Report of Dagstuhl Seminar 15511

The Graph Isomorphism Problem

Edited by

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 15511 "The Graph Isomorphism Problem". The goal of the seminar was to bring together researchers working on the numerous topics closely related to the Isomorphism Problem to foster their collaboration. To this end the participants of the seminar included researchers working on the theoretical and practical aspects of isomorphism ranging from the fields of algorithmic group theory, finite model theory, combinatorial optimization to algorithmics. A highlight of the conference was the presentation of a new quasi-polynomial time algorithm for the Graph Isomorphism Problem, providing the first improvement since 1983.

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1 Executive Summary

Pascal Schweitzer
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The Graph Isomorphism Problem remains one of the two unresolved computational problems from Garey and Johnson's list dating back to 1979 of problems with unknown complexity status. In very rough terms the problems asks to decide whether two given graphs are structurally different or one is just a perturbed variant of the other. The problem naturally arises when one is faced with the task of classifying relational structures (e.g., chemical molecules, websites and links, road networks).

While the Graph Isomorphism Problem was intensively studied from the point of view of computational complexity in the 1980s and early 1990s, in later years progress became slow and interest in the problem stalled. However, recent years have seen the emergence of a variety of results related to graph isomorphism in a number of research areas including algorithmic group theory, finite model theory, combinatorial optimization and parameterized algorithms, not to mention graph theory itself. Indeed, having been open and quite prominent for such a long time, the Graph Isomorphism Problem is repeatedly attacked with the abundance of algorithmic techniques that have been developed over the decades. While this has not led to resolution of the problem, it has led to applications of methods originally developed for the Graph Isomorphism Problem in other areas (such as machine learning and constraint satisfaction problem solving). It has also sparked fascinating concepts in complexity theory, led to a thriving compilation of techniques in algorithmic group theory, the development of software packages (such as canonical labeling tools) and perpetuating effects in algorithmic graph theory in general.

While a lot of other computational problems have a specific community associated with them, resulting in dedicated conferences, the situation for the isomorphism problem is different. This is due to the fact that the background of people working on the isomorphism problem is quite diverse which leads to infrequent encounters at regular conferences or other events. Moreover, there is a big gap between theory and practice, a phenomenon verbalized by Brendan McKay as two distinct galaxies with very few stars in between them. Indeed, the algorithms that are asymptotically fastest in theory are very different to the ones that prove to be the fastest in practical implementations. The original motivation of the seminar was to bring together researchers working on the many topics closely related to the Isomorphism Problem to foster their collaboration.

However, the face of the seminar was to change, as one of the organizers (László Babai) published a proof on the arXiv (http://arxiv.org/abs/1512.03547) on the night before the seminar that shows that graph isomorphism can be solved in quasi-polynomial time (see the abstract to the talk below). This is the first improvement over the moderately exponential algorithm for general graphs by Luks from 1983. Babai gave three intense blackboard presentations each with a duration of two hours on the new quasi-polynomial time algorithm. Apart from the presentations, there were a number of excellent talks including expository surveys on recent advances in a variety of aspects of the Graph Isomorphism Problem as detailed below.

Overall a memorable event, we hope that the seminar has encouraged future collaboration across the different areas which eventually brings us closer to the theoretical and practical resolution of the problem.

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3 Overview of Talks

3.1 Graph Indistinguishability Through Hierarchies of Relaxations

Albert Atserias (UPC – Barcelona, ES)

Joint work of Atserias, Albert; Maneva, Elitza
Main reference A. Atserias, E. N. Maneva, "Sherali-Adams Relaxations and Indistinguishability in Counting Logics", SIAM Journal on Computing, 42(1):112–137, 2013.
URL http://dx.doi.org/10.1137/120867834

As with all other problems in NP, one can write the graph isomorphism problem as a 0-1 integer linear programming feasibility problem. The straightforward relaxations into a real or rational-valued linear program leads to the concept of fractional isomorphism as first studied by Tinhofer. A natural question to ask is what the levels of the Sherali-Adams (SA) hierarchy of linear programming relaxations give when they are applied to fractional isomorphism. In this talk I will spend a significant amount of time explaining what the SA-hierarchy of relaxations is, and what the answer to this natural question is.

3.2 Graph Isomorphism in Quasipolynomial Time

László Babai (University of Chicago, US)

Main reference L. Babai, "Graph Isomorphism in Quasipolynomial Time", arXiv:1512.03547 [cs.DS], 2015.
URL http://arxiv.org/abs/1512.03547v2

We show that the Graph Isomorphism (GI) problem and the related problems of String Isomorphism (under group action) (SI) and Coset Intersection (CI) can be solved in quasipolynomial $(\exp\left((\log n)^{O(1)}\right))$ time. The best previous bound for GI was $\exp(O(\sqrt{n \log n}))$, where n is the number of vertices (Luks, 1983); for the other two problems, the bound was similar, $\exp(\widetilde{O}(\sqrt{n}))$, where n is the size of the permutation domain (Babai, 1983).

The algorithm builds on Luks's SI framework and attacks the barrier configurations for Luks's algorithm by group theoretic "local certificates" and combinatorial canonical partitioning techniques. We show that in a well–defined sense, Johnson graphs are the only obstructions to effective canonical partitioning.

Luks's barrier situation is characterized by a homomorphism φ that maps a given permutation group G onto S_k or A_k, the symmetric or alternating group of degree k, where k is not too small. We say that an element x in the permutation domain on which G acts is *affected* by φ if the φ-image of the stabilizer of x does not contain A_k. The affected/unaffected dichotomy underlies the core "local certificates" routine and is the central divide-and-conquer tool of the algorithm.

3.3 A Near-Optimal Lower Bound on the Number of Refinement Steps of the Weisfeiler-Leman Algorithm

Christoph Berkholz (HU Berlin, DE)

Joint work of Berkholz, Christoph; Nordström, Jakob

We show that there are pairs of non-isomorphic n-element relational structures that can be distinguished by the k-dimensional Weisfeiler-Leman Algorithm, but not within $n^{o(k/\log k)}$ refinement steps. This lower bound holds for all $k < n^{0.01}$ and nearly matches the n^k upper bound, the best previous lower bound was linear in n [Fürer 2001]. The hard examples are based on unsatisfiable XOR formulas (encoded as relational structures) and it remains open to prove a similar lower bound for graphs.

This result is part of an unpublished joint work with Jakob Nordström.

3.4 Query Complexity for Testing Graph Isomorphism and Related Questions

Sourav Chakraborty (Chennai Mathematical Institute, IN)

Joint work of Alon, Noga; Babai, László; Blais, Eric; Chakraborty, Sourav; Fischer, Eldar; Garcia-Soriano, David; Matsliah, Arie
Main reference L. Babai, S. Chakraborty, “Property Testing of Equivalence under a Permutation Group Action”, Electronic Colloquium on Computational Complexity (ECCC), 15(040), 2008.
URL http://eccc.hpi-web.de/eccc-reports/2008/TR08-040/index.html

We study the graph isomorphism from the point of view of query complexity. That is, how many queries to the adjacency matrix of the graph is necessary to decide if two graphs are isomorphic (or “far” from isomorphic). We also study generalizations of the graph isomorphism problem: namely the uniform hyper-graph isomorphism problem and the string isomorphism under the action of a transitive group. We also talk about the query complexity for function isomorphism.

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1 Eldar Fischer, Arie Matsliah. Testing Graph Isomorphism. *SIAM Journal on Computing*, 38(1):207–225, 2008. DOI: 10.1137/070680795

2 Noga Alon, Eric Blais, Sourav Chakraborty, David García-Soriano, Arie Matsliah. Nearly Tight Bounds for Testing Function Isomorphism. *SIAM Journal on Computing*, 42(2):459–493, 2013. DOI: 10.1137/110832677

3.5 Finding Canonical Representations for Circular-Arc Graphs

Maurice Chandoo (Leibniz Universität Hannover, DE)

Main reference M. Chandoo, "Deciding Circular-Arc Graph Isomorphism in Parameterized Logspace", in Proc. of the 33rd Symp. on Theoretical Aspects of Computer Science (STACS'16), LIPIcs, Vol. 47, pp. 26:1–26:13, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2016.
URL http://dx.doi.org/10.4230/LIPIcs.STACS.2016.26

In [1] it is shown how to find canonical representations for Helly CA graphs by reducing it to the problem of finding a canonical representation for interval graphs. The idea is to find a sequence of algebraic flips [2] that turns a Helly CA graph into an interval graph. Such a sequence corresponds to a subset of vertices, which we shall call flip set.

We show that for a large subclass of CA graphs containing HCA graphs it is quite easy to find such flip sets. More interestingly, however, is the fact that the remaining class of CA graphs have a quite restricted structure and finding flip sets for this class boils down to developing an understanding of a certain substructure in these graphs. The goal of the talk is to give a rough understanding of what these difficult CA graphs look like and what the substructure of interest is.

References

1 Johannes Köbler, Sebastian Kuhnert, and Oleg Verbitsky. Helly Circular-Arc Graph Isomorphism Is in Logspace. In *Proceedings of MFCS 2013*, vol. 8087 of LNCS, pp. 631–642, Springer, 2013. DOI: 10.1007/978-3-642-40313-2_56

2 Ross M. McConnell. Linear-Time Recognition of Circular-Arc Graphs. *Algorithmica*, 37(2):93–147, 2003. DOI: 10.1007/s00453-003-1032-7

3.6 Isomorphism through Coherent Algebras on Finite Fields

Anuj Dawar (University of Cambridge, GB)

Joint work of Dawar, Anuj; Holm, Bjarki

The k-dimensional Weisfeiler-Lehman method for distinguishing graphs is usually described in combinatorial terms as an iterative refinement procedure classifying k-tuples of vertices. The method also has an alternative characterization through coherent algebras (also called cellular algebras or coherent configuratios) of complex matrices. This was the original form proposed by Weisfeiler and Lehman, for the 2-dimensional case. In this talk, I explore a variation of the method obtained by considering coherent algebras over finite fields, instead of the complex field. This yields a family of isomorphism tests which are polynomial-time decidable and of strictly wider applicability than the Weisfeiler-Lehman method. I explore the extent and limitations of the method, showing in particular that it can decide isomorphism on graphs of colour-class size 4.

3.7 Logspace Canonizations for Graphs of Bounded Tree Width and Graphs of Bounded Genus

Michael Elberfeld (RWTH Aachen University, DE)

Finding a polynomial-time canonization algorithm for a class of graphs opens up the question of whether we can improve the algorithm with respect to its sequential and parallel runtime or memory footprint. This talk presents two recently developed canonization algorithms that have a logarithmic memory (logspace) footprint. The first applies to every class of graphs with a constant tree width [2] while the second applies to every class of graphs with a constant genus [1]. After motivating and presenting the results, we focus on the proof techniques: The first technique is an extension of Lindell's tree canonization [4] to dynamically refining tree decompositions and the second extends the idea of using universal exploration sequences for traversing 3-connected planar graphs [3] to uniquely-embeddable graphs of bounded genus.

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2. Michael Elberfeld and Pascal Schweitzer. Canonizing Graphs of Bounded Tree Width in Logspace. In *Proceedings of STACS 2016*, vol. 47 of LIPIcs, pp. 32:1–32:14, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2016. DOI: 10.4230/LIPIcs.STACS.2016.32
3. Samir Datta, Nutan Limaye, and Prajakta Nimbhorkar. 3-connected Planar Graph Isomorphism is in Log-space. In *Proceedings of FSTTCS 2008*, vol. 2 of LIPIcs, pp. 155–162, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2008. DOI: 10.4230/LIPIcs.FSTTCS.2008.1749
4. Steven Lindell. A Logspace Algorithm for Tree Canonization (Extended Abstract). In *Proceedings of STOC 1992*, pp. 400–404, ACM, 1992. DOI: 10.1145/129712.129750

3.8 Decomposition Techniques for Graph Isomorphism Testing

Martin Grohe (RWTH Aachen University, DE)

My talk was about various types of decompositions of graphs and other structures, such as tree decompositions, rank decompositions, and more generally branch decompositions of general connectivity systems, and their applications in graph isomorphism testing. Two such applications are our recent polynomial isomorphism tests for graph classes excluding a topological subgraph (with Daniel Marx) and for graph classes of bounded rank width (with Pascal Schweitzer).

3.9 Graphs Identified by the Weisfeiler-Leman Algorithm

Sandra Kiefer (RWTH Aachen University, DE)

Joint work of Kiefer, Sandra; Schweitzer, Pascal; Selman, Erkal
Main reference S. Kiefer, P. Schweitzer, E. Selman, "Graphs identified by logics with counting", arXiv:1503.08792 [cs.LO], 2015.
URL http://arxiv.org/abs/1503.08792v1

I present a classification of graphs and, more generally, finite relational structures that are identified by Color Refinement, i.e., by the 1-dimensional Weisfeiler-Leman algorithm. Using this classification, I describe how it can be decided in almost linear time whether a structure is identified by Color Refinement. The classification implies that for every identified graph, all vertex-colored versions of it are also identified. A similar statement is true for finite relational structures. The classification yields another nice result: Every class of graphs indistinguishable by Color Refinement contains a graph whose orbits are exactly the classes of the color partition of its vertex set and which has a single automorphism witnessing this fact. Considering higher-dimensional versions of the Weisfeiler-Leman algorithm, I explain why such statements are not true: I present examples of graphs of size linear in k which are identified by the 2-dimensional Weisfeiler-Leman algorithm but for which the orbit partition is strictly finer than the partition induced by the k-dimensional algorithm. These graphs have vertex-colored versions that are not identified by the k-dimensional algorithm, which can be seen using a pebble game argument.

This is joint work with Pascal Schweitzer and Erkal Selman.

3.10 Canonical Representation of Some Classes of Circular-Arc Graphs

Sebastian Kuhnert (HU Berlin, DE)

Joint work of Köbler, Johannes; Kuhnert, Sebastian; Laubner, Bastian; Verbitsky, Oleg

The frontier for efficient isomorphism testing runs right through the class of circular-arc (CA) graphs. On the one hand, the isomorphism problem is L-complete for important subclasses like interval graphs [1], proper CA graphs [2] and Helly CA graphs [3]. On the other hand, it remains open whether isomorphism of general CA graphs is in P; cf. [4].

This talk surveys the logspace algorithms of [1, 2, 3], which actually compute canonical representations for the respective graph classes. That is, for a given graph, they compute an intersection representation of the respective type such that isomorphic graphs are mapped to identical intersection models. This implies that both the recognition and the canonization problem of these graph classes are in logspace.

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3 Johannes Köbler, Sebastian Kuhnert, and Oleg Verbitsky. Helly Circular-Arc Graph Isomorphism Is in Logspace. In *MFCS 2013*, vol. 8087 of LNCS, pp. 631–642, Springer, 2013. DOI: 10.1007/978-3-642-40313-2_56
4 Andrew R. Curtis, Min Chih Lin, Ross M. McConnell, Yahav Nussbaum, Francisco J. Soulignac, Jeremy P. Spinrad, and Jayme Luiz Szwarcfiter. Isomorphism of graph classes related to the circular-ones property. *Discrete Mathematics & Theoretical Computer Science*, 15(1):157–182, 2013. http://www.dmtcs.org/dmtcs-ojs/index.php/dmtcs/article/view/2298

3.11 Representation of Groups on Graphs

Piyush P. Kurur (Indian Institute of Technology – Kanpur, IN)

Joint work of Dutta, Sagarmoy; Kurur, Piyush P.
Main reference S. Dutta, P. P. Kurur, "Representing Groups on Graphs", in Proc. of the 34th Int'l Symp. on Mathematical Foundations of Computer Science (MFCS'09), LNCS, Vol. 5734, pp. 295–306, Springer, 2009; pre-print available from author's webpage.
URL http://dx.doi.org/10.1007/978-3-642-03816-7_26
URL http://cse.iitk.ac.in/users/ppk/research/publication/DK2009.pdf

A representation of a group G on a graph X is a homomorphism from the group G to the automorphism subgroup $\mathrm{Aut}(X)$ of X. In this talk, I study the following problem: Given a group G as a Cayley table and a graph X, decide whether there is a non-trivial representation of G on X (there is always the trivial one which sends all elements of G to the identity automorphism). We call this problem the group representability problem and the main goal is to understand its relative complexity w.r.t. the graph isomorphism problem.

It turns out that graph isomorphism problem reduces to abelian group representability problem. In the other direction even solvable group representability problem reduces to graph isomorphism problem. However, nothing is know about the general problem. In particular, for a fixed non-solvable group like say A_5 we do not know the hardness of deciding whether it is representable on a graph X.

This is joint work with Sagarmoy Dutta.

3.12 Group Isomorphism via Fixed Composition Series

Eugene M. Luks (University of Oregon – Eugene, US)

Main reference E. M. Luks, "Group Isomorphism with Fixed Subnormal Chains", arXiv:1511.00151 [cs.CC], 2015.
URL http://arxiv.org/abs/1511.00151v1

In recent work, David Rosenbaum and Fabian Wagner showed that, for p-groups of order n given by Cayley tables, isomorphism-testing is in time $n^{(1/2)\log_p n+O(p)}$ time, where n is the group order; this is roughly a square-root of the classical bound. Rosenbaum subsequently extended the result to solvable groups achieving an $n^{(1/2)\log_p n+O(\log n/\log\log n)}$ time, where p is the smallest prime divisor of n. The $n^{O(p)}$ and $n^{O(\log n/\log\log n)}$ factors, respectively, are contributed by the cost of testing for isomorphisms that match fixed composition series in the two groups. Their results then follow by bounding the number of possible composition series.

We focus now on that fixed-composition-series-isomorphism subproblem and show it is in polynomial-time even for general groups. This immediately implies isomorphism-testing of groups in time $n^{(1/2)\log_q n+O(1)}$ and polynomial space, where q can even be taken to be the minimum order of a composition factor. Furthermore, an extension to fixed-composition-series-canonization together with Rosenbaum's "bidirectional collision" yields group isomorphism-testing and canonization with time and space balanced at $n^{(1/4)\log_q n+O(1)}$.

3.13 Practical Graph Isomorphism

Brendan McKay (Australian National University – Canberra, AU)

Joint work of McKay, Brendan; Piperno, Adolfo
Main reference B. D. McKay, A. Piperno, "Practical graph isomorphism, II", Journal of Symbolic Computation, 60:94–112, 2014.
URL http://dx.doi.org/10.1016/j.jsc.2013.09.003

The first practical solutions to the graph isomorphism problem appeared in 1964, mostly motivated by the problem of identifying chemical structures. Now there are a great many applications and several programs with strong performance.

The talk surveyed the development of the field, focusing mostly on the individualization-refinement paradigm that has been the most successful. In particular, we described the techniques used by the `nauty` family of programs that have been the most popular for almost 40 years [1, 2]. The manner in which the search tree is generated and pruned with the help of discovered automorphisms and invariant bounding was explained. Finally, we hinted at the innovative changes made most recently by Adolfo Piperno's program `Traces`, which is the current champion for difficult graphs, described in detail in Prof Piperno's talk.

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2 Brendan D. McKay and Adolfo Piperno. Practical graph isomorphism, II. *Journal of Symbolic Computation*, 60:94–112, 2014. DOI: 10.1016/j.jsc.2013.09.003

3.14 Fixed-Parameter Tractable Canonization and Isomorphism Test for Graphs of Bounded Treewidth

Michał Pilipczuk (University of Warsaw, PL)

Joint work of Lokshtanov, Daniel; Pilipczuk, Marcin; Pilipczuk, Michał; Saurabh, Saket
Main reference D. Lokshtanov, M. Pilipczuk, M. Pilipczuk, S. Saurabh, "Fixed-Parameter Tractable Canonization and Isomorphism Test for Graphs of Bounded Treewidth", in Proc. of the 2014 IEEE 55th Annual Symp. on Foundations of Computer Science (FOCS'14), pp. 186–195, IEEE Computer Society, 2014; to appear in SIAM Journal on Computing.
URL http://dx.doi.org/10.1109/FOCS.2014.28

During the talk we will present an algorithm for Graph Isomorphism on graphs of treewidth k that runs in time $2^{O(k^5 \log k)} \cdot n^5$. This is the first fixed-parameter algorithm for GI under this parameterization. The algorithm actually computes some form of a canonical tree decomposition of the graph, which can be of independent interest.

3.15 Some Practical Graph Isomorphism Issues

Adolfo Piperno (Sapienza University of Rome, IT)

Joint work of McKay, Brendan; Piperno, Adolfo
Main reference B. D. McKay, A. Piperno, "Practical graph isomorphism, II", Journal of Symbolic Computation, 60:94–112, 2014.
URL http://dx.doi.org/10.1016/j.jsc.2013.09.003

Traces is a tool for graph canonical labeling and automorphism group computation, included in the nauty & Traces package [1, 2, 3].

In this talk I have presented some new features of Traces, such as the possibility of treating graphs with weighted edges; some issues in the implementation of Traces heve been discussed; among these:

- preprocessing trees;
- use of the breadth first search;
- fine tuning on the use of the Schreier-Sims algorithm;
- different choices of individualized vertices.

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1 Brendan D. McKay and Adolfo Piperno. Practical graph isomorphism, II. *Journal of Symbolic Computation*, 60:94–112, 2014. http://dx.doi.org/10.1016/j.jsc.2013.09.003
2 http://cs.anu.edu.au/people/Brendan.McKay/nauty/
3 http://pallini.di.uniroma1.it/index.html

3.16 On the Isomorphism Problem for Central Cayley Graphs

Ilia Ponomarenko (Steklov Institute – St. Petersburg, RU)

A Cayley graph $\mathrm{Cay}(G, X)$ of a group G is called central if the set X is a union of conjugacy classes of G. We discuss two problems. In the first one, given a group G and a graph D with $|G|$ vertices, one should test whether D is isomorphic to a central Cayley graph of G. In the second one, we are interested in testing isomorphism of given two central graphs $\mathrm{Cay}(G, X)$ and $\mathrm{Cay}(G', X')$. Both problems are solved in polynomial time, when G is an abelian group "close" to cyclic. Concerning the second problem, we will talk on the case, when G is an almost simple group.

3.17 The Parameterized Complexity of Geometric Graph Isomorphism

Gaurav Rattan (The Institute of Mathematical Sciences, IN)

Joint work of Arvind, Vikraman; Rattan, Gaurav
Main reference V. Arvind, G. Rattan, "The Parameterized Complexity of Geometric Graph Isomorphism", in Proc. of the 9th Int'l Symp. on Parameterized and Exact Computation (IPEC'14), LNCS, Vol. 8894, pp. 51–62, Springer, 2014.
URL http://dx.doi.org/10.1007/978-3-319-13524-3_5

In this talk, we discuss our recent work on the Geometric Graph Isomorphism (GGI) problem. The problem is defined as follows: given two sets of points A and B in $\mathbb{Q}^k$, does there exist a Euclidean-distance-preserving bijection between the two sets? The dimension k of the underlying space is an important parameter of interest. We discuss our $k^{O(k)}$ FPT algorithm for this problem, and the associated canonization problem [1]. The algorithm uses techniques from lattices. We also discuss the recent work of Haviv and Regev [2] regarding isomorphism of lattices.

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2 Ishay Haviv and Oded Regev. On the Lattice Isomorphism Problem. In *SODA 2014*, pp. 391–404, SIAM, 2014. DOI: 10.1137/1.9781611973402.29

3.18 Bidirectional Collision Detection and Group Isomorphism

David J. Rosenbaum (University of Tokyo, JP)

Main reference D. J. Rosenbaum, "Bidirectional Collision Detection and Faster Deterministic Isomorphism Testing", arXiv:1304.3935 [cs.DS], 2013.
URL http://arxiv.org/abs/1304.3935v2

In this talk, we introduce bidirectional collision detection – a new algorithmic tool that applies to isomorphism testing in any class of objects that satisfies certain mild assumptions. We show that bidirectional collision detection yields a deterministic $n^{(1/2)\log n+O(1)}$ time algorithm for testing isomorphism of general groups whereas previously the $n^{\log n+O(1)}$ generator-enumeration algorithm was the best bound for several decades. Later, Laci Babai and Eugene Luks independently improved this result to $n^{(1/4)\log n+O(1)}$ using two different methods in combination with bidirectional collision detection. Faster quantum versions of our bidirectional collision detection results also exist. Although the space requirements for our algorithms are greater than those for previous isomorphism tests, we show time-space tradeoffs that interpolate between the resource requirements of our algorithms and previous work.

3.19 Parameterizations and the Graph Isomorphism Problem

Pascal Schweitzer (RWTH Aachen University, DE)

Historically, right from the beginning of the theoretical studies of the graph isomorphism problem, researchers have investigated the complexity of the isomorphism problem on restricted graph classes. Early examples are polynomial-time isomorphism tests for planar graphs and interval graphs, as well as isomorphism-completeness results for bipartite graphs, regular graphs and so on. For parameterized graph classes, such as graphs of genus at most k or graphs of degree at most k, an aim was to design fixed parameter tractable algorithms, which have a running time polynomial for each fixed k, such that the degree of the polynomial is independent of k.

In my talk I survey the results and some techniques that have been obtained over the last several decades, including fixed-parameter tractable algorithms, intermediate graph classes, and parameterizations by input similarity. I also discuss intricacies concerning techniques supposed to rule out fixed-parameter tractable algorithms and kernelization results.

3.20 Structure and Automorphisms of Primitive Coherent Configurations

Xiaorui Sun (Columbia University – New York, US)

Joint work of Sun, Xiaorui; Wilmes, John
Main reference X. Sun, J. Wilmes, "Faster Canonical Forms for Primitive Coherent Configurations", in Proc. of the 47th Annual ACM Symp. on Theory of Computing (STOC'15), pp. 693–702, ACM, 2015; pre-print available as arXiv:1510.02195v1 [math.CO].
URL http://dx.doi.org/10.1145/2746539.2746617
URL http://arxiv.org/abs/1510.02195v1

Primitive coherent configurations (PCCs) are colored directed graphs that generalize strongly regular graphs (SRGs), a class perceived as difficult for GI. Moreover, PCCs arise naturally as obstacles to combinatorial divide-and-conquer approaches for general GI.

We prove that PCCs have at most $\exp\left(O(n^{1/3})\right)$ automorphisms, with known exceptions. This is the first improvement over Babai's 1981 bound of $\exp\left(O(n^{1/2})\right)$. Our result also implies an $\exp\left(O(n^{1/3})\right)$ upper bound on the order of primitive but not doubly transitive permutation groups (with known exceptions). This bound was previously known (Cameron, 1981) only through the Classification of Finite Simple Groups.

For the analysis we develop a new combinatorial structure theory for PCCs that in particular demonstrates the presence of "clique geometries" among the constituent graphs of PCCs in certain range of the parameters.

3.21 Complexity Classes and the Graph Isomorphism Problem

Jacobo Torán (Universität Ulm, DE)

It is well known that the Graph Isomorphism problem is in NP, but not expected to be NP complete and not known to be in P. In this talk I review some of the attempts that have been made in order to provide a better classification of the problem. I give an overview on the know upper and lower bounds for the Graph Isomorphism problem in terms of complexity classes.

3.22 On Tinhofer's Linear Programming Approach to Isomorphism Testing

Oleg Verbitsky (HU Berlin, DE)

Joint work of Arvind, Vikraman; Köbler, Johannes; Rattan, Gaurav; Verbitsky, Oleg
Main reference V. Arvind, J. Köbler, G. Rattan, O. Verbitsky, "On Tinhofer's Linear Programming Approach to Isomorphism Testing", in Proc. of the 40th Int'l Symp. on Mathematical Foundations of Computer Science (MFCS'15), LNCS, Vol. 9235, pp. 26–37, Springer, 2015.
URL http://dx.doi.org/10.1007/978-3-662-48054-0_3

Exploring a linear programming approach to Graph Isomorphism, Tinhofer [1] defined the concept of a compact graph: A graph is compact if the polytope of its fractional automorphisms is integral. Tinhofer noted that isomorphism testing for compact graphs can be done quite efficiently by linear programming. However, the problem of characterizing and recognizing compact graphs in polynomial time remains an open question.

We relate this approach to the classical color-refinement (CR) procedure. We call a graph CR-definable if the CR procedure distinguishes it from any non-isomorphic graph. Babai, Erdős, and Selkow [2] showed that random graphs are CR-definable with high probability. Immerman and Lander [3] showed that the CR-definable graphs are exactly the graphs definable in two-variable first-order logic with counting quantifiers. An efficient characterization of this class of graphs has been obtained recently in [4] and [5].

Using the last result, we prove that all CR-definable graphs are compact. In other words, the applicability range for Tinhofer's linear programming approach to isomorphism testing is at least as large as for the combinatorial approach based on color refinement.

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3.23 Canonical Forms for Steiner Designs in time $v^{O(\log v)}$

John Wilmes (University of Chicago, US)

Joint work of Babai, László; Wilmes, John
Main reference L. Babai, J. Wilmes, "Quasipolynomial-time canonical form for Steiner designs", in Proc. of the 45th Annual ACM Symp. on Theory of Computing (STOC'13), pp. 261–270, ACM, 2013.
URL http://dx.doi.org/10.1145/2488608.2488642

A Steiner $S(t, k, v)$ design is a collection of v points, along with a collection of k-subsets of points, called blocks, such that every set of t points is contained in a unique block.

We produce canonical forms, and hence decide isomorphism for Steiner $S(2, k, v)$ designs in time $v^{O(\log v)}$. Previously, a quasipolynomial time-complexity bound was known for bounded k [1], while the best overall time-complexity bound was $v^{O(\sqrt{v \log v})}$ [2, 5]. A $v^{t+O(\log v)}$ time-complexity bound for Steiner $S(t, k, v)$ designs follows immediately from our result.

In fact, we analyze the individualization/refinement process on Steiner designs, and prove that $O(\log v)$ individualizations suffices to completely split an $S(2, k, v)$ design into uniquely colored vertices after naive refinement. In particular, our analysis gives a $v^{O(\log v)}$ bound on the number of automorphisms of a nontrivial Steiner design.

A simultaneous, independent proof of the same time-complexity bound was given by Chen, Sun, and Teng, and presented together with the present result at STOC'13 [4, 3].

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3.24 Group Isomorphism Is Tied up in Knots

James B. Wilson (Colorado State University, US)

After a century of attention, our understanding of isomorphisms between groups is rich and full of questions. The results have implications to Topology, Computer Science, Logic, and Algebra. Some recent projects are moving beyond established barriers while others are demonstrating why lack of progress is to be expected.

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