Zapping Zika with a Mosquito-Managing Drone: Computing Optimal Flight Patterns with Minimum Turn Cost*

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Abstract

We present results arising from the problem of sweeping a mosquito-infested area with an Unmanned Aerial Vehicle (UAV) equipped with an electrified metal grid. This is related to the Traveling Salesman Problem, the Lawn Mower Problem and, most closely, Milling with Turn Cost. Planning a good trajectory can be reduced to considering penalty and budget variants of covering a grid graph with minimum turn cost. On the theoretical side, we show the solution of a problem from The Open Problems Project that had been open for more than 15 years, and hint at approximation algorithms. On the practical side, we describe an exact method based on Integer Programming that is able to compute provably optimal instances with over 500 pixels. These solutions are actually used for practical trajectories, as demonstrated in the video.

1998 ACM Subject Classification F.2.2 [Nonnumerical Algorithms and Problems] Geometrical Problems and Computations, I.2.9 [Robotics] Autonomous Vehicles

Keywords and phrases Covering tours, turn cost, complexity, exact optimization

Digital Object Identifier 10.4230/LIPIcs.SoCG.2017.62

Category Multimedia Contribution

1 Introduction

Consider an outdoor setting that is populated by swarms of mosquitoes, with a number of known hotspots. How can we lower the danger of diseases by zapping the mosquitos with a flying drone, such as the one shown in Fig. 1?

* This work was supported by the National Science Foundation Grant No. [CNS-1646607].

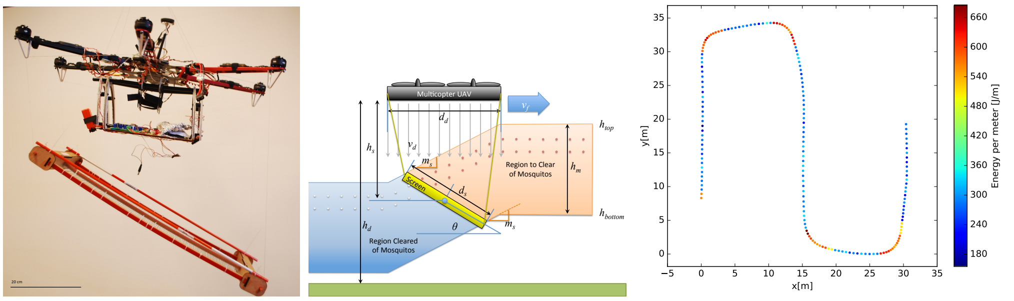


Figure 1 (Left) A drone equipped with an electrical grid for killing mosquitoes. (Middle) Physical aspects of the flying drone. (Right) Making turns is expensive.

Visiting a set of points by an optimal tour is a natural and important problem, both in theory and practice. If we are only concerned with minimizing the total distance traveled for visiting all points, we get the classic Traveling Salesman Problem (TSP). However, for path planning by a flying robot, we also incur a cost for changing direction, as illustrated in Fig. 1 (Right). This is related to the Angular-Metric TSP (AM-TSP), in which the objective is to minimize the total turn cost. In addition, we may want to focus on a subset of the points in order to provide better coverage, incurring a penalty for the uncovered ones.

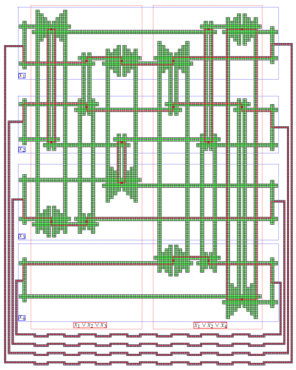
2 Related Work

The classic Traveling Salesman Problem (TSP) has enjoyed a huge amount of attention; see Cook [7] for a general overview, and Applegate et al. [2] for a more advanced textbook on computing provably exact solutions. For theoretical work on the Lawn Mower Problem, see Arkin, Fekete and Mitchell [5, 6]. Angle-restricted tour problems were studied by Fekete and Woeginger [10]. Touring points in the plane with minimal continuous turn cost was considered by Aggarwal et al. [1]. Arkin et al. [4] consider different grid-based versions of covering with turn cost, and provide a spectrum of approximation algorithms. They pose the complexity of finding a cycle cover with minimum total turn cost as an open problem, first published in the conference version in 2001 [3]; this problem gained additional attention by becoming part of The Open Problems Project [8] as Problem #53 in 2003.

3 Problems

We are given a grid graph, which arises as the dual graph from a set of pixels. We consider several different covering tour problems and their cycle cover relaxations. We identify three different ways that a pixel can be traversed, each with a different cost: straight, by a simple turn, and by a U-turn. The ratio between straight traversal and simple turns is arbitrary but fixed, while the cost of a U-turn is twice as much as for a simple turn. The following emerge for full coverage, cheap coverage of a subset, and coverage with a budget constraint.

- Given a grid graph, find a minimum-cost tour/cycle cover that covers all vertices.
- Given a grid graph and an individual penalty per vertex, find a minimum-cost tour/cycle cover in which instead of covering a vertex, its penalty may be paid.
- Given a grid graph, an individual value per vertex and a total budget, find a maximum-value tour/cycle cover of a subset of vertices, subject to the budget constraint.



	2D	3D	Hexagonal	General grids
Full cycle cover	4*	6	6	2ω
Full tour	6*	12	12	4ω
Subset cycle cover	4	6	6	2ω
Subset tour	10	14	14	$4\omega + 2$
Penalty cycle cover	6	8	8	$2(\omega + 1)$
Penalty tour	16	20	20	$4(\omega + 1) + 4$

Figure 2 (Left) The NP-completeness construction for the 1-in-3 3SAT instance. $(x_1 \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_4})$. The provided solution is $x_1 = 0, x_2 = 1, x_3 = 0, x_4 = 1$. (Right) Approximation factors of our approximation techniques. ω is the maximum number of distinct full strips (or orientations) by which a grid point is covered. ($*$) Note that Arkin et al. [4] achieve a factor of 2.5 on the number of turns for cycle cover and a factor of 3.75 on the number of turns for tours in 2-dimensional grid graphs. With distance costs it becomes a 4-approximation for cycle cover and tour.

4 Complexity and Approximation

We can prove that computing a minimum-turn cycle cover in 2-dimensional grid graphs is NP-hard. This solves Problem #53 in The Open Problems Project [8]. The hardness of the problem is not obvious: usually large parts of a solution can be easily deduced by local information and 2-factor techniques.

▶ **Theorem 1.** *It is NP-hard to find a cycle cover with a minimum number of turns in a 2-dimensional grid graph.*

See Fig. 2 (Left) for an idea of the construction. For a full proof, see Krupke [11] and the upcoming paper [9]. In addition, we can give a number of approximation algorithms; see Fig. 2 (Right) for an overview, and [11, 9] for details.

5 Integer Programming

The following is an Integer Programming formulation for finding an optimal budget cycle cover with turn cost.

$$\max \sum_{v \in V} d_v (1 - y_v) \tag{1}$$

$$\text{s.t.} \quad 1 \leq 4y_v + \sum_{\{u,w\} \subseteq N(v)} x_{v,\{u,w\}} \leq 4 \qquad \forall v \in V \tag{2}$$

$$2x_{v,\{w\}} + \sum_{u \in N(v), u \neq w} x_{v,\{w,u\}} = 2x_{w,\{v\}} + \sum_{u \in N(w), u \neq v} x_{w,\{u,v\}} \qquad \forall \{v,w\} \in E \tag{3}$$

$$\sum_{v \in V} \sum_{\{u,w\} \subseteq N(v)} \text{cost}(uvw) x_{v,\{u,w\}} \leq B \tag{4}$$

$$x_{v,\{u,w\}} \in \mathbb{N}_0, y_v \in \{0,1\} \qquad \forall v \in V, \{u,w\} \subseteq N(v)$$

Variable $x_{v,\{u,w\}}$ counts the traversals of v with end points in u and w, while y_v indicates whether a variable is left uncovered. The objective function in Eq. (1) sums up the densities

d_v of all pixels for which the *not covered* variable is false. Eq. (2) enforces a pixel to be covered or the *not covered* variable to be set to true. Arkin et al. [4] showed that no pixel needs to be visited more than four times, otherwise a simple local optimization can be performed. Eq. (3) enforces the transitions between two adjacent pixels to match. Eq. (4) limits the costs of the tour to the budget B (the battery runtime).

The separation of subtours is more complicated than for the classic TSP, because there may be subtours that cross but are not connected (due to turn cost); moreover, instead of connecting two subtours, one subtour can also be discarded. The following can separate any given solution with multiple subtours. Let C be the pixels of a selected subtour. Let p be a pixel in C not traversed by other subtours, and another covered pixel $p' \notin C$. C_s are the pixels that are traversed by the subtour without turning. $T(v)$ describes the 90°-turn variables of a pixel v. x' refers to the variable assignment in the current solution.

$$y_p + y_{p'} + \sum_{\substack{\{u,w\}\subseteq N(p)\\ x'_{p,\{u,w\}}=0}} x_{p,\{u,w\}} + \sum_{\substack{t\in T(v)\\ v\in C_s\setminus\{p\}}} t + \sum_{\substack{v\in C\setminus(C_s\cup\{p\})\\ u\neq w\in V\\ x'_{v,\{u,w\}}=0}} x_{v,\{u,w\}} \geq 1 \tag{5}$$

6 The Video

The video opens with an introduction of the challenge of controlling mosquitoes with a UAV, followed by a discussion of geometric modeling aspects, leading to finding minimum-turn covering tours and the closely related minimum-turn cycle covers. The complexity of the latter is a long-standing open problem, whose solution is stated. For purposes of coverage with flying drones, the further variants with penalties and budget constraint are introduced, followed by a sketch of an approximation approach. An exact method based on Integer Programming is described next, which is able to solve relevant real-world instances to provable optimality. The video concludes by showing how such trajectories are used by the drone.

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