who arrive late incur a huge cost. Each edge has a capacity and a delay.

We start considering a very simple network with just one edge and we prove that the game has no pure Nash equilibrium. The worst mixed Nash equilibrium is symmetric and socially quite bad, since each player pays her minmax cost. The best Nash equilibrium is only marginally better. The social optimum is achieved by spreading players at capacity over time. The price of Anarchy is approximately 2.

There exists a correlated equilibrium whose cost is quite close to the optimum. Therefore a planner could (almost) achieve efficiency in equilibrium by providing a suitable correlation device.

We plan to extend the results to more general networks.

3.10 Computing Earliest Arrival Flows in Polynomial Space

Miriam Schlöter (TU Berlin, DE)

License  Creative Commons BY 3.0 Unported license
© Miriam Schlöter

In the last years, the number of evacuations for example in areas endangered by natural disasters has increased. Thus, a better evacuation planning before the emergency occurs is of great interest. The core of evacuation planning is captured by earliest arrival flows. Given a network \mathcal{N} with capacities and transit times on the arcs, a subset of source nodes with supplies and a single sink node, an earliest arrival flow is a dynamic flow in \mathcal{N} such that the total amount of flow that has arrived at the sink is maximal for all points in time.

In networks with a single source earliest arrival flows can be computed in polynomial space using the successive shortest path algorithm.

For networks with multiple sources Nadine Baumann and Martin Skutella developed an algorithm to compute earliest arrival flows [1]. Their algorithm consists of two parts: At first the earliest arrival pattern is computed and after that using the breakpoints of the pattern the actual earliest arrival flow is derived.

While the first part of the algorithm only works on the original network, the second part requires attaching an additional sink to the network for every breakpoint of the earliest arrival pattern. As the earliest arrival pattern in the worst case can have exponentially many breakpoints, the resulting network also can get exponentially large.

During the computation of the pattern the times at which the sources run empty in an earliest arrival flow are computed. Mainly making use of these times, we present an algorithm to compute earliest arrival flows in networks with multiple sources which only requires polynomial expansion of the original network.

References

- 1 N. Baumann. *Solving evacuations problems efficiently – earliest arrival flows with multiple sources*. In FOCS Proceedings, 399–408, 2006.

3.11 Atomic Selfish Routing over Time

Daniel Schmand (RWTH Aachen, DE)

License  Creative Commons BY 3.0 Unported license
© Daniel Schmand

In this talk we discuss a model for atomic selfish routing games over time. We assume that we are given some demand which travels unsplittable and selfishly from a given origin to a destination over time. In addition to that we are given capacities on the directed edges, which are an upper bound on the amount of flow that is allowed to enter an edge at a certain time. This may force some flow to wait on intermediate nodes of the route.

In the first part of the talk we try to discuss the model and the question how to design good scheduling rules on the edges in order to minimize the sum of arrival times in a social optimum.

In the second part of the talk we assume that we are given some scheduling rules on the edges and discuss the inefficiency and existence of Nash equilibria depending on the chosen scheduling rules. We present some open questions that may be a start for future research.

3.12 Dynamic Atomic Congestion Games with Seasonal Flows

Marc Schroeder (Maastricht University, NL)

License  Creative Commons BY 3.0 Unported license
© Marc Schroeder

We propose a model of discrete time dynamic congestion games with atomic players and a single source-destination pair. The latencies of edges are composed by free-flow transit times and possible queuing time due to capacity constraints.

We give a precise description of the dynamics induced by the individual strategies of players and of the corresponding costs, either when the traffic is controlled by a planner, or when players act selfishly. Importantly, we model seasonalities by assuming that departure flows fluctuate periodically over time.

Our main contributions are two-fold. First, we introduce a measure that captures the queues induced by periodicity of inflows. For socially optimal flows, this measure is the increase in costs compared to uniform departures. The same holds for equilibrium flows, if the network is parallel. In general the analysis is more intricate. We even provide an example in which periodic departures induce lower equilibrium costs than the uniform departures. Second, we illustrate a new dynamic version of Braess’s paradox: the presence of initial queues in a network may decrease the long-run costs in equilibrium. This paradox may arise even in networks for which no Braess’s paradox was previously known.

3.13 Inefficiency caused by Risk Aversion in Selfish Routing

Nicolás E. Stier-Moses (Facebook – Menlo Park, US)

License © Creative Commons BY 3.0 Unported license
© Nicolás E. Stier-Moses

Joint work of Lianeas, Thanasis; Nikolova, Evdokia; Stier-Moses, Nicolás E.

Main reference E. Nikolova, N. E. Stier-Moses, “The Burden of Risk Aversion in Mean-Risk Selfish Routing,” in Proc. of the 16th ACM Conf. on Economics and Computation (EC’15), pp. 489–506, ACM, 2015.

URL <http://dx.doi.org/10.1145/2764468.2764485>

Considering congestion games with uncertain delays, we compute the inefficiency introduced in network routing by risk-averse agents. At equilibrium, agents may select paths that do not minimize the expected latency so as to obtain lower variability. A social planner, who is likely to be more risk neutral than agents because it operates at a longer time-scale, quantifies social cost with the total expected delay along routes. From that perspective, agents may make suboptimal decisions that degrade long-term quality.

We define the price of risk aversion (PRA) as the worst-case ratio of the social cost at a risk-averse Wardrop equilibrium to that agents are risk-neutral. For networks with general delay functions and a single source-sink pair, we show that the PRA depends linearly on the agents’ risk tolerance and on the degree of variability present in the network. In contrast to the price of anarchy, in general the PRA increases when the network gets larger but it does not depend on the shape of the delay functions. To get this result we rely on a combinatorial proof that employs alternating paths that are reminiscent of those used in max-flow algorithms. For series-parallel (SP) graphs, the PRA becomes independent of the network topology and its size. As a result of independent interest, we prove that for SP networks with deterministic delays, Wardrop equilibria maximize the shortest-path objective among all feasible flows.

3.14 Traffic signal optimization and user equilibria

Martin Strehler (TU Cottbus, DE)

License © Creative Commons BY 3.0 Unported license
© Martin Strehler

Joint work of Strehler, Martin; Köhler, Ekkehard

We will present a cyclically time-expanded network model for simultaneously optimizing traffic assignment and traffic signals in an urban road network with a special focus on user equilibria. The model itself combines ideas from static and dynamic models. Being a completely linear approach, this allows the use of strict mathematical programming techniques. However, since a linear model may be too restrictive for real world applications, we investigate in particular travel times and latency functions. A comparison of travel times with simulation tools like MATSim and VISSIM demonstrates the applicability of our approach. These results give rise to numerous open questions concerning user equilibria.

3.15 New iterative algorithm for the Link Transmission Model: DTA designed in support of optimization procedures

Chris M. J. Tampère (KU Leuven, BE)

License © Creative Commons BY 3.0 Unported license
© Chris M. J. Tampère

Main reference W. W. E. Himpe, R. Corthout, C. M. J. Tampère, “An efficient iterative link transmission model.” Transportation Research Part B: under review (R1).

URL <http://tinyurl.com/o65gbfc>

Dynamic Traffic Assignment (DTA) models are often part of some bi-level optimization, e.g. calibration, optimal control, network design. If the optimization is to be solved iteratively, the DTA needs to be repetitively computed on the same network under marginally different inputs.

We present a DTA algorithm capable of calculating equilibria in medium to large networks consistent with LWR theory in feasible time. It is efficient especially when repeatedly calculating solutions on the same network, because both the Dynamic Network Loading (DNL) component and the Dynamic Shortest Path (DSP) calculation are done using a previously computed solution as a warm start. For the DNL this is achieved through a novel iterative algorithm. Rather than computing traffic states consistently at once with short (CFL-compliant) time steps, we iteratively compute a fixed point solution between forward propagation of multiple, destination-based commodity flows and back-propagation of congestion waves until consistency. The potential of the approach is illustrated on case studies of OD-estimation and anticipatory ramp metering, that take the DNL and DTA respectively as a constraint, the sensitivity of which is calculated numerically through finite differencing.

3.16 Dynamic network loading for congested networks in DTA: survey of theory, models and properties

Chris M. J. Tampère (KU Leuven, BE)

License  Creative Commons BY 3.0 Unported license
© Chris M. J. Tampère

This presentation gives an overview of DNL as a supply model for DTA models. It shows the importance of the link and node models that are combined into the DNL. The DNL models in literature are categorized into a point-queue and a physical queue family, based on whether they only carry demand information downstream, or in addition also propagate supply constraints (congestion) upstream. Whereas this has been widely recognized for the link model, the role of an adequate node model is usually underestimated.

Node models combine the task of propagating consistently demand and supply constraints imposed by all connected link ends, with imposing internal constraints themselves (resulting from conflict points where traffic in different directions cannot simultaneously pass). It is shown that such node models exist, but they only have unique solutions under prohibitively strict conditions. The properties and behavior of physical queue models with state-of-the-art node models are illustrated using some pathological cases that have been described in the literature. These cases may have multiple stable solutions with transitions between them being triggered by simple events, they may exhibit oscillatory solutions even under stationary boundary flows, or may diverge to gridlock, a network state with zero flow and infinite travel time. The presentation concludes with an overview of properties of the travel time operator and DTA solution depending on the chosen DNL model.

3.17 Uniqueness of Nash Equilibria in Atomic Splittable Congestion Games

Veerle Timmermans (Maastricht University, NL)

License  Creative Commons BY 3.0 Unported license
© Veerle Timmermans

Joint work of Timmermans, Veerle; Harks, Tobias

Congestion games were first introduced by Rosenthal in 1973 [1]. In these games a finite set of players competes over a finite set of resources. In a pure strategy every player chooses a subset of resources, and the cost for every resource depends on the number of other players choosing this resource. We consider the splittable variant, where every player has a positive demand, and in a pure strategy a player can split this demand over several strategies. Furthermore our cost functions are player-specific.

We introduce the class of two-sided matching matroids, a class where every pair of bases contains a perfect matching in their symmetric difference. We prove that when the strategy space of every player consists of bases of a two-sided matching matroid, the Nash equilibrium is unique, no matter how these strategy spaces are interweaved. We show that this class contains laminar matroids, transversal matroids and graphic matroids of generalized series parallel graphs.

References

- 1 R. Rosenthal. *A class of games possessing pure-strategy Nash equilibria*. International Journal of Game Theory, 2(1):65–67 (1973).

3.18 Transportation Networks: Some Useful Tools for Analysing “Flows over time”?

Dave P. Watling (University of Leeds, GB)

License © Creative Commons BY 3.0 Unported license

© Dave P. Watling

Joint work of Watling, Dave P.; O’Hare, Steven; Connors, Richard; Sumalee, Agachai; Shepherd, Simon; Koh, Andrew; Rasmussen, Thomas; Nielsen, Otto; Prato, Carlo

The purpose of the presentation is to describe some issues arising in the field of static traffic network analysis (using non-atomic congestion games), which may be useful in analyzing problems in which network flows vary over time. Four issues are considered:

1. The Price of Anarchy (PoA) for such static games may vary considerably with a scaling of the origin-destination (s - t) demands, and the nature of this variation (e.g. in terms of the shape of relationship of PoA versus scale factor) differs across networks. Some recent research is briefly described which attempts to characterize “transition phases” in such a relationship (as the set of minimal cost paths or minimal marginal cost paths expands or contracts).
2. An alternative route choice model, representing travellers’ mis-perceptions of travel costs/delays, which smoothly distributes flows over all permitted paths. This model is parameterized in a way that in one limit, the flows that arise approximate Wardrop equilibrium flows to an arbitrary accuracy.
3. Applications of this alternative route choice model for (a) analysing problems of network design (where the resulting mathematical program with equilibrium constraints is smooth in the design variables, given a smooth objective), and (b) modelling competitions between two cities in setting tolls, subject to travellers choosing route and whether to travel (an equilibrium problem with equilibrium constraints).
4. A second alternative route choice model in which relaxed conditions allow traffic to be distributed (in equilibrium) smoothly over only a sub-set of the permitted paths, with some paths unused.

3.19 Equilibrium Computation for Linear Complementarity Problems

Bernhard von Stengel (London School of Economics, GB)

License © Creative Commons BY 3.0 Unported license

© Bernhard von Stengel

The linear complementarity problem (LCP) generalizes linear programming (via the complementary slackness conditions of a pair of optimal primal and dual solutions) and finding Nash equilibria of bimatrix games. Lemke’s classical complementary pivoting algorithm finds a solution to an LCP in many cases.

We give an exposition of Lemke’s algorithm in geometric and algebraic terms, and explain the intrinsic local orientation of the computed path which is naturally defined in terms of signs of determinants. For traffic networks with affine delay functions, a Wardrop equilibrium is the solution to an LCP with the number of paths as its dimension. For details see [1].

References

- 1 Richard W. Cottle, Jong-Shi Pang, Richard E. Stone (1992), *The Linear Complementarity Problem*. Academic Press.

4 Open Problems

4.1 The Complexity of Computing a Maximum Robust Flow

Jannik Matuschke (TU Berlin, DE)

License  Creative Commons BY 3.0 Unported license
© Jannik Matuschke

Consider a capacitated network with source s and sink t , and an integer k . Two players F and C play the following zero-sum game: First F specifies an flow on s - t -paths within the arc capacities, then C interdicts the flow by destroying a set S of k arcs. F 's goal is to maximize the amount of surviving flow, i.e., the flow on paths that do not intersect S . For several years, computing the value of this very basic maximum flow interdiction game was believed to be NP-hard, even when k is fixed to 2. However, recently an error in the proof was discovered, and the complexity of the problem is open again.

4.2 Strategic or non-strategic blockages

Kai Nagel (TU Berlin, DE)

License  Creative Commons BY 3.0 Unported license
© Kai Nagel

We have seen in several presentations, especially from the transport engineering side, that blockages play an important role, especially when they spill back across nodes. Yet, such models seem to be difficult to solve for the mathematical approaches that are presented during the seminar. This presentation introduces some specific flow models, many of them explained in more detail by Chris Tampère in his review talk. The presentation then asks the question if it might be possible to come up with model classes and/or specific problem formations that might be more amenable to the tools that are currently discussed and/or if one could identify directions for progress.

The general direction of the presentation was prepared together with Ekkehard Köhler, Martin Strehler, Chris Tampère.

4.3 A polyhedral approach to thin flows

Neil Olver (VU University of Amsterdam, NL)

License  Creative Commons BY 3.0 Unported license
© Neil Olver

Koch and Skutella have given a structural characterization of equilibria in the deterministic queueing model, which aims to capture crucial dynamic aspects of traffic. They show that the equilibria flow can be decomposed into phases, and each phase can be described as a solution to a problem they dub “thin flows with resetting”. A number of fascinating questions remain open; in particular, it is not known if thin flows with resetting can be computed in polynomial time.

Koch and Skutella give an efficient algorithm for a special case of the problem, “thin flows without resetting”, showing how it can be solved by solving multiple maximum flow problems. I will present an alternate viewpoint and show that thin flows without resetting can be seen as the optimal solution of a certain linear program. The (as of yet unrealized)

hope is that this polyhedral perspective may be useful in attacking the general problem with resetting edges.

4.4 Selfish Routing with Uncertainty – The Role of Information

Alexander Skopalik (Universität Paderborn, DE)

License © Creative Commons BY 3.0 Unported license
© Alexander Skopalik

We discuss open problems related to uncertainty and the role of information in selfish routing.

We consider a model of selfish traffic routing in which users are uncertain about the overall demand in the system. Instead users only have a belief (i.e., a probability distribution) about the current state of the system. Thus, users choose routes to minimize their expected cost.

We seek to understand the role of information about the state of the system on the performance of the system.

An easy example shows that equilibrium flow is improved if players have full information in comparison to having no information. We exhibit a – somewhat counter intuitive – example in which traffic flow is worse if users have information as opposed to the flow without information.

By combining the ideas of the two examples, we can show that there exist examples in which it is optimal to provide only partial information to the users.

This raises questions regarding the optimal signaling mechanism, its performance influence and computational complexity.

4.5 Open Question Session: “Flows over time” but not as we know it?

Dave P. Watling (University of Leeds, GB)

License © Creative Commons BY 3.0 Unported license
© Dave P. Watling

Joint work of Watling, Dave P.; Cantarella, Giulio E.

Main reference D. P. Watling, G. E. Cantarella, “Modelling sources of variation in transportation systems: Theoretical foundations of day-to-day dynamic models,” *Transportmetrica B: Transport Dynamics*, 1(1):3–32, 2013.

URL <http://dx.doi.org/10.1080/21680566.2013.785372>

The focus of “flows over time” has been on the analysis of the kind of dynamics that would be observed within a day (“within-day dynamics”), exploring equilibrium properties arising from these time-dependent interactions. An alternative field of study in the transportation community is the study of “day-to-day dynamics”, which describe the transient process of adaptations of travellers in their choices (which may or may not converge to some equilibrium). There are several classes of such models, and examples are given based on

1. a differential equation representing continuous-time, deterministic dynamics;
2. a difference equation representing discrete-time, deterministic dynamics;
3. a markov process representing discrete-time, stochastic dynamics.

The question is whether existing work in studying flows over time is extensible or could be adapted to understand the transient, non-equilibrium dynamics of such systems, in addition to the within-day dynamics conventionally.

Participants

- Umang Bhaskar
TIFR Mumbai, IN
- Roberto Cominetti
University of Chile – Santiago de Chile, CL
- José R. Correa
University of Chile – Santiago de Chile, CL
- Martin Gairing
University of Liverpool, GB
- Yannai A. Gonczarowski
The Hebrew University of Jerusalem, IL
- Tobias Harks
Maastricht University, NL
- Martin Hofer
Universität des Saarlandes, DE
- Max Klimm
TU Berlin, DE
- Ekkehard Köhler
TU Cottbus, DE
- Felix König
TomTom – Berlin, DE
- Jannik Matuschke
TU Berlin, DE
- Kai Nagel
TU Berlin, DE
- Neil Olver
VU University of Amsterdam, NL
- Britta Peis
RWTH Aachen, DE
- Rahul Savani
University of Liverpool, GB
- Marco Scarsini
LUISS Guido Carli – Rome, IT
- Miriam Schlöter
TU Berlin, DE
- Daniel Schmand
RWTH Aachen, DE
- Marc Schröder
Maastricht University, NL
- Alexander Skopalik
Universität Paderborn, DE
- Martin Skutella
TU Berlin, DE
- Nicolás E. Stier-Moses
Facebook – Menlo Park, US
- Martin Strehler
TU Cottbus, DE
- Chris M. J. Tampère
KU Leuven, BE
- Veerle Timmermans
Maastricht University, NL
- Laura Vargas-Koch
RWTH Aachen, DE
- Bernhard von Stengel
London School of Economics, GB
- Dave P. Watling
University of Leeds, GB

