Report of Dagstuhl Seminar 15441

Duality in Computer Science

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Abstract

This report documents the programme and outcomes of Dagstuhl Seminar 15441 'Duality in Computer Science'. This seminar served as a follow-up seminar to the seminar 'Duality in Computer Science' (Dagstuhl Seminar 13311). In this seminar, we focused on applications of duality to semantics for probability in computation, to algebra and coalgebra, and on applications in complexity theory. A key objective of this seminar was to bring together researchers from these communities within computer science as well as from mathematics with the goal of uncovering commonalities, forging new collaborations, and sharing tools and techniques between areas based on their common use of topological methods and duality.

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1 Executive Summary

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Aims of the seminar

Duality allows one to move between an algebraic world of properties and a spacial world of individuals and their dynamics, thereby leading to a change of perspective that may, and often does, lead to new insights. Because computer science is fundamentally concerned both with specification of programs and the dynamics of their executions, dualities have given rise to active research in a number of areas of theoretical computer science. In this seminar we particularly wanted to concentrate on applications of duality in semantics for continuous data with special focus on probability in computation, algebra and coalgebra, and applications in complexity theory.

The seminar

Our call for participation was exceptionally successful and right up to the actual start of the meeting we were in danger of exceeding the number of places allocated. We see this as a vindication of our aim of bringing these researchers together for exchanging ideas centred around the common topic of duality. The talks offered fell quite naturally into groupings which allowed us to adopt a fairly thematic programme structure:

Day 1, morning session: Duality and classical algebra. Talks by Libor Barto, Michael Pinsker, Max Dickmann, and Marcus Tressl.

Day 1, afternoon session: Duality and categories. Talks by Paul Taylor, Steve Vickers, and Pino Rosolini.

Day 2, morning session: Duality and topology. Talks by Matthew de Brecht, Mathias Schröder, Reinhold Heckmann, and Jean Goubault-Larrecq.

Day 2, afternoon session: Alternative views on duality. Talks by Niels Schwartz, George Hansoul, Rob Myers, and Alexander Kurz.

Day 3, morning session: Duality and coalgebra. Talks by Adriana Balan, Dirk Pattinson, Ulrich Berger, and Samuel J. van Gool.

Day 4, morning session: Duality and domain theory. Talks by Jimmie Lawson, Abbas Edalat, Achim Jung, and Klaus Keimel.

Day 4, afternoon session: Duality and logic. Talks by Peter Schuster, Martín Escardó, Vladimir Shavrukov, and Vasco Brattka.

Day 5, morning session: Duality and probability. Talks by Willem Fouché, Dexter Kozen, Daniela Petrişan, and Drew Moshier.

Final thoughts

As always, Dagstuhl staff were incredibly efficient and helpful which allowed all of us, including the organisers, to focus on the exchange of ideas and plans for joint work. We are sincerely grateful to them for their hospitality and professionalism.

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3 Overview of Talks

3.1 Some aspects of positive coalgebraic logic

Adriana Balan (University Politehnica of Bucharest, RO), Alexander Kurz (University of Leicester, GB), and Jiří Velebil

Main reference A. Balan, A. Kurz, J. Velebil, "Positive fragments of coalgebraic logics," Logical Methods in Computer Science, 11(3):1–51, 2015.
URL http://dx.doi.org/10.2168/LMCS-11(3:18)2015
URL http://arxiv.org/pdf/1402.5922

Positive modal logic was introduced in an influential 1995 paper of Dunn [2] as the positive fragment of standard modal logic, using modal formulas built only from atomic propositions, conjunction, disjunction, box and diamond. The corresponding semantics is based on Kripke frames, equivalently, on coalgebras for the powerset functor. In the present talk we will show how to generalize Dunn's result from Kripke frames to coalgebras for a (weak-pullback preserving) functor on the category of sets, using two well-known dual adjunctions: between sets and Boolean algebras, respectively between posets and bounded distributive lattices. To ensure that the resulting positive coalgebraic logic will indeed extend the logic of distributive lattices by monotone modal operations, we have to work in an ordered-enriched context.

This is joint work with A. Kurz and J. Velebil [1].

References

1 A. Balan, A. Kurz, and J. Velebil. Positive Fragments of Coalgebraic Logics. *Log. Methods Comput. Sci.*, 11(3:18):1–51, 2015.
2 J. M. Dunn. Positive Modal Logic. *Studia Logica*, 55(2):301–317, 1995.

3.2 The CSP Basics Revisited

Libor Barto (Charles University – Prague, CZ)

Joint work of Barto, Libor; Opršal, Jakub; Pinsker, Michal
Main reference L. Barto, J. Opršal, M. Pinsker, "The wonderland of reflections", Unpublished manuscript, Oct. 2015.
URL http://www.karlin.mff.cuni.cz/~barto/Articles/0_doubleshrink.pdf

There are three reductions used to compare the complexity of constraint satisfaction problems (CSPs) over two relational structures: a) reduction by means of pp-interpretation, b) via homomorphic equivalence, c) by adding constants to cores. A fundamental fact for the CSP theory is that pp-interpretations between relational structures correspond to continuous homomoprhisms between the algebraic alter egos of these structures – their polymorphism clones. We extend this fact to include the other two reductions.

As a corollary, we get that the complexity of the CSP over a finite structure X depends only on height 1 identities satisfied by polymorphisms of X. The role of continuity for CSPs over infinite relational structures will be discussed as well.

3.3 Duality and Concurrency in Program Extraction

Ulrich Berger (Swansea University, GB)

Program extraction from proofs combines logic and programming to a method for creating provably correct software [1]. In this talk we look at two important principles, one from logic and one from programming, that so far were out of reach for program extraction, and propose ways to change this.

The principle in question from logic is duality in categories, more specifically, the duality of initial algebras and terminal coalgebras which correspond to induction and coinduction. In constructive logic or type theory (the usual formalisms behind program extraction) this duality seems broken, and while induction are constructively well-understood the constructive theory of coinduction is still in a state of flux.

The programming principle in question is concurrency, that is, the possibility that computations are executing simultaneously, are potentially interacting with each other, and may deliver conflicting results. In addition, some of the simultaneous computations may diverge. In traditional program extraction all computations terminate and there is no concurrency.

We will introduce a formal system for program extraction that reinstalls the duality between induction and coinduction and allows concurrency. The latter will be illustrated by applying program extraction to Tsuiki's infinite Gray-code [2].

References

1 U. Berger, K. Miyamoto, H. Schwichtenberg, and M. Seisenberger. Minlog – a tool for program extraction for supporting algebra and coalgebra. In *CALCO-Tools*, volume 6859 of *LNCS*, pages 393–399. Springer, 2011.

2 H. Tsuiki. Real Number Computation through Gray Code Embedding. *Theor. Comput. Sci.*, 284(2):467–485, 2002.

3.4 Residuals in the Weihrauch lattice and their applications

Vasco Brattka (Universität der Bundeswehr – München, DE)

Joint work of Vasco Brattka with Arno Pauly and Matthew Hendtlass, Alexander P. Kreuzer
Main reference V. Brattka, M. Hendtlass, A. P. Kreuzer, "On the Uniform Computational Content of Computability Theory," arXiv:1501.00433v2 [math.LO], 2015.
URL http://arxiv.org/abs/1501.00433v2

We discuss the algebraic structure of the Weihrauch lattice. Besides infimum and supremum, the lattice comes equipped with a product and a compositional product as well as two natural closure operators, the star product and the parallelization. Hence, the Weihrauch lattice carries a rich algebraic structure and with a certain subset of these operations it is, for instance, a Kleene algebra. In this talk we address the fact that none of the operations inf, sup and product have left or right residuals, but the compositional product $*$ has a one-sided residual that we call implication $\rightarrow$. This residual has a number of interesting applications and can be used to characterize computational problems such as Martin-Löf randomness MLR or cohesiveness COH. For instance, we obtain $\mathrm{MLR} \equiv_{\mathrm{W}} (\mathrm{C_N} \rightarrow \mathrm{WWKL})$ and $\mathrm{COH} \equiv_{\mathrm{W}} (\lim \rightarrow \mathrm{KL})$, where $\mathrm{C_N}$ denotes choice on the Natural numbers, KL denotes König's Lemma and WWKL Weak Weak König's Lemma.

3.5 Duality theory for quasi-Polish and represented spaces

Matthew de Brecht (NICT – Osaka, JP)

We will introduce some recent duality-related research within three categories of increasing generality.

The first topic concerns the category of quasi-Polish spaces. An important result by R. Heckmann shows that these spaces dually correspond to countably presented frames. Stated differently, quasi-Polish spaces can be viewed as the spaces of models of countably axiomatized propositional geometric theories. We will introduce joint work with T. Kawai showing that this correspondance naturally extends to the standard powerspace / powerlocale constructions. This provides a connection between quasi-Polish co-algebras and propositional geometric modal logic.

The second topic concerns the (cartesian closed) category of QCB-spaces. Recent work by M. Schröder and V. Selivanov organized these spaces into a hierarchy according to the complexity of their admissible representations. A separate but related hierarchy was later defined by Schroder, Selivanov, and the current author which characterizes spaces according to the complexity of defining a basis for their topology. Based on the hierarchies, we will raise the question of whether the duality characterizations of quasi-Polish spaces have natural generalizations within the category of QCB-spaces.

The third topic concerns the (locally cartesian closed) category of (Baire-) represented spaces. We will introduce some joint work with A. Pauly which uses Sierpinski-like objects to classify Borel subsets of spaces. We show that some of the basic results of P. Taylor's Abstract Stone Duality have natural interpretations in this setting. In particular, we provide an "abstract" version of the Jayne-Rogers theorem, which is of interest in descriptive set theory.

This work was supported by JSPS Core-to-Core Program, A. Advanced Research Networks and by JSPS KAKENHI Grant Number 15K15940.

3.6 Exploring the domain-theoretic content of some mathematical structures

Max Dickmann (University of Paris VII, FR)

The first half of the talk was devoted to:

1. Indicate the relationship between the topology of a spectral space and the Scott topology of its specialization order.
2. Characterize in terms of order the spectrality of the Scott topology on forests and root systems, and determine its relation with the coarse lower topology in these and other, related cases.

In the second half we stated some outstanding domain-theoretic properties of the following structures:

- The spaces of preorders and of quadratic modules of (commutative, unitary) rings where -1 is not a sum of squares.

- Various classes of prime ideals of the ring of continuous, real-valued functions on a topological space.
- Root systems of finite Krull dimension.

The results reported in this talk will appear, among many other related results, in Chapter 11 of the forthcoming book [1].

References

1 M. Dickmann, N. Schwartz, M. Tressl. *Spectral Spaces*. Cambridge University Press (approx. 450 pages; to appear in 2017).

3.7 Differentiation in Logical Form

Abbas Edalat (Imperial College London, GB)

Joint work of Mehrdad Maleki
Main reference A. Edalat, "A continuous derivative for real-valued functions," in S. B. Cooper, B. Lowe, A. Sorbi (eds.), "New Computational Paradigms, Changing Conceptions of What is Computable," LNCS, Vol. 4497, pp. 493–519, Springer, 2008; pre-print available from author's webpage.
URL http://dx.doi.org/10.1007/978-3-540-73001-9_26
URL http://www.doc.ic.ac.uk/~ae/papers/banachff.pdf
Main reference A. Edalat, A. Lieutier, D. Pattinson, "A Computational Model for Multi-Variable Differential Calculus," Information and Computation, Vol. 224, pp. 22–45, 2013.
URL http://dx.doi.org/10.1016/j.ic.2012.11.006

We introduce a point free, localic representation of the Clarke gradient of a real valued locally Lipschitz map on a finite dimensional Euclidean space. Our work is based on the domain-theoretic derivation of the Clarke gradient, called the L-derivative, which defines the Clarke gradient of a Lipschitz map as the supremum of the single-step functions that correspond to the single-ties the function satisfies. Here, a single-tie is a collection of locally Lipschitz maps that have a generalised compact, convex set-valued Lipschitz constant in an open neighbourhood in the Euclidean space. We introduce the notion of a strong single-tie which has the following property: The L-derivative of a Lipschitz map is the supremum of single-step functions, each way-below the L-derivative, that correspond to the strong single-ties the function satisfies. For the localic representation of the L-derivative using approximable mappings, we define a strong knot of approximable mappings, with respect to a given open neighbourhood and and an open set, which satisfies the following stone duality: If a Lipschitz map is in the strong tie of an open neighbourhood and a compact convex set, then the approximable mapping representing the Lipschitz map is in the knot of the open neighbourhood and any open set containing the compact convex set. Conversely, if an approximable mapping is in the strong knot of an open neighbourhood and an open set then there exists a compact convex subset of the open set such that the Lipschitz map corresponding to the approximable mapping is in the strong tie of the open neighbourhood and any open set containing the compact convex set. We then show that this duality enables us to obtain the L-derivative in localic framework.

3.8 Compactly generated Hausdorff locales

Martin H. Escardo (University of Birmingham, GB)

We say that a Hausdorff locale is compactly generated if it is the colimit of the diagram of its compact sublocales connected by inclusions. We show that this is the case if and only if the natural map of its frame of opens into the second Lawson dual is an isomorphism. More generally, for any Hausdorff locale, the second dual of the frame of opens gives the frame of opens of the colimit. In order to arrive at this conclusion, we generalize the Hofmann–Mislove–Johnstone theorem and some results regarding the patch construction for stably locally compact locales. In particular, the colimit is the patch of the first Lawson dual of the frame.

3.9 Topological dualities in structural Ramsey theory

Willem L. Fouché (UNISA – Pretoria, ZA)

Many results in classical Ramsey theory and the theory of well quasi-orderings can be proven by topological means, or be expressed as topological-dynamical phenomena.

Thierry Coquand has given many examples of such topological (-dynamical) expressions of combinatorial phenomena (Kruskal a la Nash Williams, the countable version of van der Waerden), which are classically equivalent to statements involving well-quasi-orderings in point-free topology, or locales, and which are, thus formulated, constructively provable.

I studied topological versions of these combinatorial phenomena from the viewpoint of Gelfand duality of commutative C^*-algebras, the latter duality being provable in constructive mathematics, when adequately phrased, and having, therefore, interesting computational content.

References

1. Andreas Blass. Prime ideals yield almost maximal ideals. *Fund. Math*, 127:57–66, 1987.
2. W. L. Fouché. Symmetry and Ramsey degrees of finite relational structures. *J. Comb. Theory A*, 85:135–147, 1997.
3. P. Freyd All topoi are localic, or Why permutation models prevail. *J. Pure Appl. Alg.*, 46:49–58, 1987.
4. A. S. Kechris, V. G. Pestov, and S. Todorcevic. Fraïssé limits, Ramsey theory, and topological dynamics of automorphism groups. *Geometric And Functional Analysis*, 15:106–189, 2005.

3.10 Pointfree convergence

Jean Goubault-Larrecq (ENS – Cachan, FR)

Joint work of Jean Goubault-Larrecq and Frédéric Mynard

The objective of this work is to define a pointfree analogue of convergence spaces, mirroring the theory of locales as a pointfree analogue of topological spaces. The latter rests on a famous duality, which evolved over the ages from the work of Marshall Stone: there is a functor $\mathcal{O}_{\mathrm{Stone}}$ from the category **Top** of topological spaces to the category of locales, that is, to the opposite of the category of frames, and $\mathcal{O}_{\mathrm{Stone}}$ is left adjoint to a functor $\mathsf{pt}_{\mathrm{Stone}}$. This reduces to a duality between sober spaces and spatial lattices.

Convergence spaces, due to G. Choquet, form a strictly larger category than **Top**, and one that has some features that **Top** lacks, for example Cartesian closure. We build a similar adjunction between the category **Conv** of convergence spaces and a category of so-called *focales*, study its associated monad, and relate this adjunction to the Stone adjunction mentioned above.

Precisely, we define a convergence lattice as a bounded lattice L together with a monotone map lim from the poset of filters on L to L. Every convergence space X gives rise to a convergence lattice $\mathcal{L}(X)$, obtained as its powerset. This defines a functor $\mathcal{L}$ from the category of convergence spaces to the opposite of the category of convergence lattices, and that is what we call the category of focales.

We show that this functor has a right adjoint pt. For a convergence lattice L, $\mathsf{pt}L$ is the convergence space of all compact prime filters of L, with some suitable notion of convergence.

Any adjunction gives rise to a monad. In the case of the Stone adjunction, the result monad is the sobrification monad. We study the monad associated with our $\mathcal{L} \dashv \mathsf{pt}$ adjunction, yielding two possible notions of sobriety for convergence spaces:

- the *temperate* convergence spaces X are those that can be written as the image of some convergence lattice L by the monad functor, and we characterize them as those spaces that are replete, tiled, and separated with respect to some so-called designated limit function; we can take for L the convergence lattice of all *tiles* on X—tiles are a fundamental notion to our new theory;
- the *tee-totalers* are those such that the unit of the monad is an isomorphism; we characterize them as those convergence spaces where every compact ultrafilter is principal, or equivalently where every subset is a tile.

Every tee-totaler is temperate, and every temperate convergence space is quasi-sober, where quasi-sobriety means that every compact ultrafilter has a generic point. All three notions are equivalent for T_0 topological spaces.

Reinhold Heckmann presented another list of possible notions of sobriety for Ω-embedded convergence spaces at this Dagstuhl workshop. There are striking similarities in some of our definitions. We have started to compare our notions, but haven't yet reached a conclusion.

In a second part, we study the relationship between our adjunction $\mathcal{L} \dashv \mathsf{pt}$ and the familiar Stone adjunction between topological spaces and locales. The two adjunctions fit nicely into a commuting diagram of four adjunctions, as we have shown. The last open problem that remained was solved by the audience at Dagstuhl.

3.11 Mereotopological distributive lattices

Georges Hansoul (University of Liège, BE)

Mereotopological structures have been devised to algebraize Whitehead's conception of space and time (by regions and intervals rather than points and instants). They also serve as models for the RCC (region connection calculus) and some logics such as MTML. These are structures over a signature containing mereological (i.e. boolean) symbols and symbols of a more topological nature ($a \prec b$: a is a non-tangential part of b: $\bar{a} \subseteq b^\circ$; $a\mathcal{C}b$: a is in contact with b: $\bar{a} \cap \bar{b} \neq 0$; …). Among these structures, well known are contact algebras, for which we give two definitions. Let B be a boolean algebra. If $\mathcal{C} \subseteq B^2$, then $\mathcal{C}(a, -)$ denotes $\{b \mid a\mathcal{C}b\}$ and $\not{\mathcal{C}}$ is $B^2 \setminus \mathcal{C}$.

1. A structure $\mathcal{B} = (B, \mathcal{C})$ is a *contact algebra* if $\mathcal{C} \subseteq B^2$ satisfies:
   (i) $\not{\mathcal{C}}(a, -)$ and $\not{\mathcal{C}}(-, a)$ are ideals for each $a \in B$;
   (ii) $a\mathcal{C}a \to a = 0$;
   (iii) $\mathcal{C}$ is symmetric;
   (iv) $a \mapsto \mathcal{C}(a, -)$ is one-to-one.
5. A structure $\mathcal{B} = (B, \prec)$ is a *proximity algebra* if $\prec \subseteq B^2$ satisfies:
   (i) $\prec (a, -)$ is a filter and $\prec (-, a)$ an ideal for each $a \in B$;
   (ii) $a \prec b \to a \leq b$;
   (iii) $a \prec b \to \neg a \prec \neg b$ ($\neg$ is for complement);
   (iv) $a = \vee\{b \mid b \prec a\}$.

These definitions are equivalent through the link: $(*) \quad a \prec b \quad \text{iff} \quad (a \not{\mathcal{C}} \neg b)$.

And one of the most fruitful results in the theory is a representation theorem which states ([1]) that any contact algebra may be densely embedded in a *standard* contact algera, that is a contact algebra $(RO(Y), \mathcal{C}_\tau)$ where $Y = (Y, \tau)$ is a topological space, $RO(Y)$ is the boolean algebra of all regular open subsets of Y and, for $O, V \in RO(Y)$, $O\mathcal{C}_\tau V$ iff $\bar{O} \cap \bar{V} \neq \emptyset$.

This result is in fact very closed to a result obtained long before by de Vries for completely different purposes and celebrating a duality between compact spaces and de Vries algebras (complete proximity algebras satisfying $a \prec b \to \exists\, c,\ a \prec c \prec b$).

In [2] was raised the need to extend the theory to structures where the mereological part is no longer boolean and $\prec$, $\mathcal{C}$ are no longer interdefinable. They give some representation theorem for contact distributive lattices $(L, \mathcal{C})$. On the other hand, Bezhanishvili and Harding extend de Vries duality to *proximity frames* $(L, \prec)$. But what is really asked in [2] is a study for mixed structures $(L, \mathcal{C}, \prec)$. Two problems are examined: 1) when can we say that $\mathcal{C}$ and $\prec$ are linked (that is, convey to a unique conception of time or space) and 2) do we have representation theorems?

A *mereotopological lattice* is a structure $\mathcal{L} = (L, \mathcal{C}, \prec)$ where L is a bounded distributive lattice and $\mathcal{C}$, $\prec$ satisfies their respective axiom (i).

▶ **Theorem 1.** *For a mereotopological lattice $\mathcal{L}$, the following are equivalent:*
1. *for each $a \in L$, $\prec (a, -)$ and $\not{\mathcal{C}}(a, -)$ are kernel and cokernel of some congruence on L;*
2. (a) *$a \prec c$ and $a\mathcal{C}b \to a\mathcal{C}(b \wedge c)$ and* (b) *$a \prec b \vee c \to a \prec c$ or $a\mathcal{C}b$.*

▶ **Theorem 2.** *Let $\mathcal{L}$ be a mereotopological lattice satisfying $\prec$(ii), $\prec$(iv), $a\mathcal{C}b$ iff $\exists\, c$, $a \prec c$ and $b \wedge c = 0$, and $\mathcal{C}$ is symmetric.*

Then there exists a bitopological space $Y = (Y, \tau_1, \tau_2)$ and a dense embedding $\mathcal{L} \to RO_{12}(Y)$ where $O \in RO_{12}(Y)$ if $O = \mathrm{Int}_{\tau_2}\, cl_{\tau_1}(O)$, $O \prec U$ iff $cl_{\tau_1}(O) \subseteq U$ and $O\mathcal{C}U$ iff $cl_{\tau_1}(O) \cap cl_{\tau_1}(U) \neq \emptyset$.

References

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2 Düntsch I., MacCaull W., Vakarelov D., Winter M., Distributive contact lattices: Topological representations, The Journal of Logic and Algebraic Programming 76 (2008), 18–34.

3.12 Notions of Sobriety for Convergence Spaces

Reinhold Heckmann (AbsInt – Saarbrücken, DE)

In this talk, we discuss and compare two notions of sobriety for convergence spaces, and relate them to repleteness. The first ("E-sober") has a very general definition in terms of a regular subspace of the double exponential monad w.r.t. Sierpinski space (as considered by Rosolini for equilogical spaces and Taylor in ASD), but has the drawback that almost no closure properties can be shown for the class of E-sober spaces. The second ("A-sober") has a quite special definition based directly on filters and convergence, but has the advantage that the class of "A-sober" spaces is closed under product, regular subspace, and exponentiation. Finally, we compare the two notions of sobriety with repleteness. Every E-sober space is replete and every replete space is A-sober. For topological spaces, both notions of sobriety coincide with the traditional sobriety, and so, topological spaces are replete in the category of convergence spaces if and only if they are sober.

3.13 On the Ho-Zhao problem

Achim Jung (University of Birmingham, GB)

Joint work of Achim Jung, Weng Kin Ho, and Xiaoyong Xi

Given a poset P, the set $\Gamma(P)$ of all Scott closed sets ordered by inclusion forms a complete lattice which is order-dual to the frame of open sets. A subcategory $\mathbf{C}$ of $\mathbf{Pos}_d$ (the category of posets and Scott-continuous maps) is said to be Γ-faithful if for any posets P and Q in $\mathbf{C}$, $\Gamma(P) \cong \Gamma(P)$ implies $P \cong Q$. It is known that the category of all continuous dcpo's and the category of bounded complete dcpo's are Γ-faithful, while $\mathbf{Pos}_d$ is not. Ho & Zhao (2009) asked whether the category $\mathbf{DCPO}$ of dcpo's is Γ-faithful. In this talk I presented an example that shows that this is not the case, thus settling the Ho-Zhao problem in the negative.

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3.14 The Cuntz semigroup for C*-algebras and domain theory

Klaus Keimel (TU Darmstadt, DE)

Domain theory has its origins in Mathematics and Theoretical Computer Science. Mathematically it combines order and topology. Its central concepts have their origin in the idea of approximating ideal objects by their relatively finite or, more generally, relatively compact parts.

The development of domain theory in recent years was mainly motivated by question in denotational semantics and the theory of computation. But since in 2008 Coward, Elliott and Ivannescu have introduced a new invariant for C*-algebras, domain theoretical notions and methods are used in the theory of C*-algebras in connection with the Cuntz semigroup.

The talk was largely expository. It presented those notions of domain theory that seem to be relevant for the theory of Cuntz semigroups and have sometimes been developed independently in both communities. It also contains a new aspect in presenting results of Elliott, Robert and Santiago on the cone of traces of a C*-algebra as a particular case of the dual of an abstract Cuntz semigroup.

3.15 Kolmogorov Extension, Martingale Convergence, and Compositionality of Processes

Dexter Kozen (Cornell University, US)

We show that the Kolmogorov extension theorem and the Doob martingale convergence theorem are two aspects of a common generalization, namely a colimit-like construction in a category of Radon spaces and reversible Markov kernels. The construction provides a compositional denotational semantics for standard iteration operators in probabilistic programming languages, e.g. Kleene star or while loops, as a limit of finite approximants, even in the absence of a natural partial order.

3.16 Stone Duality for Categories of Relations

Alexander Kurz (University of Leicester, GB)

Joint work of Achim Jung, Alexander Kurz, and Andrew Moshier

We show that the dual equivalence between proximity lattices and compact ordered Hausdorff spaces can be recovered by three well-known categorical constructions. First, the duality of distributive lattices and Priestley spaces is extended to weakening-closed relations. Second, idempotents are split. Third, maps are recovered as adjoint pairs of relations. Adjoint pairs homming into 2 are Dedekind cuts, thus giving an abstract account of extending duality from the zero-dimensional to the Hausdorff situation. The three steps can be described in purely category theoretic terms and can be applied to a range of different dualities.

3.17 Pointwise Directed Families and Continuity of Function Spaces

Jimmie D. Lawson (Louisiana State University – Baton Rouge, US)

(Up-)directed sets play an important role in domain theory and elsewhere, e.g., lower semicontinuous functions, idempotent analysis. In the study of function spaces $[X \to P]$, P a poset, X. Xi and J. Liang in 2009 introduced a weaker notion of directedness for functions, namely a family of functions F such that for each x in X, the set $\{f(x) \mid f \in F\}$ is a directed subset. Such a family of functions is called pointwise directed. Basic properties of pointwise directed families will be presented; the principal application is showing that certain new examples of classes of function spaces are continuous domains.

3.18 Natural Duality for Representable Generalized Orthoalgebras

M. Andrew Moshier (Chapman University – Orange, US)

Joint work of M. Andrew Moshier and P. Jipsen

Separation Logic is meant to formalize allocation of resources (memory, processors, and so on). A resource may be allocated or not and cannot be allocated twice. Heaps constitute a typical class of structures that can be modelled this way, where a heap is the collection of all partial functions from a set L (locations) to V (values). In a heap, $\perp$ is the entirely undefined function and $\oplus$ is merging of partial functions ony when they have disjoint domains. Heaps, as well as other sorts of resources, constitute partial cancellative, positive commutative monoid (in the literature, known as generealized orthoalgebras [GAOs]). We study this idea from the perspective of universal algebra and natural duality.

Consider the simplest posssible heap P_1 consisting of the partial functions from $\{1\}$ to $\{1\}$. We can write this as $\{\perp, \top\}$ with $\perp$ the undefined function, $\top$ the defined function and the partial operation $\oplus$ for which $\top \oplus \top$ is undefined and $x \oplus y$ behaves like join otherwise. We consider the quasivariety $ISP(P_1)$. As a confirmation that this quasivariety captures something useful, we note that all heaps belong to $ISP(P_1)$. Of course, there are other structures in the quasivariety that are not simple heaps, but they are all representable as heaps with allocation constraints rouggly of the form "if $h(l_1)$ is allocated then $h(l_2)$ is also."

We show that $ISP(P_1)$ is not finitely axiomatizable by quasi-equations. We then show that in spite of this, $ISP(P_1)$ is naturally dualizable at the finite level. That is, there is a structure Q_1 on the same underlying set $\{\perp, \top\}$ so that $ISP_\omega(P_1)$ is dually equivalent to $ISP_\omega(Q_1)$ (P_ω meaning we take only finite products) via the usual construction of hom-sets into the dualizing object. As this is work in progress, we discuss in some detail the problems involved in extending the finite-level duality to a duality for all of $ISP(P_1)$.

3.19 Program Equivalence is Coinductive

Dirk Pattinson (Australian National University – Canberra, AU)

We describe computational models, notably Turing and counter machines, as state transition systems with side effects. Side effects are expressed via an algebraic signature and interpreted over comodels for that signature; basically, comodels describe the memory model while the transition system captures the control structure. Completeness of equational reasoning over comodels is known to be a subtle issue. We identify a criterion on equational theories and classes of comodels that guarantees completeness, over the given class of comodels, of the standard equational calculus, and show that this criterion is satisfied in our leading examples. Based on such a complete equational axiomatization of the memory model, we then give a complete inductive-coinductive calculus for program equivalence in the full computational model. This calculus is phrased in terms of simulation between states, where a state simulates another if it has at least the same terminating computations, with the same cumulative effect on global state.

3.20 Coinduction up-to techniques in a fibrational setting

Daniela Petrişan (University of Paris VII, FR)

Joint work of Filippo Bonchi, Daniela Petrişan, Damien Pous, and Jurriaan Rot

Bisimulation is used in concurrency theory as a proof method for establishing behavioural equivalence of processes. Up-to techniques can be seen as a means of optimizing proofs by coinduction. For example, to establish that two processes are equivalent one can exhibit a smaller relation, which is not a bisimulation, but rather a bisimulation up to a certain technique, say 'up-to contextual closure'. However, the up-to technique at issue has to be sound, in the sense that any bisimulation up-to should be included in a bisimulation.

In this talk, I will present a general coalgebraic framework for proving the soundness of a wide range of up-to techniques for coinductive unary predicates, as well as for bisimulations. The specific up-to techniques are obtained using liftings of functors to appropriate categories of relations or predicates. In the case of bisimulations with silent moves the situation is more complex. Even for simple examples like CCS, the weak transition system gives rise to a lax bialgebra, rather than a bialgebra. In order to prove that up-to context is a sound technique we have to account for this laxness. The flexibility and modularity of our approach, due in part to using a fibrational setting, pays off: I will show how to obtain such results by changing the base category to preorders.

This is joint work with Filippo Bonchi, Damien Pous and Jurriaan Rot and is based on the papers [1] and [2].

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3.21 Uniform Birkhoff

Michael Pinsker (Charles University – Prague, CZ)

Joint work of Michael Pinsker and Mai Gehrke

I will present a joint work with Mai Gehrke which characterizes pseudovarieties of finitely generated algebras, i.e., classes of finitely generated algebras closed under homomorphic images, subalgebras, and finite products. The result involves both algebraic and topological considerations, and is a common generalization of a description of pseudovarietes of finite algebras due to Eilenberg, Schuetzenberger, Reiterman and Banaschewski, as well as the description of the pseudovariety generated by a single oligomorphic algebra due to Bodirsky and myself. I will also outline the connection of pseudovarieties with Constraint Satisfaction Problems.

3.22 Frames as equilogical algebras

Giuseppe Rosolini (University of Genova, IT)

Joint work of Giuseppe Rosolini, Giulia Frosoni, and Alessio Santamaria

We present a connection between frames and algebras for the double exponential monad on the Sierpinski space. I spoke about this in the last Dagstuhl seminar on Duality and we are now close to a complete solution of the problem. Instrumental for the presentation is Dana Scott's category of equilogical spaces, see [1, 2].

I also connect our work on algebras for the double exponential monad with the work of Paul Taylor on Abstract Stone Duality [3] and with that of Steve Vickers of algebras for the double powerlocale monad [4].

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3.23 Towards a theory of sequentially locally convex QCB-spaces

Matthias Schröder (TU Darmstadt, DE)

We develop a theory of sequentially locally convex QCB-spaces. These are vector spaces equipped with a QCB-topology that arises as the sequentialisation of a locally convex topology. Remember that QCB-spaces form the class of topological spaces which can be handled by the representation based approach to Computable Analysis; a locally convex space is a topological vector space such that its topology is induced by a family of seminorms.

We investigate the class of sequentially locally convex co-Polish spaces. Co-Polish spaces are Hausdorff spaces such that the Scott-topology on the lattice of opens is quasi-Polish. They admit an admissible representation with a locally compact domain. Spaces equipped with such a representation allow for a *Simple Complexity Theory*, meaning the measurement of time complexity in terms of (1) a *discrete* (rather than a continuous) parameter on the input and (2) the output precision.

We present a duality result between the class of sequentially locally convex co-Polish spaces and and the class of separable metrisable locally convex spaces.

3.24 Eliminating Disjunctions by Disjunction Elimination

Peter M. Schuster (University of Verona, IT)

Completeness theorems, or contrapositive forms of Zorn's Lemma, are often invoked in elementary contexts in which the corresponding conservation theorems would suffice. Unlike the former, the latter are syntactical a priori and thus predisposed to proofs with finite methods only. To get a universal constructive conservation theorem for definite Horn clauses, we use Scott's multi-conclusion entailment relations as extending Tarskian single-conclusion consequence relations, or algebraic closure operators. In a nutshell, the extra multi-conclusion axioms can be reduced to rules for the underlying single-conclusion relation that in many contexts turn out to hold. Thanks to a sandwich criterion due to Scott, the method can also be proved optimal.

Applications include the separation and extension theorems known under the names of Krull-Lindenbaum, Artin-Schreier, Szpilrajn and Hahn-Banach. Related work can be found, for example, in locale theory (Mulvey-Pelletier 1991), dynamical algebra (Coste-Lombardi-Roy 2001, Lombardi 1997-8), formal topology (Cederquist-Coquand-Negri 1998) and proof theory (Coquand-Negri-von Plato 2004, Negri-von Plato 2011).

3.25 Locales as spectral spaces

Niels Schwartz (Universität Passau, DE)

Recently a new presentation of locales has been introduced, cf. [4]. Traditionally, locales are the objects of $\mathtt{Fr}^{\mathrm{op}}$, the opposite of the category of frames, [3]. Stone duality is an anti-equivalence between the category $\mathtt{BDLat}$ of bounded distributive lattices and the category $\mathtt{Spec}$ of spectral spaces, [2], [1]. Frames are bounded distributive lattices and frame homomorphisms are bounded lattice homomorphisms. Therefore Stone duality associates with every frame L its spectrum $\mathrm{Spec}(L)$ and with every frame homomorphism $\varphi : L \to M$ the spectral map $\mathrm{Spec}(\varphi) : \mathrm{Spec}(M) \to \mathrm{Spec}(L)$. The spectra of frames are called *locales* and the Stone duals of frame homomorphisms are called *localic maps*. The locales and the localic maps form a subcategory $\mathtt{Loc} \subset \mathtt{Spec}$, which is called the *category of locales*.

Frames are the main objects in the study of pointfree topology. The new approach to locales introduces points into pointfree topology and makes it possible to view pointfree topology as a part of classical point set topology.

The lecture gives a brief explanation of basic facts about topological locales from [4] and current research directions, cf. [5], [6] and other forthcoming papers.

Locales are special spectral spaces and localic maps are special spectral maps. The following results characterize locales and localic maps by a simple topological condition. The characterization uses the *patch topology* or *constructible topology* of a spectral space, [2], [1]. The patch space of a spectral space X is denoted by X_{con}, the patch closure of a subset $M \subseteq X$ is denoted by $\overline{M}^{\mathrm{con}}$.

▶ **Theorem 1.** *A spectral space X is a locale if and only if $\overline{U}^{\mathrm{con}}$ is open for every open set $U \subseteq X$.*

▶ **Theorem 2.** *A spectral map $f : X \to Y$ between locales is a localic map if and only if $\overline{f^{-1}(V)}^{\mathrm{con}} = f^{-1}(\overline{V}^{\mathrm{con}})$ for each open $V \subseteq Y$.*

These results show that the formation of the patch closure of an open set is of basic importance for locales. Let X be a locale, $\mathcal{O}(X)$ its frame of open sets and $\mathring{\mathcal{K}}(X)$ its lattice of quasi-compact open sets.

▶ **Theorem 3.** *For a locale X the map $U \mapsto \overline{U}^{\mathrm{con}}$ is a nucleus of the frame $\mathcal{O}(X)$. Its image is the frame $\mathring{\mathcal{K}}(X)$. (This nucleus is called the* natural nucleus *of $\mathcal{O}(X)$.)*

These results are the starting point of a topological theory of locales. By Stone duality the category $\mathtt{Loc}$ is equivalent to $\mathtt{Fr}^{\mathrm{op}}$. So there are numerous questions about the translation of facts about locales into facts about the objects of $\mathtt{Fr}^{\mathrm{op}}$. For example, recall that the nuclei of a frame L are considered as the *sublocales* of the locale L, [3]. In the topological theory of locales a subset A of a locale X is called a *localic subspace* if it is a spectral subspace (i.e., is closed for the patch topology, [1]), is a locale with the subspace topology and the inclusion map $A \to X$ is a localic map.

▶ **Theorem 4.** *Let X be a locale and $A \subseteq X$ a spectral subspace. Then the following conditions are equivalent.*

1. *A is a localic subspace.*
2. *If $U \subseteq A$ and $V \subseteq X$ are open and $U = V \cap A$ then $\overline{U}^{\mathrm{con}} = \overline{V}^{\mathrm{con}} \cap A$.*
3. *If $C \subseteq A$ is constructible then $\overline{C}^{X}$, the closure in X, is constructible.*

The following category theoretic facts describe the relation of the category `Loc` with the categories `Top` of topological spaces and `Spec` of spectral spaces.

▶ **Theorem 5.** *The subcategory* `Spec` $\subset$ `Top` *is reflective. If X is a topological space then the locale $L(X) = \mathrm{Spec}(\mathcal{O}(X))$ is the reflection of X. The reflection map $S_X : X \to L(X)$ sends $x \in X$ to the prime ideal $\{O \in \mathcal{O}(X) \mid x \notin O\}$.*

▶ **Theorem 6.** *The subcategroy* `Loc` $\subset$ `Spec` *is coreflective. The coreflection of a spectral space X is the locale $L(X) = \mathrm{Spec}(\mathcal{O}(X))$. The coreflection map $R_X : L(X) \to X$ is the Stone dual of the inclusion homomorphism $\overset{\circ}{\mathcal{K}}(X) \to \mathcal{O}(X)$.*

Following these results it is an obvious question how properties of a topological space X are connected with properties of the spectral reflection $L(X)$. As examples of this type of results we mention:

▶ **Theorem 7.** *Let X be a spectral map. Then the following conditions are equivalent.*
1. *The reflection map S_X is a homeomorphism.*
2. *The reflection map is a spectral map.*
3. *X is Noetherian.*

▶ **Theorem 8.** *A topological space X is quasi-compact if and only if the spectral reflection $L(X)$ is quasi-compact.*

The spectral reflection can be used to construct the Stone-Čech compactification of a normal topological space and the Gleason cover of a compact space. In the literature various topological spaces have been associated with a partially ordered set. The spectral reflection can be used to relate many of these constructions to each other and to produce new constructions.

Finally, there are category theoretic questions about locales. First one notes that `Loc` is complete and cocomplete since the category of frames is known to be complete and cocomplete. This leads to the question how limits and colimits in `Loc` can be computed. As `Loc` is coreflective in `Spec` it follows from general principles that the colimit of a diagram in `Loc` coincides with the colimit computed in `Spec`. Not all colimits in `Spec` are easy to determine. Therefore this answer does not solve all problems concerning colimits. But the situation with limits is more complicated. For example, given two locales X and Y, a product exists in each of the categories `Spec` and `Loc`. But these products are distinct. The product in `Spec` is the same as in `Top`, [2]. (This also follows from the fact that `Spec` is reflective in `Top`.) The product in `Loc` can be constructed in the following way.

First one forms the spectral product $X \times Y$, which is a spectral space, but usually not a locale. Let p_X and p_Y be the projections onto the components. One applies the localic coreflection and obtains the spectral map $R_{X\times Y} : L(X \times Y) \to X \times Y$. Now the domain and the codomains of the maps $p_X \circ R_{X\times Y} : L(X \times Y) \to X$ and $p_Y \circ R_{X\times Y} : L(X \times Y) \to Y$ are locales. But the maps are only spectral and are usually not localic. To complete the construction, there is a largest localic subspace $Z \subseteq L(X \times Y)$ such that $p_X \circ R_{X\times Y}|_Z$ and $p_Y \circ R_{X\times Y}|_Z$ are localic maps. Now the locale Z with these localic maps as projections is the localic product of X and Y.

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3.26 Duality for r.e. sets with applications

Vladimir Shavrukov (Utrecht, NL)

We begin with some basic specifics of the dual space $(E^*)_*$ of the lattice E^* of r.e. sets (mod finite). Then we introduce some ultrapower-like models of arithmetic that allow us to have a closer look at the individual points of $(E^*)_*$ (=prime filters of E^*). We show how to apply all of this to obtain a result in the classical recursion-theoretic tradition on r-maximal and hyperhypersimple r.e. sets.

3.27 A Chu-like extension of topological spaces

Paul Taylor (Birmingham, UK)

Main reference P. Taylor, "A Chu-like Extension of Topological Spaces," Slides, 2015.
URL http://paultaylor.eu/slides/15-Dagstuhl.pdf

Dana Scott's *equilogical spaces* provide the best known construction of a cartesian closed category with all finite limits in which the category of sober topological spaces is embedded. An equilogical space $X \equiv A/\sim$ is defined using a partial equivalence relation $\sim$ on the underlying set of an algebraic lattice A. Then the simplest exponential

$$\Sigma^{(A/\sim)} \;\equiv\; \Sigma^A/\cong \qquad \text{is given by} \qquad \phi \cong \psi \;\equiv\; \forall xy : A.(x \sim y) \Rightarrow (\phi x = \psi y),$$

after which the formulae become more and more complicated. We would like to break up this construction into parts with simpler universal properties in order to understand and compute these exponentials.

The categorical methods that have been used (local cartesian closure, exact completion, embedding in toposes) are in my view *set theory* – tools for the study of *discrete* structures. My objective is to find the *intrinsic* logic of *topology* and higher *computability* theory.

The alternative that we propose here is inspired by Martín Escardó's analogy that equilogical spaces are to topological spaces as complex numbers are to real ones and Steven Vickers' result that the double powerlocale is a monad on the category of locales that reduces to the double exponential $\Sigma^{\Sigma^{(-)}}$ on locally compact spaces. However, we start from the category of *sober topological spaces* because that of locales does not obey the necessary categorical properties.

Recall that *Chu spaces* provide models of Linear Logic with an *involutive negation* that interchanges two components. In our case we replace the identity by a *monad*, for which we obtain a "*square root*" (*cf.* $\sqrt{-1}$), written \$, and a *tensor product* $\otimes$ such that maps

$$X \to \$Y, \qquad Y \to \$X \qquad \text{and} \qquad X \otimes Y \to \Sigma \equiv \$\mathbf{1}$$

are in natural bijection, **1** being the terminal object. In fact $\otimes$ has projections but not in general diagonals.

We may also calculate equalisers, coequalisers and hence (four kinds of) *image factorisation*. That into regular epis and monos defines a *coreflective subcategory*, whose objects do have diagonals, so that $\otimes$ is the categorical product and \$ the exponential. Restricting the subcategory further to those objects that are generated by \$ and finite limits yields general exponentials.

This construction is actually founded on *equideductive logic* rather than point–set topology. With this new tool it may be easier to examine higher exponentials and translate principles from recursion theory to equideductive logic. On the other hand, the Chu-like category may turn out to be a better setting than the cartesian closed one in which to do this study.

In fact, the talk as given diverged somewhat from the slides and explored equideductive logic and its possible relationship to constructive descriptive set theory, since other participants of the seminar such as Matthew de Brecht were interested in this.

3.28 Stone Duality as a Topological Construction

Marcus Tressl (Manchester University, GB)

We present a purely topological construction of Stone Duality passing both ways from a bounded distributive (semi-) lattice to its spectrum and back, using purely topological constructions, such as: Sobrification, Desobrification (passing to the subspace of locally closed points), Hyperspaces and Inverse spaces.

3.29 Monadic second order logic as a model companion of modal logic

Samuel J. van Gool (City University of New York, US)

Joint work of Samuel J. van Gool and Silvio Ghilardi
Main reference S. Ghilardi, S. J. van Gool, "A model-theoretic characterization of monadic second order logic on infinite words," arXiv:1503.08936v1 [math.LO], 2015.
URL http://arxiv.org/abs/1503.08936v1

We exhibit a connection between monadic second order logics and modal logics, making use of the language of first-order model theory; specifically, model companions. Model companions stem from A. Robinson's work on model completions: a model companion of a universal theory T is an extension T^* of T which has the same universal consequences (i.e., T^* is a co-theory of T), but in addition allows for the elimination of quantifier alternations (i.e., T^* is model-complete). Monadic second order logic and modal logic are two different formalisms that can be used to describe the same classes of models, such as infinite words, or infinite trees. Both logics, when interpreted in appropriate power set structures, can actually be viewed as first-order theories. We show in several specific cases that the first-order theory corresponding to MSO is the model companion of the first-order theory corresponding to modal logic. During the talk, I focused on MSO on infinite words, and showed how it can be characterized as the model companion of linear temporal logic with an initial element. I also reported on our ongoing work of extending these results to infinite trees.

3.30 Stone duality without the duality

Steven J. Vickers (University of Birmingham, GB)

The original Stone dualities bring out a wonderful connection between the discrete world of algebra and logic and the continuous world of point-set topology, and they lead to the dual adjunction between frames and topological spaces. Without choice principles the dualities fail, but they still guide the development of point-free topology, replacing point-set structure by algebra. What is left of the original dualities is then less exciting: for example, Stone locales are dual to Boolean algebras. However, they still mark the connection between impredicative world (needing powersets) of frames and the predicative world of frame presentations. It is at this point that duality begins to lose its duality: for how do we define *predicatively* the structures on presentations that correspond to arbitrary maps between the locales? What does it mean to "respect the relations" in the presentations? A general impredicative technique comes from understanding that a classifying topos, or the topos of sheaves over X, is the "geometric mathematics generated by a generic point of X", and that to construct a point of Y in that mathematics is to define a locale map from X to Y. The geometric constraints on the reasoning guarantee that the generic point transformation is continuous. I shall sketch a predicative version of this that is often adequate, in which the geometric structure of Grothendieck toposes is replaced by the arithmetic structure of Joyal's Arithmetic Universes. They have a finitary, coherent internal logic, but also the ability to form types for free algebras including the natural numbers. This allows some infinities (as needed for geometric logic) to be part of the internal structure in a finitary way.

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