

Geometric Modelling, Interoperability and New Challenges

Edited by

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 17221 “Geometric Modelling, Interoperability and New Challenges”. While previous Dagstuhl seminars on geometric modeling were focused on basic research, this seminar was focused on applications of geometric modeling to four topic areas: big data and cloud computing, multi-material additive manufacturing, isogeometric analysis, and design optimization. For this purpose we brought together participants from industry urgently in need of better solutions, researchers in the above application areas, and researchers in the geometric modeling community.

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Edited in cooperation with Georg Muntingh

1 Executive Summary

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This report documents the program and the outcomes of Dagstuhl Seminar 17221 “Geometric Modelling, Interoperability and New Challenges”.

The importance of accurate geometric models of shapes, both naturally occurring and man-made, is rapidly growing with the maturing of novel manufacturing technologies and novel analysis technologies. The advent of big data challenges and cloud-based computing serves to confound the distribution and remote access of these geometric models. While previous Dagstuhl seminars on geometric modeling were focused on basic research, this seminar was focused on applications of geometric modeling. We selected four core application areas that stretch the underlying mathematical underpinnings of the discipline to its limits and beyond:



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- Big data and Cloud computing
- Multi-material additive manufacturing (3D Printing)
- Isogeometric analysis
- Design optimization

The seminar provided a forum for leading researchers to present new ideas, to exchange scientific insights, and to bring together practical applications and basic research. The previous seminar explored the theory of shape representations, shape transformations, and computational models for each. This seminar explored application of this theory to the four above-mentioned application challenges, strengthening the reliability and performance of applications in engineering, manufacturing, and scientific exploration. The goal of the seminar was to establish a common understanding between the Geometric Modeling research community and the above application fields by addressing the following questions:

- How to handle geometric descriptions that cannot be processed by one computer but have to be distributed among many computers in the cloud?
- How to process huge 3D data sets that may be noisy and incomplete into clean, scalable, and easy to access 3D environments?
- How to turn big, dispersed maybe noisy and incomplete geometric data into clean, scalable, and easy to access 3D information that can be used for change detection and decision making?
- How to represent, control, and process complex, anisotropic, internal material enabled by additive manufacturing, and how to design for additive manufacturing?
- How to perform topology optimization of the internal structure of objects to enable additive manufacturing to reach its full potential?
- How to introduce analysis-based design in CAD systems with isogeometric analysis in mind?

To answer these questions we brought together participants from industry urgently in need of better solutions, researchers in the application areas represented by the four topics, and researchers in the geometric modeling community whose interests align with the four topic areas. The scientific presentations lasted 15 to 20 minutes. Senior researchers gave 4 overview talks on the 4 themes of the seminar. Perspective working groups were organized for each of the four topics.

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3 Overview of Talks

3.1 Planar Pythagorean-Hodograph B-spline Curves

Guðrun Albrecht (Universidad Nacional de Colombia – Medellín, CO)

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Joint work of Guðrun Albrecht, Carolina Vittoria Beccari, Jean-Charles Canonne, Lucia Romani

We introduce the new class of planar Pythagorean-Hodograph (PH) B-Spline curves. They can be seen as a generalization of the well-known class of planar Pythagorean-Hodograph (PH) Bézier curves, presented by R. Farouki and T. Sakkalis in 1990, including the latter ones as special cases. Pythagorean-Hodograph B-Spline curves are nonuniform parametric B-Spline curves whose arc-length is a B-Spline function as well. An important consequence of this special property is that the offsets of Pythagorean-Hodograph B-Spline curves are non-uniform rational B-Spline (NURBS) curves. Thus, although Pythagorean-Hodograph B-Spline curves have fewer degrees of freedom than general B-Spline curves of the same degree, they offer unique advantages for computer-aided design and manufacturing, robotics, motion control, path planning, computer graphics, animation, and related fields. After providing a general definition for this new class of planar parametric curves, we present useful formulae for their construction and discuss their remarkable attractive properties. Then we provide a method to determine within the set of all PH B-Splines the one that is closest to a given reference spline having the same degree and knot partition.

3.2 The Role of Volume Meshes in 3D Modeling for Additive Manufacturing

Marco Attene (CNR – Genova, IT)

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Joint work of Daniela Cabiddu, Marco Attene

Main reference D. Cabiddu, M. Attene, “ ϵ -maps: Characterizing, detecting and thickening thin features in geometric models”, *Computers & Graphics*, Vol. 66, pp. 143–153, Elsevier, 2017.

URL <https://doi.org/10.1016/j.cag.2017.05.014>

Classical product development pipelines include a design phase and a process planning that precedes the actual fabrication. When moving from design to process planning, smooth NURBS-based surfaces are typically tessellated, and the resulting triangle meshes are analyzed and prepared for the final slicing step. This talk highlights the limitations of this surface-based approach, and discusses how explicit volume meshes can be exploited to effectively resolve a number of tasks, including the geometric analysis to ensure compatibility with the target printer, and the proper description of objects constituted of multiple and graded materials.

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3.3 Big Data Impact on Geometric Modeling

Chandrajit Bajaj (University of Texas – Austin, US)

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Joint work of Benedikt Bauer, Radhakrishna Bettadapura, Antje Vollrath, Abhishek Bhowmick, Eshan Chattopadhyay, David Zuckerman, Andrew Gillette, Alexander Rand

URL <https://cvcweb.ices.utexas.edu/cvcwp/publications/>

We revisit classic problems of geometric modeling in this age of “Big Data” and present scalable solutions with accuracy / speed tradeoffs. The scalable techniques include dimension reduction of spline spaces, generating low-discrepancy samplings of product spaces with quasi-polynomial number of samples (as opposed to exponential in the product dimension), approximate non-uniform fast Fourier transforms, shape optimized dictionaries, and approximate geometric optimization. The latter is by reduction to Semi-Definite Programming (SDP), and methods for generating almost regular, congruent tiled arrangements using a new generative class of polyhedra. Applications of these scalable techniques are to functional data and shape approximation, geometric shape similarity, complementarity matching and constructing multi-component assemblies.

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3.4 Low Rank Approximation for Planar Domain Parameterization

Falai Chen (Univ. of Science & Technology of China – Anhui, CN)

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Joint work of Maodong Pan, Falai Chen

Construction of spline surfaces from given boundary curves is one of the classical problems in computer aided geometric design, which regains much attention in iso-geometric analysis in recent years and is called domain parameterization. However, for most of the state-of-the-art parametrization methods, the rank of the spline parameterization is usually large for complex computational domains, which results in higher computational cost in solving numerical PDEs. In this talk, we propose a low-rank representation for the spline parameterization of planar domains using low-rank tensor approximation technique, and apply quasiconformal mapping as the framework of the spline parameterization. Under given correspondence of boundary curves, a quasi-conformal map with low rank and low distortion between a unit square and the computational domain can be obtained by solving a non-linear optimization problem. We propose an efficient algorithm to compute the quasi-conformal map by solving

two convex optimization problems alternatively. Experimental results show that our approach can produce a low-rank parametric spline representation of planar domains.

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3.5 Introductory Talk on Additive Manufacturing

Tor Dokken (SINTEF – Oslo, NO)

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The first part of the talk addressed Additive Manufacturing (AM) from the perspective of manufacturing. Differences between AM, subtractive manufacturing and formative manufacturing were explained. The evolution of AM was outlined, fundamentals of AM explained including the formal definition of AM by ISO/ASTM 52900. A short description of the seven fundamentally different AM process categories provided. In the second part of the talk the digital perspective of AM was in focus exemplified by the EC H2020 R&I Action CAXMan – Computer Aided Technologies for Additive Manufacturing. In CAXMan a 3-variate isogeometric approach for simulation based design targeting AM is addressed. CAXMan has a strong focus on the challenges of interoperability of design, simulation, process planning and actual manufacturing for AM, and development of suitable standards supporting AM.

3.6 Precise Construction of Micro-structures and Porous Geometry via Functional Composition

Gershon Elber (Technion – Haifa, IL)

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We introduce a modeling constructor for micro-structures and porous geometry via curve-trivariate, surface-trivariate and trivariate-trivariate function (symbolic) compositions. By using 1-, 2- and 3-manifold based tiles and paving them multiple times inside the domain of a 3-manifold deforming trivariate function, smooth, precise and watertight, yet general, porous/micro-structure geometry might be constructed, via composition. The tiles are demonstrated to be either polygonal meshes, (a set of) Bézier or B-spline curves, (a set of) Bézier or B-spline (trimmed) surfaces, (a set of) Bézier or B-spline (trimmed) trivariates or any combination thereof, whereas the 3-manifold deforming function is either a Bézier or a B-spline trivariate. We briefly lay down the theoretical foundations, only to demonstrate the power of this modeling constructor in practice, and also present a few 3D printed tangible examples. We will then discuss these results and conclude with some future directions and limitations.

3.7 Isogeometric Finite Element Methods for Shape Optimization

Daniela Fußeder (MTU Aero Engines – München, DE)

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Joint work of Bernd Simeon, Daniela Fußeder

We present a framework for shape optimization with isogeometric analysis (IGA) from an optimal control perspective: In this formulation both the state and the control space are discretized by B-splines which means practically that two (possibly different) meshes may be used, one for simulation and one for optimization.

As a result, interpolation errors of the state and the control may be addressed separately, yet through the connection of IGA a tight relation between geometry and analysis is maintained.

Moreover, several options like optimizing weights in a non-uniform rational B-spline geometry, using local refinement or exploiting higher polynomial degrees of basis functions are shown.

3.8 THB-spline Simplification and Scattered Data Approximation

Carlotta Giannelli (University of Firenze, IT)

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Joint work of Cesare Bracco, Carlotta Giannelli, Alessandra Sestini

The geometric processing of unstructured large data sets may be effectively addressed by developing reliable adaptive schemes that guarantee high-quality approximations in terms of compact representations. By focusing on spline representations, adaptivity may easily be achieved by considering multilevel extensions of the standard B-spline model, where the tensor-product structure is (locally) preserved level by level.

We present two adaptive schemes that exploit the capabilities of truncated hierarchical B-splines (THB-splines) to reduce the computational costs connected with the reconstruction of large data sets. In particular, we introduce a THB-spline simplification algorithm defined in terms of an efficient data reduction operator. Subsequently, in order to deal with scattered data of high complexity and different nature, we present an adaptive data fitting scheme as extension of local least-squares approximations to hierarchical spline constructions. The performances of the two methods will be illustrated by a selection of examples, including real data sets describing different terrain configurations.

References

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3.9 Visualization of Uncertainty-aware Geometry

Christina Gillmann (TU Kaiserslautern, DE)

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Joint work of Christina Gillmann, Thomas Wischgoll, Jose Tiberio Hernandez, Hans Hagen, Bernd Hamann

Images usually contain a variety of image errors such as lense flair, artifacts and blur. This errors are aggravating the application of image processing methods and reduce the quality of geometry extraction. To solve this problem, we present a novel description of geometry. In contrast to the classical definition of geometry we not solely consider points that can be composed to higher order objects. In addition each point obtains a support function describing the possibility that a point exists and also a range where this point is allowed to be located. To be able to generate this geometry we also extend well known geometry extraction techniques to output the requested points and also their support functions. These support functions are based on uncertainty image metrics and can also be used to adapt the extracted geometry thus the resulting points obtain qualitatively higher support functions.

3.10 De Boor Suitable T-splines

Ron Goldman (Rice University – Houston, US)

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Joint work of Yang Zhang, Visit Pataranutaporn, Ron Goldman

We describe necessary and sufficient conditions on the T-mesh for an Analysis Suitable T-spline to have a local de Boor recursive evaluation algorithm.

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3.11 Making Multidisciplinary Design Optimization Work

Thomas A. Grandine (The Boeing Company – Seattle, US)

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In many engineering intensive design activities (e.g. airplanes and high performance automobiles), the shapes of the engineered products can have a tremendous effect on their engineering performance. Determining these shapes is often best accomplished by formulating optimization problems which attempt to maximize engineering performance subject to certain

constraints. For example, one might wish to minimize the weight of an airplane subject to constraints on being able to carry a certain number of passengers a prescribed distance in a certain amount of time. Solving design problems this way imposes severe restrictions on geometric modeling methods as well as the analysis codes which determine that engineering performance. This talk will explore the interplay between optimization and the requirements it imposes on analysis and geometric design.

3.12 Drall-based Ruled Surface Modeling

Hans Hagen (TU Kaiserslautern, DE) and Benjamin Karer (TU Kaiserslautern, DE)

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In continuum mechanics, ruled surfaces have shown considerable potential as candidates to modeling thin narrow strips of inextensible elastic material. Yet, the surface definitions usually applied in this domain are often limited in their generality or require complicated constraints to guarantee important geometrical properties. We introduce a model for the analytically exact definition of arbitrary ruled surfaces featuring trivial constraints for properties like the developability of a surface or the isometry of a deformation. The model is based on the movement of two coupled moving frames of reference whose transition equations are governed by a one-parametric minimal system of invariants of ruled surfaces. Based on these invariants, we derive a bending energy integral for arbitrary ruled surfaces of finite width. We demonstrate our model's descriptive power by several examples. Being general, accurate, and parallelizable, our model is well suited for high-detail modeling of arbitrarily shaped inextensible elastic surface strips.

3.13 Shape from Sensors

Stefanie Hahmann (INRIA Grenoble Rhône-Alpes, FR)

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Joint work of Tibor Stanko, Stefanie Hahmann, Georges-Pierre Bonneau, Nathalie Saguin-Sprynski
Main reference T. Stanko, S. Hahmann, G.-P. Bonneau, N. Saguin-Sprynski, "Shape from sensors: Curve networks on surfaces from 3D orientations", *Computers & Graphics*, 66:74–84, Elsevier, 2017.
URL <https://doi.org/10.1016/j.cag.2017.05.013>

We present a novel framework for acquisition and reconstruction of 3D curves using orientations provided by inertial sensors. While the idea of sensor shape reconstruction is not new, we present the first method for creating well-connected networks with cell complex topology using only orientation and distance measurements and a set of user-defined constraints. By working directly with orientations, our method robustly resolves problems arising from data inconsistency and sensor noise. Although originally designed for reconstruction of physical shapes, the framework can be used for "sketching" new shapes directly in 3D space. We test the performance of the method using two types of acquisition devices: a standard smartphone, and a custom-made prototype for measuring orientations and distances.

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3.14 An Error Bound for Interpolation by Low Rank Functions

Bert Jüttler (Universität Linz, AT)

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It has been observed recently that tensor-product spline surfaces with low rank coefficients provide advantages for efficient numerical integration, which is important in the context of matrix assembly in isogeometric analysis [1]. By exploiting the low-rank structure one may efficiently perform multivariate integration by a executing a sequence of univariate quadrature operations. This fact has motivated us to study the problem of creating such surfaces from given boundary curves. More precisely, we propose a method for interpolation by rank-2 functions and apply it to the problem of creating surface patches from four given boundary curves [2]. In addition, we discuss the generalization to rank- n functions and analyze the relationship to the method of cross approximation [3]. In particular we derive a new error bound for interpolation by rank- n functions. Joint work with Nira Dyn and Dominik Mokriš.

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3.15 Shape Preserving Interpolation on Surfaces

Panagiotis Kaklis (The University of Strathclyde – Glasgow, GB) and Alexandros Ginnis

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We introduce a criterion for G2 shape-preserving interpolation of points on surfaces, based on the behaviour of the corresponding composite geodesic interpolant in the neighbourhood of the data points. Two alternative families of splines are proposed, combining appropriately nu- or non-uniform-degree splines with preimages of the geodesic segments, for conforming to the shape-preservation criterion for sufficiently large values of the nu or interval-degree parameters. Preliminary numerical results are given using the nu-splines-based interpolants for shape-preserving interpolation on cylinders, spheres and free-form surfaces.

3.16 Truncated Hierarchical Loop Subdivision Surfaces and Application in Isogeometric Analysis

Hongmei Kang (Univ. of Science & Technology of China – Anhui, CN), Falai Chen (Univ. of Science & Technology of China – Anhui, CN), and Xin Li

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Joint work of Falai Chen, Hongmei Kang

Main reference H. Kang, X. Li, F. Chen, J. Deng, “Truncated Hierarchical Loop Subdivision Surfaces and application in isogeometric analysis”, *Computers & Mathematics with Applications*, 72(8):2041–2055, Elsevier, 2016.

URL <https://doi.org/10.1016/j.camwa.2016.06.045>

Subdivision Surface provides an efficient way to represent free-form surfaces with arbitrary topology. Loop subdivision is a subdivision scheme for triangular meshes, which is C2 continuous except at a finite number of extraordinary vertices with G1 continuous. In this paper we propose the Truncated Hierarchical Loop Subdivision Surface (THLSS), which generalizes truncated hierarchical B-splines to arbitrary topological triangular meshes. THLSS basis functions are linearly independent, form a partition of unity, and are locally refinable. THLSS also preserves the geometry during adaptive h-refinement and thus inherits the surface continuity of Loop subdivision surface. Adaptive isogeometric analysis is performed with the THLSS basis functions on several complex models with extraordinary vertices to show the potential application of THLSS.

3.17 Branched Covering Surfaces

Konrad Polthier (Freie Universität Berlin)

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© Konrad Polthier (Freie Universität Berlin)

Joint work of Konrad Polthier, Konstantin Poelke, Matthias Nieser, Ulrich Reitebuch, Faniry Razafindrazaka

Multivalued functions and differential forms naturally lead to the concept of covering surfaces and more generally of covering manifolds. This talk will review, illustrate and discretize basic concepts of branched covering surfaces starting from complex analysis, surface theory up to their recent appearance in geometry processing algorithms. Applications will touch artistic surface modeling, geometry retargeting, surface and volume parameterization, and novel weaved surface representations.

3.18 Assembly Design of Wind-up Toys

Ligang Liu (Univ. of Science & Technology of China – Anhui, CN)

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Joint work of Ligang Liu, Peng Song, Xiaofei Wang, Xiao Tang, Chi-Wing Fu, Hongfei Xu, Niloy J. Mitra

Wind-up toys are mechanical assemblies that perform intriguing motions driven by a simple spring motor. Due to the limited motor force and small body size, wind-up toys often employ higher pair joints of less frictional contacts and connector parts of nontrivial shapes to transfer motions. These unique characteristics make them hard to design and fabricate as compared to other automata. In this paper, we present a computational system to aid the

design of wind-up toys, focusing on constructing a compact internal wind-up mechanism to realize user-requested part motions. Our key contributions include an analytical modeling of a wide variety of elemental mechanisms found in common wind-up toys, including their geometry and kinematics, conceptual design of wind-up mechanisms by computing motion transfer trees that support the requested part motions, automatic construction of wind-up mechanisms by connecting multiple elemental mechanisms, and an optimization on the geometry of parts and joints with an objective of compacting the mechanism, reducing its weight, and avoiding collision. We use our system to design wind-up toys of various forms, fabricate a number of them using 3D printing, and show the functionality of various results.

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3.19 Generalized Splines on T-meshes

Tom Lyche (University of Oslo, NO)

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Joint work of Bracco Cesaro, Tom Lyche, Carla Manni, Fabio Roman, Hendrik Speleers

Splines play a central role in IgA. We consider a generalization of polynomial splines known as Tchebycheffian splines. With such splines one can obtain nonrational representation of conic sections and Tchebycheffian B-splines have the important properties of the polynomial ones. Multivariate extensions of Tchebycheffian splines can be obtained via the tensor-product approach or by considering more general T-mesh structures. In this talk we consider Tchebycheffian spline spaces over planar T-meshes and analyze their dimensions, extending the results presented in [1, 2, 3].

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3.20 Spectral Analysis of Matrices from NURBS Isogeometric Methods

Carla Manni (University of Rome “Tor Vergata”, IT)

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Joint work of Carla Manni, Carlo Garoni, Stefano Serra-Capizzano, Hendrik Speleers

When discretizing a linear PDE by a linear numerical method, the computation of the numerical solution reduces to solving a linear system. The size of this system grows when we refine the discretization mesh. We are then in the presence of a sequence of linear systems

with increasing size. It is usually observed in practice that the corresponding sequence of discretization matrices enjoys an asymptotic spectral distribution. Roughly speaking, this means that there exists a function, say f , such that the eigenvalues of the considered sequence of matrices behave like a sampling of f over an equispaced grid on the domain of f .

In this talk we analyze the spectral properties of discretization matrices arising from isogeometric Galerkin and collocation methods, based on d -variate NURBS of given degrees and applied to general second-order elliptic partial differential equations defined on a d -dimensional domain. This extends the results obtained for B-spline based isogeometric methods [1, 3].

The provided spectral information can be exploited for designing algorithms with convergence speed independent of the fineness parameters and also substantially independent of the degrees of the used NURBS, as in [2] for the B-spline case.

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3.21 Multi-scale Differential Analysis of Complex Shapes

Nicolas Mellado (Paul Sabatier University – Toulouse, FR)

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Joint work of Thibault Lejembre, Nicolas Mellado

Main reference N. Mellado, G. Guennebaud, P. Barla, P. Reuter, C. Schlick, “Growing Least Squares for the Analysis of Manifolds in Scale-Space”, *Computer Graphics Forum*, 31(5):1691–1701, Wiley, 2012.

URL <http://dx.doi.org/10.1111/j.1467-8659.2012.03174.x>

With the proliferation of acquisition devices, gathering massive volumes of 3D data is now easy. Processing such large masses of point-clouds, however, remains a challenge, because of noise, varying sampling, missing data, and shapes complexity.

This talk will present a multi-scale shape analysis framework that aims at detecting arbitrary geometric features as local stability of the surface derivatives. Derivatives are obtained by differentiating implicit surfaces reconstructed at multiple scales from the point-cloud. Inspired by scale-space analysis, this framework is an attempt to provide similar analysis tools for unstructured 3d point-clouds, e.g. feature and pertinent scale extraction. This talk will also review how this framework can be used to improve point-cloud registration, shape abstraction and primitive approximation (point-cloud to CAD).

3.22 Navigation in 3D Networked Virtual Environments: Sprite Trees and 3D Bookmarks

Géraldine Morin (University of Toulouse, FR)

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Joint work of Géraldine Morin, Minhui Zhu, Thomas Forgiione, Wei Tsang Ooi, Axel Carlier, Vincent Charvillat
Main reference M. Zhu, G. Morin, V. Charvillat, W. T. Ooi, “Sprite tree: an efficient image-based representation for networked virtual environments”, *The Visual Computer*, 33(11):1385–1402, Springer, 2017.

URL <http://dx.doi.org/10.1007/s00371-016-1286-0>

In this talk, we present how to adapt the 3D representation of large Networked Virtual Environments (NVE) to usage and navigation. Accessing and navigating through such large dataset is not easy for light clients with limited capacities, that is, with a limited rendering capacity, and a low bandwidth. First, we propose an alternative image based representation for the 3D content, generated by peers and collected in a sprite tree. We show that this representation leads to reducing both the data size but also the rendering time. Second, we show that we can further ease user navigation by proposing 3D bookmarks. These bookmarks also improve the predictability of the navigation: we benefit from the predictability in the streaming strategy and therefore can offer a better quality of service to the user.

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3.23 B-spline-like Bases for C^2 Cubics on the Powell-Sabin 12-Split

Georg Muntingh (SINTEF – Oslo, NO) and Tom Lyche (University of Oslo, NO)

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For spaces of constant, linear, and quadratic splines of maximal smoothness on the Powell-Sabin 12-split of a triangle, Cohen, Lyche and Riesenfeld recently discovered so-called S-bases. These are simplex spline bases with B-spline-like properties on a single macrotriangle, which are tied together across macrotriangles in a Bézier-like manner.

In this talk we give a formal definition of an S-basis in terms of certain basic properties. We proceed to investigate the existence of S-bases for the aforementioned spaces and additionally the cubic case, resulting in an exhaustive list. From their nature as simplex splines, we derive simple differentiation and recurrence formulas to other S-bases. We establish a Marsden identity that gives rise to various quasi-interpolants and domain points forming an intuitive control net, in terms of which conditions for C^0 , C^1 , and C^2 -smoothness are derived.

Although the cubic bases can only be used to define smooth surfaces over specific triangulations, we envision applications for local constructions, such as hybrid meshes and extra-ordinary points, with the potential to be used in isogeometric analysis.

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3.24 T-junctions in Spline Surfaces

Jörg Peters (*University of Florida – Gainesville, US*)

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Joint work of Kestutis Karciauskas, Daniele Panozzo, Jörg Peters

T-junctions occur in quad-meshes where surface strips start or terminate. While hierarchical splines are naturally suited for introducing T-junctions into quad meshes, they are not naturally suited for generating surfaces from quad meshes with T-junctions: some (many?) quad meshes with T-junctions do not admit a choice of knot-intervals for smooth hierarchical splines.

A simple geometric continuous spline construction, GT-splines, solves the problem, covering T-junctions by two or four patches of degree bi-4. GT-splines complement multi-sided surface constructions in generating free-form surfaces with adaptive layout.

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3.25 Isogeometric Analysis on Triangulations

Xiaoping Qian (*University of Wisconsin – Madison, US*)

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Joint work of Xiaoping Qian, Songtao Xia

In this talk, I will present a triangulation based isogeometric analysis approach. In this approach, both the geometry and physical fields are represented by bivariate/trivariate splines in Bernstein–Bézier form over the triangulation. We describe procedures for domain parameterization and triangulation from given physical domains, and procedures for constructing Cr-smooth basis functions over the domain. As a result, this approach can achieve automated meshing of objects of complex topologies, allow highly localized refinement, and obtain optimal convergence rates.

3.26 Moments of Sets with Refinable Boundary

Ulrich Reif (*TU Darmstadt, DE*)

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Joint work of Jan Hakenberg, Ulrich Reif

Main reference J. Hakenberg, U. Reif, “On the volume of sets bounded by refinable functions”, *Applied Mathematics and Computation*, 272:2–19, Elsevier, 2016.

URL <https://doi.org/10.1016/j.amc.2015.06.050>

We present a method for determining moments (volume, center of mass, inertia tensor, etc.) for objects which are bounded by refinable functions, such as 2d sets bounded by subdivision curves and 3d sets bounded by subdivision surfaces. The approach is based on the solution of an eigenequation resulting from the interrelation between a certain multilinear form and its refined counterparts.

3.27 Weights for Stabilised Least Squares Fitting

Malcolm A. Sabin (*Numerical Geometry Ltd. – Cambridge, GB*)

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There is a set of well established ideas which seem to fit together in a way which is simple, powerful and general. Powerful because they seem to offer the combination of achieving desirable goals with a surprising level of possible automation. General because they seem to apply to a very wide subset of approximation theory applications. However, I have not yet seen these ideas presented in this combination before. Despite the generality available, the talk describes an experiment in the simplest possible context, that of scalar interpolation of univariate data, with promising results.

3.28 Spline-based Quadrature Schemes in Isogeometric Analysis for Boundary Element Methods

Maria Lucia Sampoli (*University of Siena, IT*), Francesco Calabrò, and Giancarlo Sangalli (*University of Pavia, IT*)

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© Maria Lucia Sampoli, Francesco Calabrò, and Giancarlo Sangalli
Joint work of Alessandra Aimi, Francesco Calabrò, Mauro Diligenti, Maria Lucia Sampoli, Giancarlo Sangalli, Alessandra Sestini

Boundary Element Methods (BEMs) are schemes studied since the mid '80, for the numerical solution of those Boundary Valued Problems, which can be transformed to Boundary Integral Equations. Indeed, if the fundamental solution of the differential operator is known, a wide class of elliptic, parabolic, hyperbolic, interior and exterior problems can be reformulated by integral equations defined on the boundary of the given domain, whose solution is successively obtained by collocation or Galerkin procedures. On the other side, the new Isogeometric analysis approach (IgA), establishes a strict relation between the geometry of the problem domain and the approximate solution representation, giving surprising computational advantages. In the IgA setting a new formulation of BEMs has been studied, where the discretization spaces are splines spaces represented in B-spline form ([1, 2]). In order to take all the possible benefits from using B-splines instead of Lagrangian basis, an important point is the development of specific new quadrature formulas for efficiently implementing the assembly phase of the method ([3]).

In this talk the problem of constructing appropriate and accurate quadrature rules, tailored on B-splines, for Boundary Integral Equations is addressed.

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- 3 A. Aimi, F. Calabrò, M. Diligenti, M. L. Sampoli, G. Sangalli, A. Sestini, Efficient assembly based on B-spline tailored quadrature rules for the IgA-SGBEM, Submitted to *Computer Methods in Applied Mechanics and Engineering*, (under revision)

3.29 Introductory Talk on Isogeometric Analysis

Giancarlo Sangalli (University of Pavia, IT)

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Joint work of P. Antolín, A. Buffa, A. Collin, G. Elber, M. Kapl, M. Martinelli, F. Massarwi, R. Vázquez, T. Takacs, M. Tani, et al.

This presentation is an overview of the main recent results on the numerical analysis and mathematical foundations of isogeometric analysis, introduced the seminal 2005 paper by Thomas J.R. Hughes, Austin Cottrell and Jury Bazilevs as a new approach to the discretization of partial differential equations (PDEs). Isogeometric analysis is a collection of methods that use splines, or some of their extensions such as NURBS (non-uniform rational B-splines), T-splines, LR-splines, hierarchical splines, etc. as functions to build approximation spaces which are then used to solve partial differential equations numerically.

The main challenge isogeometric analysis wants to address was to improve the interoperability between CAD and PDE solvers, and for this reason it uses CAD mathematical primitives, that is, splines and extensions, to represent PDE unknowns as well. Full interoperability is a challenging aim, for a number of reasons. For example, CAD modellers give a parametrization only of the boundary of geometrical objects as collections of manifolds in three-dimensional space, while the approximation spaces need to be defined on a three-dimensional volume that represents the computational domain. Moreover, these manifolds are often obtained by trimming. In this respect, isogeometric methods have taken the first fundamental steps towards a satisfactory solution and have undoubtedly created strong interest in these questions, promoting interaction between different scientific communities.

3.30 Interactive Deformation of Virtual Paper

Camille Schreck (IST Austria – Klosterneuburg, AT)

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Joint work of Camille Schreck, Damien Rohmer, Stefanie Hahmann, Marie-Paule Cani, Shuo Jin, Charlie Wang, Jean-François Bloch

Main reference C. Schreck, D. Rohmer, S. Hahmann, M.-P. Cani, S. Jin, C. Wang, J.-F. Bloch, “Nonsmooth Developable Geometry for Interactively Animating Paper Crumpling”, *ACM Trans. Graph.*, 35(1):10:1–10:18, ACM, 2015.

URL <http://dx.doi.org/10.1145/2829948>

Although paper is a very common material in our every-day life, it can hardly be found in 3D virtual environments. Indeed, due to its fibrous structure, paper material exhibits complex deformations and sound behaviour which are hard to reproduce efficiently using standard methods. Most notably, the deforming surface of a sheet of paper is constantly isometric to its 2D pattern, and may be crumpled or torn leading to sharp and fine geometrical features. We propose to combine usual physics-based simulation with new procedural, geometric methods in order to take advantage of prior knowledge to efficiently model a deforming sheet of paper. Our goals are to reproduce a plausible behavior of paper rather than an entirely physically accurate one in order to enable a user to interactively deform and create animation of virtual paper.

3.31 A Framework for Isogeometric Analysis on Unstructured Quadrilateral Meshes

Hendrik Speleers (University of Rome “Tor Vergata”, IT)

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Joint work of Deepesh Toshniwal, Hendrik Speleers, Thomas J. R. Hughes

Main reference D. Toshniwal, H. Speleers, and T. J. R. Hughes, “Smooth cubic spline spaces on unstructured quadrilateral meshes with particular emphasis on extraordinary points: Geometric design and isogeometric analysis considerations”, *Computer Methods in Applied Mechanics and Engineering*, 327:411–458, Elsevier, 2017.

URL <https://doi.org/10.1016/j.cma.2017.06.008>

CAD representations of arbitrary genus geometries with a finite number of tensor-product polynomial patches invariably lead to surface representations with unstructured quadrilateral meshes and, thus, extraordinary vertices. Construction of smooth splines over such unstructured quadrilateral meshes is of considerable interest within the field of isogeometric analysis, and a myriad of approaches have been explored that focus on the design and analysis of such geometries.

In this talk we present an alternative approach towards construction of smooth splines over unstructured quadrilateral meshes. Acknowledging the differing requirements posed by design (e.g., the convenience of an intuitive control net) and analysis (e.g., good approximation behavior), we propose the construction of a separate, smooth spline space for each while ensuring isogeometric compatibility. A key ingredient in the approach is the use of singular parameterizations at extraordinary vertices. We demonstrate the versatility of the approach with several applications in design and analysis. The constructed spline spaces show superior approximation behavior, and seem to be well behaved even at the singularities.

3.32 Using Machine Learning Techniques in Geometric Modeling

Georg Umlauf (HTWG Konstanz, DE)

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In this talk two applications of machine learning approaches to geometric modeling problems were presented: knot placement for b-spline approximation and shape classification of point clouds.

Selecting knot values to receive good approximation results is a challenging task. Proposed approaches range from parametric averaging to genetic algorithms. We propose the use of Support Vector Machines (SVMs) for finding suitable knot vectors in B-spline curve approximation. The SVMs are trained to distinguish between locations along the curve that are well or not well suited as knots in the parametric domain. This score is based on different geometric features of a parameters corresponding point in the point cloud. A score weighted averaging technique is used to produce the final knot vector. We further propose a method to use the score weighted averaging technique for t-spline surface approximation.

In the reverse engineering process one has to classify parts of point clouds with the correct type of geometric primitive. Features based on different geometric properties like point relations, normals, and curvature information can be used to train SVMs. These geometric features are estimated in the local neighborhood of a point of the point cloud. The multitude of different features makes an in-depth comparison necessary. Instead unsupervised learning

methods, e.g. auto-encoders, can be used to learn the relevant features simultaneously with the classification task. These automatic features can be visualized to interpret the learned classification criteria.

3.33 A Multi-sided Bézier Patch with a Simple Control Structure

Tamas Várady (Budapest University of Technology and Economics, HU)

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Joint work of Tamas Várady, Peter Salvi

Generalized Bézier (GB) patches represent a new multi-sided scheme; they are compatible with quadrilateral Bézier patches and inherit many of their properties. Boundaries and corresponding cross-derivatives (ribbons) are specified as standard Bézier surfaces of arbitrary degrees. The surface is defined over a convex polygonal domain; local coordinates are computed using generalized barycentric coordinates. The control structure is simple and intuitive, control points are associated with a combination of bi-parametric Bernstein functions, multiplied by rational terms.

The input ribbons may have different degrees, but when the final patch is built, the representation will have a uniform degree. Interior control points – other than those specified by the user – are placed automatically by a special degree elevation algorithm, or can be used for shape optimization or approximation of point clouds. GB patches connect to adjacent Bézier surfaces with G1 or G2 continuity. Several examples will be shown to illustrate the proposed scheme.

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3.34 Shape Optimization for Fine-scale Structure Design for 3D Printing

Denis Zorin (New York University, US)

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Additive fabrication technologies are limited by the types of material they can print: while the technologies are continuously improving, still only a relatively small discrete set of materials can be used in each printed object. At the same time, the low cost of introducing geometric complexity suggests the alternative of controlling the elastic material properties by producing microstructures, which can achieve behaviors significantly differing from the solid printing material.

The presentation focused on results related to design of printable microstructures, including identifying topologies allowing for maximal coverage of effective material properties, printability constraints, stress concentration minimization and worst-case load analysis.

4 Working groups

4.1 Additive Manufacturing Focus Group

Tor Dokken (SINTEF – Oslo, NO), Thomas A. Grandine (The Boeing Company – Seattle, US), and Georg Muntingh (SINTEF – Oslo, NO)

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In a discussion session on the interactions between additive manufacturing, computer-aided geometric design, and isogeometric analysis, we identified future challenges and main areas of interests.

The following participants were the most active in the discussion:

- Tor Dokken (SINTEF – Oslo, NO)
- Ron Goldman (Rice University – Houston, US)
- Thomas A. Grandine (The Boeing Company – Seattle, US)
- Jens Gravesen (Technical University of Denmark – Lyngby, DK)
- Bert Jüttler (Universität Linz, AT)
- Géraldine Morin (University of Toulouse, FR)
- Georg Muntingh (SINTEF – Oslo, NO)
- Malcolm A. Sabin (Numerical Geometry Ltd. – Cambridge, GB)
- Giancarlo Sangalli (University of Pavia, IT)
- Timothy L. Strotman (Siemens – Milford, US)

IGA for AM

What will it take, in practice, to incorporate IGA into optimal design of highly engineered products? Although the technology is very promising, there are (as of yet) very few inroads in actual industry practices. Trivariate IGA for AM is a promising direction.

Tools for design and visualisation in AM

In the past there was a focus on developing tools for design and visualisation of B-rep models. The solids now encountered in additive manufacturing require a much richer inner structure, to represent aspects such as densities, multimaterial properties, and microstructures. There is a similar need for tools for design and visualisation of objects with these properties, requiring new techniques and technologies.

Multiscale modelling for AM

Early research is being carried out on printing of structural members for very large objects, such as buildings and jet airplanes. There are major challenges in controlling the process well enough, in making the technology affordable at that scale, and in the design processes. In particular, the scale of the final printed part and the scale of the nanostructure from which it is constructed can span several orders of magnitude, raising the specter of hard mathematical challenges properly handling multiscale modelling. Moreover, multiscale design paradigms are still in their infancy, and much important research remains to be done to understand the most promising and effective technologies for dealing with this.

CAGD for AM

How to make the CAGD representation relevant for AM? In general the physics define the design and the geometry is the fall-out of the design, i.e., form follows function. Hence CAGD will have to focus on the paradigms of the individual AM processes. For instance, trimming is a natural construction in the context of subtractive manufacturing. Is a different paradigm needed for additive manufacturing? An emerging paradigm in AM is multiscale design based on microstructures of objects with a complex topology. The design of such microstructures representing the rich inner structure and properties (density, topology, elastic textures, hardness, etc.) of additively manufactured parts will be a challenge in CAGD.

Functional composition

The tiling of existing design into hexahedral microstructures requires hex-meshing. It is an old trick to generate such meshes by precomposing a spline map representing a trimmed surface by a map from the unit square to the trimmed parameter domain. The same idea has been used for generating porous geometry using tileable microstructures, keeping the number of control points relatively low. An issue is that functional composition causes the degree of the splines to go up, which can be a problem for the analysis. These compositions of spline mappings are interesting in many ways in applications, and the time is ripe to make this part of the CAGD discussion.

4.2 Isogeometric Analysis Focus Group

Tor Dokken (SINTEF – Oslo, NO), Thomas A. Grandine (The Boeing Company – Seattle, US), and Georg Muntingh (SINTEF – Oslo, NO)

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Joint work of The participants in the Dagstuhl seminar

Isogeometric analysis (IGA) is a highly promising technology, and yet few industrial inroads have been made. Part of the reason for this is that it represents a disruptive technology, i.e., it requires substantial changes to the industrial flow of digital information in at least two places, both CAD and CAE. Moreover, in the general case it requires thinking about geometric modelling from a volumetric point of view rather than traditional boundary-represented solid modelling. It may prove fruitful to explore boundary element methods and shell methods for structures as a means of getting started, since those types of analysis only require surface models. As with the introduction of other transformative technologies in industry, the path forward will be paved with substantial and successful pilot projects and programs that build customer demand for the technology.

Status of IGA as of 2017

Isogeometric analysis was introduced by Professor Tom Hughes in 2005, and it has not progressed as fast as expected at the time. The community is growing too slow: The first IGA conference in Austin, Texas (2011) attracted 100 participants, while the conference in Pavia, Italy (2017) attracted 187 persons. Success has been reported on IGA for shell formulations, for instance in the LS-Dyna integration. In addition, work on volumetric IGA has started, based on LR B-splines, T-splines and the FEAP-prototype. The Interdisciplinary

Geometric Activities in IGA have brought people from CAD and analysis together, such as in this Dagstuhl seminar.

What is blocking the progress of IGA?

- Early claims that IGA would solve interoperability between Analysis and CAD blocks progress, as interoperability issues still exists – although the use of B-splines in IGA helps.
- Today CAD-systems are based on boundary-representations (shells), and CAD must move to mathematical volumes to properly support volumetric IGA. There is a technology gap between 2-variate CAD and 3-variate analysis. The legacy of existing CAD-models is blocking a move as well.
- In CAD-systems the mathematics is hidden from the user. This makes it difficult to design for IGA.
- There are well-established CAD/FEM-solutions, conservative vendors and customers. Introduction of new technology in systems with many user seats can be painful. Current codes that work well cannot easily be adapted to IGA. The analysis vendors of IGA must be convinced to use IGA.
- For many IGA application areas well-established (industrial) FEM-based solutions exist. So far hundreds of person-years have been invested in IGA research. This competes with thousands of person-years invested in FEM research.
- Most of the hard problems in analysis are vector-valued, whereas IGA has focused on scalar-valued problems. Scalar-interpolants form a tiny niche.
- As of now no industrial funding has been triggered, which is for instance the case for machine learning. Industry wants, in general, fast results.
- IGA has been very good for science. Lots of researchers now understand higher order elements and how to adapt them to analysis frameworks. In practice, there are bottlenecks. For instance, the basis functions necessary for a simple geometry cannot be used to solve Maxwell's equations.
- Most of the work on, for instance, proving error estimates is done theoretically, and it is not used in practice.

How can the progress of IGA be improved?

Killer applications for IGA must be identified, where FEA is outperformed. The following research directions are promising.

- There are (topology) optimization challenges that can only be solved using a tight coupling of geometry and analysis, as provided by IGA.
- Modelling and simulation in medicine, for instance in prosthesis design. Dental implants and hearing aids have a direct connection to additive manufacturing. These require a shift to analysis based on novel, volumetric representations, and IGA can fill this need.
- There have been IGA work programmes where several geometers and analysts have been put together, solving certain equations not previously solved by FEM.

Note that although killer applications are necessary, they are not sufficient. It is not realistic to expect the industry to replace their systems anytime soon, as long as current systems are adequate. The following additional measures can be taken to expedite the industrial uptake of IGA.

- Improve the CAD end of IGA.

- Focus on areas where traditional CAD and FEM are not sufficient, such as additive manufacturing.
- Prepare for IGA in standards, such as STEP ISO 10303.
- Apply to areas beyond engineering. In computer science, for instance, the focus is on discrete geometries, but these are not sufficient for analysis.
- Increase the teaching of geometry, by getting IGA into the computer science, applied maths and engineering curricula, which have traditionally had a focus on discrete geometries.
- Expand the scope of IGA, to increase the total IGA research activity. This can be compared to machine learning, where much of the funding comes from industries. This would also ensure a focus on the practical side, to complement the disproportionate theoretical focus in IGA.
- Start from the models resulting from analysis, and improve these using CAD concepts.

4.3 Big Data Focus Group

Thomas A. Grandine (The Boeing Company – Seattle, US), Tor Dokken (SINTEF – Oslo, NO), and Georg Muntingh (SINTEF – Oslo, NO)

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In a discussion session on Big Data in geometric modelling, we identified future challenges and main areas of interests. This includes both the application of Big Data algorithms to modelling and the application of geometric algorithms to Big Data problems. In this report we briefly summarize this discussion.

The following participants were the most active in the discussion:

- Chandrajit Bajaj (University of Texas – Austin, US)
- Heidi Dahl (SINTEF – Oslo, NO)
- Thomas A. Grandine (The Boeing Company – Seattle, US)
- Panagiotis Kaklis (The University of Strathclyde – Glasgow, GB)
- Benjamin Karer (TU Kaiserslautern, DE)
- Georg Muntingh (SINTEF – Oslo, NO)
- Jörg Peters (University of Florida – Gainesville, US)

Design of microstructures

In additive manufacturing the vast freedom of design comes with a high complexity. To deal with this complexity, one strategy is to introduce an intermediate scale, for instance based on cells (e.g. voxels), each of which contains a microstructure. Optimization then becomes feasible, as it can proceed in terms of the parameters of the individual microstructures, to determine a heterogeneous object with optimal properties not attainable by the bulk material. In this sense properly designed microstructures can bridge the gap in complexity. Such multiscale design and manufacturing has the potential to create vast amounts of intermediate data, even though the initial and final design is of limit size.

The digital twin

Another example of big geometric data is the digital twin, which is a concept well known for instance in the aerospace and shipping industry. For each airplane/ship that leaves

the factory/ship yard, there is a full-blown virtual model of several terabytes in size. This digital twin is systematically updated with all modifications that happen during its lifecycle, which typically lasts at least 15 years. Such modifications include information on hazardous materials, repairs, etc. This information is often acquired by scanning the internal parts of ships.

Industrial Big Data

Another example of Big Data comes in the form of operational and sensor data, such as pressures and velocities. Although these are not of a geometrical nature per se, they typically have a spatial distribution that should be taken into account. Copious amounts of such data is produced and collected, but how should it be processed? Drawing an analogy to newspapers: will you keep a complete archive, or will you identify clippings? How can we grab and organize the relevant informational content? In certain cases the government demands that collected data remains unprocessed, and there is a sense that one cannot lose any data. More often the goal should be to extract relevant information from compressed data with high accuracy.

Factory models

As factories grow in size, so does their complexity. This complexity can be managed by dynamically building large geometric models of the factory. Such models can track everything that moves, including the parts that are to be assembled or produced. If a change to this flow of parts is proposed, this can be simulated first and checked for viability.

A challenge here is that buildings of large size are themselves subject to changes. For instance, rain can cause the ceiling to move by centimeters, moving and misaligning the sensors, for which millimeter precision is desired.

Earth science and data from space

Data collected by space missions, from the far universe to the solar system and observational science, is another area with a strong interest for manipulating Big Data. Space agencies are also very interested in tracking space debris.

The further our sensors extend, from underground to outer space, the more data is collected. Often the problems encountered have a lot in common, but sometimes the presence of unique features makes new techniques necessary.

Design recognition

How can one automatically recognize past designs for the purpose of future design? A large industrial group has implemented a number of such systems, and within a few years each has fallen into disuse. They keep reimplementing and discarding it. What is preventing the adoption? Is it fundamental? Is it part of the drive to create something new? One reason is that assembling to order works, unless you keep changing the ingredients. Such updates are not local, and everything that was previously in balance has to change. There is a need for machine learning that plays nice with all aspects involved, including geometric structures and compressed sensing.

5 Open problems

5.1 Technical position paper on trivariates

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A Brep does not fail to describe a solid. What it fails to do is to provide a boundary-aligned parametrisation of a non-trivial solid or of the cells of a Whitney stratification.

Both for additive manufacture and for IGA of solids we need a parametrisation to provide a ‘scaffolding’ for the description of internal structure.

Within a Brep modeller there is actually a clean structure. There are topological entities, linked by a BOUNDS relation, such as vertices, edges, faces, cells.

Each of the topological entities is given shape and position by embedding it in a geometrical entity which can be expressed by a LIESIN relation.

These two relations are pretty well all the information the topology needs. The geometry also needs representations of shape, and although a rich modeller may have alternatives, it is possible for NURBS to provide all the embeddings. Except that the solid cells don’t need any explicit embedding. The default is \mathbb{R}^3 , and we can see a cell as being bounded by faces as trimming just like the trimming of a face by edges.

There are problems with trimming, and for many purposes it might be more convenient to have a boundary-aligned parametrisation in both bivariate and trivariate cells. If you want to do this in a way which both

- (i) has the trivariate parametrisation compatible at the boundary with the bivariate parametrisations of the faces, and
 - (ii) avoids serious distortion at the corners of the parametrisation,
- we need to split the solid cells into the bricks for which there is a natural trivariate parametrisation. This is exactly the hex meshing problem which Sandia have been addressing for many years.

I observe that

- (a) there are extraordinary edges analogous to the extraordinary vertices of a subdivision surface (but I am not assuming subdivision geometry!)
- (b) it is not necessary to use valencies other than 3, 4 and 5.
- (c) if the valencies are so limited, the number of possible ways in which these extraordinary edges can meet is limited to about a dozen.

The actual geometry can be defined either

- (i) by using subdivision,
- (ii) by using the analogue of finite-filling quads which still maintain the necessary continuity, or
- (iii) by going to the dual where there are ordinary edges, but a dozen or so different ‘ n -faced cells’.

Of these only the first supports nested spaces of basis functions to meet the full requirements of field analysis. This may be a weak requirement.

Current CAD systems do not handle faces in this way, and so trying to get them to support it for solid cells would be inviting them to put a serious self-inconsistency into the theoretical model which underpins the code. Adding trimmed \mathbb{R}^3 would almost certainly be a much easier thing for them to implement.

Another issue is that the tensor product representations that we have are considerably better at representing spatial frequencies which are aligned with the iso-parametric directions than with those that are skew. Using the same basis for both the position field and the under-load displacement field (in the case of elasticity analysis) is making the assumption that the dominant spatial frequencies of the two are well-aligned. Trying to use the same basis for the optimisation displacement fields is, of course, a non-starter because of sheer volume of data.

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