Optimal Reachability in Weighted Timed Automata and Games*

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Abstract

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1 Overview of the Talk

Toward the development of more reliable computerized systems, expressive models are designed, targetting application to automatic verification (model-checking). As part of this effort, timed automata have been proposed in the early nineties [2] as a powerful and suitable model to reason about (the correctness of) real-time computerized systems. Timed automata extend finite-state automata with several clocks, which can be used to enforce timing constraints between various events in the system. They provide a convenient formalism and enjoy reasonably-efficient algorithms (*e.g.* reachability can be decided using polynomial space), which explains the enormous interest that they provoked in the community of formal methods. Timed games [4] extend timed automata with a way of modelling systems interacting with external, uncontrollable components: some transitions of the automaton cannot be forced or prevented to happen. The reachability problem then asks whether there is a strategy (or controller) to reach a given state, whatever the (uncontrollable) environment does. This problem can also be decided, in exponential time.

Timed automata and games are not powerful enough for representing quantities like resources, prices, temperature, etc. The more general model of hybrid automata [14] allows for accurate modelling of such quantities using hybrid variables. The evolution of these variables follow differential equations, depending on the state of the system, and this unfortunately makes the reachability problem undecidable, even in the very restricted case of stopwatches (stopwatches are clocks that can be stopped, and hence, automata with stopwatches only are the simplest hybrid automata one can think of).

Weighted (or priced) timed automata [3, 5] and games [15, 1, 9] have been proposed in the early 2000's as an intermediary model for modelling resource consumption or allocation problems in real-time systems (*e.g.* optimal scheduling [6]). As opposed to (linear) hybrid

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systems, an execution in a weighted timed model is simply one in the underlying timed model: the extra quantitative information is just an observer of the system, and it does not modify the possible behaviours of the system.

In this talk, we will investigate the models of weighted timed automata and games, and we will mostly focus on the important optimal reachability problem: given a target location, we want to compute the optimal (*i.e.* smallest) cost for reaching a target location, and a corresponding strategy. We will survey the main results that have been obtained on that problem, from the primary results of [3, 5, 16, 13, 8, 17, 7] to the most recent developments [11, 10]. We will also mention our new tool TiAMo, which can be downloaded at `https://git.lsv.fr/colange/tiamo`. We will finally show that weighted timed automata and games have applications beyond that of model-checking [12].

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