On Definable and Recognizable Properties of Graphs of Bounded Treewidth

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Abstract

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1 Overview of the talk

The foundational observation of the field of automata and logic is that on many well-behaved classes of structures the notion of *recognizability* of a structure's property by a finite-state automaton is equivalent to *definability* of this property in *monadic second-order logic* (MSO). This equivalence holds for properties of words and trees, both finite and infinite, and provides means for the algorithmic treatment of MSO-definable properties in these classes of structures.

It is natural to ask what aspects of a class of structures imply that the notions of MSO-definability and recognizability by (appropriately defined) finite automata coincide on this class. In early 90s, Courcelle [2] proved that on any class of structures of bounded treewidth, that is, where structures roughly look like trees with trunks of width bounded by a constant, one implication holds: MSO-definability implies recognizability. This fundamental result already provides most of the desired algorithmic corollaries, most notably that MSO-definable properties of structures of bounded treewidth can be decided in linear fixed-parameter time, where treewidth is the parameter. However, the reverse implication became known as the *Courcelle's conjecture* and remained open until very recently, despite multiple attempts and some incomplete proofs [3, 4]. Finally, last year together with Bojańczyk we resolved the conjecture in affirmative [1].

The main obstacle when approaching the Courcelle's conjecture is that the constructed MSO formula expressing the recognizable property in question has to work only on the structure, and not on its tree decomposition certifying the constant upper bound on the treewidth. More precisely, if we were given such a tree decomposition, then we could just existentially quantify an accepting run of the automaton recognizing the property. However, while graphs of treewidth 1, that is, forests, can be perfectly understood, for structures of larger treewidth computing an optimum-width tree decomposition is a highly nontrivial combinatorial task, and an appropriate tree decomposition cannot be immediately defined from the structure. Therefore, the main technical contribution of [1] is a proof that an approximate tree decomposition of a structure can be constructed by means of a *nondeterministic MSO transduction*, which is a formalism that captures MSO-definable

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transformations of relational structures. The crucial ingredient of this proof is an application of Simon's factorization forest theorem [5].

During the talk, we will discuss the relation between MSO-definability and recognizability on various classes of structures, in particular on classes of bounded treewidth. We will also give a sketch of the proof of the Courcelle's conjecture, focusing on the role played by Simon's factorization forest theorem.

References

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