

Report from Dagstuhl Seminar 17361

Finite and Algorithmic Model Theory

Edited by

Anuj Dawar¹, Erich Grädel², Phokion G. Kolaitis³, and
Thomas Schwentick⁴

1 University of Cambridge, GB, anuj.dawar@cl.cam.ac.uk

2 RWTH Aachen, DE, graedel@informatik.rwth-aachen.de

3 University of California – Santa Cruz, US, kolaitis@cs.ucsc.edu

4 TU Dortmund, DE, thomas.schwentick@tu-dortmund.de

Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 17361 “Finite and Algorithmic Model Theory”.

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1 Executive Summary

Anuj Dawar

Erich Grädel

Phokion G. Kolaitis

Thomas Schwentick

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Topic and Goals of the Seminar

Finite and Algorithmic Model Theory (FAMT) encompasses a number of research themes united around common methods for analysing the expressive power of logical formalisms on structures that are either finite or can be finitely represented. These are precisely the structures that can serve as inputs to computation and, for this reason, FAMT is intimately connected to computer science. Over the past decades the subject has developed through a close interaction between theoretical computer science and closely related areas of mathematics, including logic and combinatorics, and a strong research community has been forged, with a common research agenda which has influenced several important areas of computer science. The last Dagstuhl-like meeting of this research community before this seminar had been Aux Houches in 2012.

The principal goals of the seminar have been the following:

1. To identify fresh research challenges in the area of finite and algorithmic model theory, arising from the main application areas and to make new connections between core research in FAMT and emerging application areas, such as logic and learning.



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2. To transfer knowledge from emerging methods and techniques in core FAMT to application areas.
3. To strengthen the research community in FAMT, especially by integrating younger members into it.

Organisation and Activities

The organisers developed a schedule consisting of three invited one-hour survey talks, more focussed regular contributions proposed by the participants, an open problem session, and a final discussion about the state and perspectives on the future of FAMT. The three survey talks were given by

- Wied Pakusa (Oxford) on recent achievements concerning the quest for a logic for polynomial time, focussing on Rank Logic and on Choiceless Polynomial Time,
- Dan Suci (Washington) on highlights of the connections between FAMT and databases.
- Martin Grohe (Aachen) on new developments in machine learning and connections to FAMT.

In addition, 22 other participants gave regular talks on their recent work on topics of FAMT. A further social highlight was a superb concert on Thursday evening performed by Jan Van den Bussche (violin), Wolfgang Thomas (violin) and Jouko Väänänen (piano).

Outcomes

The seminar exceeded our expectations in achieving our principal goals. The invited talk by Pakusa gave an overview on the status of the ongoing pursuit for a logic for PTIME, and demonstrated the depth and technical sophistication that FAMT has reached. This talk was complemented by several insightful presentations on new and deep work on core topics of finite model theory. A particular highlight was the double presentation by Torunczyk and Siebertz, who brought in new methods from stability theory to the study of the finite model theory of sparse structures.

The invited talks by Grohe and Suci explored new connections between the core methods of finite model theory and emerging areas of applications. These talks were also complemented by several other talks on new directions in FAMT and on interactions of FAMT with other areas. In particular, Grädel's talk focussed on new work in the area of database provenance, while Atserias' talk discussed a priori unexpected connections between constraint satisfaction and quantum information theory.

Overall, the presentations at the seminar were highly stimulating, and we know through discussions during and after the seminar that the new work presented has motivated others to take up the explorations of these questions. Based on the feedback received, we believe that this seminar will serve as a catalyst for new research directions in FAMT.

The organizers regard the seminar as very successful. As reflected in the final discussion, there was a consistent sentiment expressed by the participants that the FAMT community is in very healthy state. There are interesting new developments and exciting results in different directions, there is a strengthening of traditional connections to areas, such as databases and verification, but also new connections are emerging with such areas as knowledge representation, learning theory, logics for dependence and independence, and quantum information theory. Finally, and perhaps more importantly, there is an infusion of

several outstanding young researchers who have the interest and hold the promise to advance FAMT in the years to come.

The participants clearly expressed the wish to have a next meeting of the FAMT community, be it in Dagstuhl or elsewhere, within the next two to three years.

The organizers are grateful to the Scientific Directorate of the Center for its support of this workshop and the staff of Schloss Dagstuhl for the perfect organisation of our stay and their hospitality.

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3 Overview of Talks

3.1 Advice Automatic Structures and Uniformly Automatic Classes

Fariad Abu Zaid (TU Ilmenau, DE)

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Joint work of Fariad Abu Zaid, Erich Grädel, Frederic Reinhardt
Main reference Fariad Abu Zaid, Erich Grädel, Frederic Reinhardt: “Advice Automatic Structures and Uniformly Automatic Classes”, in Proc. of the 26th EACSL Annual Conference on Computer Science Logic, CSL 2017, August 20-24, 2017, Stockholm, Sweden, LIPIcs, Vol. 82, pp. 35:1–35:20, Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2017.
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We study structures that are automatic with advice. These are structures that admit a presentation by finite automata (over finite or infinite words or trees) with access to an additional input, called an advice. Over finite words, a standard example of a structure that is automatic with advice, but not automatic in the classical sense, is the additive group of rational numbers.

By using a set of advices rather than a single advice, this leads to the new concept of a parameterised automatic presentation as a means to uniformly represent a whole class of structures. The decidability of the first-order theory of such a uniformly automatic class reduces to the decidability of the monadic second-order theory of the set of advices that are used in the presentation. Such decidability results also hold for extensions of first-order logic by regularity preserving quantifiers, such as cardinality quantifiers and Ramsey quantifiers.

To investigate the power of this concept, we present examples of structures and classes of structures that are automatic with advice but not without advice, and we prove classification theorems for the structures with an advice automatic presentation for several algebraic domains. In particular, we prove that the class of all torsion-free Abelian groups of rank one is uniformly omega-automatic and that there are uniform omega-tree-automatic presentation of the class of all Abelian groups up to elementary equivalence and the class of all countable divisible Abelian groups. On the other hand we show that every uniformly omega-automatic class of Abelian groups must have bounded rank.

While for certain domains, such as trees and Abelian groups, it turns out that automatic presentations with advice are capable of presenting significantly more complex structures than ordinary automatic presentations, there are other domains, such as Boolean algebras, where this is provably not the case. Further, advice seems not be of much help for representing some particularly relevant examples of structures with decidable theories, most notably the field of reals.

3.2 Generalized Satisfiability Problems via Operator Assignments

Albert Atserias (UPC – Barcelona, ES)

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Joint work of Albert Atserias, Phokion G. Kolaitis, Simone Severini

Main reference Albert Atserias, Phokion G. Kolaitis, Simone Severini: “Generalized Satisfiability Problems via Operator Assignments”, in Proc. of the Fundamentals of Computation Theory - 21st International Symposium, FCT 2017, Bordeaux, France, September 11-13, 2017, Proceedings, Lecture Notes in Computer Science, Vol. 10472, pp. 56–68, Springer, 2017.

URL https://doi.org/10.1007/978-3-662-55751-8_6

Schaefer introduced a framework for generalized satisfiability problems on the Boolean domain and characterized the computational complexity of such problems. We investigate an algebraization of Schaefer’s framework in which the Fourier transform is used to represent constraints by multilinear polynomials in a unique way. The polynomial representation of constraints gives rise to a relaxation of the notion of satisfiability in which the values to variables are linear operators on some Hilbert space. For the case of constraints given by a system of linear equations over the two-element field, this relaxation has received considerable attention in the foundations of quantum mechanics, where such constructions as the Mermin-Peres magic square show that there are systems that have no solutions in the Boolean domain, but have solutions via operator assignments on some finite-dimensional Hilbert space. We obtain a complete characterization of the classes of Boolean relations for which there is a gap between satisfiability in the Boolean domain and the relaxation of satisfiability via operator assignments. To establish our main result, we adapt the notion of primitive-positive definability (pp-definability) to our setting, a notion that has been used extensively in the study of constraint satisfaction problems. Here, we show that pp-definability gives rise to gadget reductions that preserve satisfiability gaps. We also present several additional applications of this method. In particular and perhaps surprisingly, we show that the relaxed notion of pp-definability in which the quantified variables are allowed to range over operator assignments gives no additional expressive power in defining Boolean relations.

3.3 On computability and tractability for infinite sets

Mikolaj Bojanczyk (University of Warsaw, PL)

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Joint work of Mikolaj Bojanczyk, Szymon Toruńczyk

We propose a definition for computable functions on definable sets. Definable sets are possibly infinite data structures that can be defined using a fixed underlying relational structure. We show that, under suitable assumptions on the underlying structure, a programming language called definable while programs captures exactly the computable functions. Next, we introduce a complexity class called fixed dimension polynomial time, which intuitively speaking describes polynomial computation on definable sets. We show that this complexity class contains all functions computed by definable while programs with suitably defined resource bounds. Proving the converse inclusion would prove that Choiceless Polynomial Time with Counting captures order invariant polynomial time on finite graphs.

3.4 Conservative Extensions in Fragments of First-Order Logic

Carsten Lutz (Universität Bremen, DE)

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Joint work of Jean Christoph Jung, Mauricio Martel, Thomas Schneider, Frank Wolter

Main reference Jean Christoph Jung, Carsten Lutz, Mauricio Martel, Thomas Schneider, Frank Wolter: “Conservative Extensions in Guarded and Two-Variable Fragments”, in Proc. of the 44th International Colloquium on Automata, Languages, and Programming, ICALP 2017, July 10-14, 2017, Warsaw, Poland, LIPIcs, Vol. 80, pp. 108:1–108:14, Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2017.

URL <http://dx.doi.org/10.4230/LIPIcs.ICALP.2017.108>

In the area of description logic (DL), deciding whether a logical theory is a conservative extension of another theory is a fundamental reasoning task with applications in ontology modularity and reuse, ontology versioning, and ontology summarization. It is known that conservative extensions are decidable in many DLs and that they can often be characterized elegantly in term of model theoretic notions. In this talk, we consider the decidability of conservative extensions in more expressive decidable fragments of first-order logic such as the two-variable fragment and the guarded fragment. We show undecidability for these two fragments and decidability for the two-variable guarded fragment. The latter rests on a model-theoretic characterization that is considerably more complex than for many standard DLs.

3.5 Expressive Power of Entity-Linking Frameworks

Ronald Fagin (IBM Almaden Center – San Jose, US)

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Joint work of Douglas Burdick, Lucian Popa, Phokion G. Kolaitis, Ronald Fagin, Wang-Chiew Tan

Main reference Douglas Burdick, Ronald Fagin, Phokion G. Kolaitis, Lucian Popa, Wang Chiew Tan: “Expressive Power of Entity-Linking Frameworks”, in Proc. of the 20th International Conference on Database Theory, ICDT 2017, March 21-24, 2017, Venice, Italy, LIPIcs, Vol. 68, pp. 10:1–10:18, Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2017.

URL <http://dx.doi.org/10.4230/LIPIcs.ICDT.2017.10>

We develop a unifying approach to declarative entity linking by introducing the notion of an entity linking framework and an accompanying notion of the certain links in such a framework. In an entity linking framework, logic-based constraints are used to express properties of the desired link relations in terms of source relations and, possibly, in terms of other link relations. The definition of the certain links in such a framework makes use of weighted repairs and consistent answers in inconsistent databases. We demonstrate the modeling capabilities of this approach by showing that numerous concrete entity linking scenarios can be cast as such entity linking frameworks for suitable choices of constraints and weights. By using the certain links as a measure of expressive power, we investigate the relative expressive power of several entity linking frameworks and obtain sharp comparisons. In particular, we show that we gain expressive power if we allow constraints that capture non-recursive collective entity resolution, where link relations may depend on other link relations (and not just on source relations). Moreover, we show that an increase in expressive power also takes place when we allow constraints that incorporate preferences as an additional mechanism for expressing “goodness” of links.

3.6 Machine Learning and Algorithmic Model Theory

Martin Grohe (RWTH Aachen, DE)

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Joint work of Martin Grohe, Christof Löding, Martin Ritzert

After giving some general background in machine learning, I introduced a declarative model theoretic framework for learning. Then I talked about recent positive and negative learnability results that we obtained within this framework for learning models defined in first-order and monadic second-order logic.

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- 2 M. Grohe and M. Ritzert. Learning first-order definable concepts over structures of small degree. In *Proceedings of the 32nd ACM-IEEE Symposium on Logic in Computer Science*, 2017.

3.7 Provenance Analysis for Logic and Games

Erich Grädel (RWTH Aachen, DE)

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Provenance analysis for database transformations is used to track the provenance or dependence of computed facts from different input items. For positive query languages, such as (unions of) conjunctive queries or datalog, it has been shown that provenance analysis can be done via interpretations in commutative semirings, to answer questions about the trust in, the cost of, or the required clearance level for derived facts, or the number of derivation trees that are available for establishing such a fact.

We generalize this analysis to logics that include negation, such as full FO and LFP, exploiting connections between logic and games. Beyond the already familiar applications to query evaluation (and hence logic), provenance analysis also has interesting interpretations in finite and infinite games to answer more subtle questions than just who wins the game, such the number or costs of winning strategies, or issues such as confidence and trust in game-theoretic settings. The mathematical basis of this approach are interpretations of logics and games in ω -commutative or absorptive semirings of polynomials or power series.

This is joint work with Val Tannen.

3.8 Dependence logic vs. constraint satisfaction

Lauri Hella (*University of Tampere, FI*)

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Joint work of Lauri Hella, Phokion G. Kolaitis

Main reference Lauri Hella, Phokion G. Kolaitis: “Dependence Logic vs. Constraint Satisfaction”, in Proc. of the 25th EACSL Annual Conference on Computer Science Logic, CSL 2016, August 29 - September 1, 2016, Marseille, France, LIPIcs, Vol. 62, pp. 14:1–14:17, Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2016.

URL <http://dx.doi.org/10.4230/LIPIcs.CSL.2016.14>

During the past decade, dependence logic has emerged as a formalism suitable for expressing and analyzing notions of dependence and independence that arise in different scientific areas. The sentences of dependence logic have the same expressive power as those of existential second-order logic, hence dependence logic captures NP on the class of all finite structures. We identify a natural fragment of universal dependence logic and show that, in a precise sense, it captures constraint satisfaction. This tight connection between dependence logic and constraint satisfaction contributes to the descriptive complexity of constraint satisfaction and elucidates the expressive power of universal dependence logic.

3.9 Combinatorial Properties of the Weisfeiler-Leman Algorithm

Sandra Kiefer (*RWTH Aachen, DE*)

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Joint work of Brendan McKay, Iliia Ponomarenko, Pascal Schweitzer, Erkal Selman

Main reference Sandra Kiefer, Iliia Ponomarenko, Pascal Schweitzer: “The Weisfeiler-Leman dimension of planar graphs is at most 3”, in Proc. of the 32nd Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2017, Reykjavik, Iceland, June 20-23, 2017, pp. 1–12, IEEE Computer Society, 2017.

URL <http://dx.doi.org/10.1109/LICS.2017.8005107>

The Weisfeiler-Leman algorithm is a combinatorial procedure that plays a crucial role both in theoretical and practical research on the graph isomorphism problem. For every k there is a k -dimensional version of the algorithm, which repeatedly refines a partition of the set of k -tuples of vertices of the input graph. The final partition can often be used to distinguish non-isomorphic graphs and for every pair of graphs, there is some dimension of the algorithm that decides isomorphism of the two graphs. We have established upper bounds on this dimension for some graph classes.

By the famous correspondence by Cai, Fürer and Immerman, a graph is identified by the k -dimensional Weisfeiler-Leman algorithm if and only if it is definable in $C^{(k+1)}$, first order logic with counting and restricted to $k + 1$ variables. Thus, our dimension bounds also yield bounds on the logical complexity of the graph classes.

In order to gain a better understanding of its dynamics and precise complexity, we have also studied the number of iterations of the algorithm until stabilization, which corresponds to the quantifier depth of the corresponding counting logic.

In this talk, I gave an introduction to the mechanisms of the algorithm, presented some of our results in this field (see also [1], [2], [3]) and related them to open questions for future work.

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3.10 Green's Relations in Finite Automata

Manfred Kufleitner (Universität Stuttgart, DE)

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Joint work of Lukas Fleischer, Manfred Kufleitner

Main reference Lukas Fleischer, Manfred Kufleitner: “Green’s Relations in Finite Transformation Semigroups”, in Proc. of the Computer Science - Theory and Applications - 12th International Computer Science Symposium in Russia, CSR 2017, Kazan, Russia, June 8-12, 2017, Proceedings, Lecture Notes in Computer Science, Vol. 10304, pp. 112–125, Springer, 2017.

URL https://doi.org/10.1007/978-3-319-58747-9_12

Green’s relations are a fundamental tool in the structure theory of semigroups. They can be defined by reachability in the (right/left/two-sided) Cayley graph. The equivalence classes of Green’s relations then correspond to the strongly connected components. We study the complexity of Green’s relations in semigroups generated by transformations on a finite set. Our first result shows that, in the worst case, the number of equivalence classes is in the same order of magnitude as the number of elements. Another important parameter is the maximal length of a chain of strongly connected components. Our second result (the main contribution) is an exponential lower bound for this parameter. There is a simple construction for an arbitrary set of generators. However, the proof for a constant size alphabet is rather involved. We also investigate the special cases of unary and binary alphabets. All these results are extended to deterministic finite automata and their syntactic semigroups. The technical report is available on arXiv [1].

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3.11 Locality of counting logics

Dietrich Kuske (TU Ilmenau, DE)

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Joint work of Dietrich Kuske, Nicole Schweikardt

Main reference Dietrich Kuske, Nicole Schweikardt: “First-Order Logic with Counting: At Least, Weak Hanf Normal Forms Always Exist and Can Be Computed!”, CoRR, Vol. abs/1703.01122, 2017.

URL <http://arxiv.org/abs/1703.01122>

We introduce the logic FOCN(\mathbb{P}) which extends first-order logic by counting and by numerical predicates from a set \mathbb{P} , and which can be viewed as a natural generalisation of various counting logics that have been studied in the literature.

We obtain a locality result showing that every FOCN(\mathbb{P})-formula can be transformed into a formula in Hanf normal form that is equivalent on all finite structures of degree at most d . A formula is in Hanf normal form if it is a Boolean combination of formulas describing the neighbourhood around its tuple of free variables and arithmetic sentences

with predicates from \mathbb{P} over atomic statements describing the number of realisations of a type with a single centre. The transformation into Hanf normal form can be achieved in time elementary in d and the size of the input formula. From this locality result, we infer the following applications:

1. The Hanf-locality rank of first-order formulas of bounded quantifier alternation depth only grows polynomially with the formula size.
2. The model checking problem for the fragment $\text{FOC}(\mathbb{P})$ of $\text{FOCN}(\mathbb{P})$ on structures of bounded degree is fixed-parameter tractable (with elementary parameter dependence).
3. The query evaluation problem for fixed queries from $\text{FOC}(\mathbb{P})$ over fully dynamic databases of degree at most d can be solved efficiently: there is a dynamic algorithm that can enumerate the tuples in the query result with constant delay, and that allows to compute the size of the query result and to test if a given tuple belongs to the query result within constant time after every database update.

3.12 Static Analysis of Agent-Based Models

James F. Lynch (Clarkson University – Potsdam, US)

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Agent-based models are a class of dynamical systems whose states are populations of discrete elements called agents. The states evolve as the agents interact according to rules which may be probabilistic. Computer scientists have developed languages and software tools for specifying and analysing agent-based models, in particular agent-based models of biochemical reaction networks. One problem they have investigated is how to determine if an agent-based model can be abstracted to a simpler model that preserves relevant aspects of its behavior.

We formalize agent-based models as stochastic processes whose states are metafinite models, and we define a notion of abstraction. Our main results are conditions that imply an abstraction is sound, and further conditions that imply it preserves the Markov property.

This work extends earlier work of the author,

- A Logical Characterization of Individual-Based Models, Proc. 23rd Ann. IEEE Symp. on Logic in Computer Science, 379-390, 2008.

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3.13 Common Knowledge? Cayley!

Martin Otto

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Joint work of Felix Canavoi, Martin Otto

Main reference Felix Canavoi, Martin Otto: “Common knowledge and multi-scale locality analysis in Cayley structures”, in Proc. of the 32nd Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2017, Reykjavik, Iceland, June 20-23, 2017, pp. 1–12, IEEE Computer Society, 2017.

URL <http://dx.doi.org/10.1109/LICS.2017.8005072>

The usual epistemic setting of S5 structures (multi-modal Kripke structures with equivalence relations) forms an elementary class. Up to bisimulation it is first-order interpretable in a dual class of plain vertex-coloured graphs. Bisimilar companions with acyclicity properties that make them suitable for a locality-based Ehrenfeucht-Fraïssé analysis can be obtained in products with Cayley groups of large girth [2]. The extension of this scenario that deals with common knowledge (with accessibility relations for coalitions of agents based on reachability/transitive closures) is inherently non-elementary and seems averse to locality based techniques. Up to bisimulation, however, this common knowledge setting admits a natural and direct algebraic interpretation in Cayley groups. It turns out that (finite) Cayley groups with non-trivial acyclicity properties offer bisimilar companions that lend themselves to a locality based Ehrenfeucht-Fraïssé analysis despite the long-range, multi-scale reachability relations of common knowledge. Characterisation theorems of common knowledge logic based on this approach were presented in [1]; in this talk I would focus on the underlying concepts and methods.

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3.14 Proof Complexity of Constraint Satisfaction Problems

Joanna Ochremiak (University Paris-Diderot, FR)

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Joint work of Albert Atserias, Joanna Ochremiak

Main reference Albert Atserias, Joanna Ochremiak: “Proof Complexity Meets Algebra”, in Proc. of the 44th International Colloquium on Automata, Languages, and Programming, ICALP 2017, July 10-14, 2017, Warsaw, Poland, LIPIcs, Vol. 80, pp. 110:1–110:14, Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2017.

URL <http://dx.doi.org/10.4230/LIPIcs.ICALP.2017.110>

I showed that the most studied propositional and semi-algebraic proof systems behave well with respect to the standard CSP reductions. As an application I presented two unconditional gap theorems, which say that CSPs that admit small size refutations in some classical proof systems are exactly the CSPs which can be solved by local consistency methods. Finally, I gave examples of proof systems with good behaviour with respect to reductions and simultaneously small size refutations beyond bounded width.

3.15 Rank Logic and Choiceless Polynomial Time

Wied Pakusa (University of Oxford, GB)

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The search for a logic which captures polynomial time remains one of the most important challenges in the area of finite model theory. The task is to find a logical system which can express precisely those properties of finite structures, say of finite graphs, that can be decided by polynomial-time algorithms. In my talk I survey recent results about the expressive power of Rank Logic and Choiceless Polynomial Time. To date, these two logics are considered to be the most promising candidates for capturing polynomial time.

3.16 VC-density of nowhere dense graph classes

Sebastian Siebertz (University of Warsaw, PL)

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Joint work of Michał Pilipczuk, Sebastian Siebertz, Szymon Toruńczyk

Main reference Michał Pilipczuk, Sebastian Siebertz, Szymon Toruńczyk: “On Wideness and Stability”, CoRR, Vol. abs/1705.09336, 2017.

URL <http://arxiv.org/abs/1705.09336>

The notion of *Vapnik-Chervonenkis dimension*, short *VC-dimension*, was introduced by Vapnik and Chervonenkis [2] and independently by Shelah [9] (a formula has the *independence property* if and only if it has infinite VC-dimension) and has found important applications in statistical learning theory, logic, discrete and computational geometry and many other areas.

Formally, VC-dimension is defined as follows. Let A be a set and let $\mathcal{F} \subseteq \mathcal{P}ow(A)$ be a family of subsets of A . For a set $X \subseteq A$ let $X \cap \mathcal{F} := \{X \cap F : F \in \mathcal{F}\}$. The set X is *shattered* by \mathcal{F} if $X \cap \mathcal{F} = \mathcal{P}ow(X)$. The VC-dimension of \mathcal{F} is the maximum size of a set X that is shattered by \mathcal{F} . One of the main uses of VC-dimension is the Sauer-Shelah-Lemma [2, 8, 10], which states that the cardinality of a set family \mathcal{F} on a ground set A with VC-dimension d satisfies $|\mathcal{F}| \leq \sum_{i=0}^d \binom{|A|}{i} \in \mathcal{O}(|A|^d)$. This motivates the definition of *VC-density*, which describes the asymptotic growth of finite set families (as the size of the ground set A goes to infinity).

VC-density in model theory studies the asymptotic growth of arbitrary finite definable families. More precisely, let $\psi(\bar{x}, \bar{y})$ be a first-order formula, where \bar{x} is an m -tuple and \bar{y} is an n -tuple of variables. Let \mathfrak{A} be a structure and let A be a set of elements of \mathfrak{A} . Then the set of ψ -types or ψ -traces over A in \mathfrak{A} is the set

$$S_\psi(\mathfrak{A}, A) = \{\{\bar{a} \in A^m : \mathfrak{A} \models \psi(\bar{a}, \bar{b})\} : \bar{b} \in V(\mathfrak{A})^n\}.$$

In my talk I will present the following theorem (which will appear in [7]) concerning the number of types in sparse graph classes, which characterizes exactly the monotone graph classes of minimal VC-density.

► **Theorem 1.** *Let \mathcal{C} be a class of graphs and let $\psi(\bar{x}, \bar{y})$ be a first-order formula, where \bar{x} is an m -tuple and \bar{y} is an n -tuple of variables.*

1. *If \mathcal{C} is nowhere dense, then for every $\epsilon > 0$ there exists a constant c such that for every $G \in \mathcal{C}$ and every $A \subseteq V(G)$, we have $|S_\psi(A, G)| \leq c \cdot |A|^{n+\epsilon}$.*

2. If \mathcal{C} has bounded expansion, then there exists a constant c such that for every $G \in \mathcal{C}$ and every $A \subseteq V(G)$, we have $|S_\psi(A, G)| \leq c \cdot |A|^n$.

The main tools applied to prove the theorem are Gaifman's Locality Theorem [5], a closure lemma developed in [3, 4] and new bounds characterizing nowhere dense graph classes as uniform quasi-wide [6, 7]. Our result generalizes the recent result of Bobkov [1] that nowhere dense classes are dp-minimal (a notion defined by Shelah in [11]).

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3.17 Finite and Algorithmic Model Theory

Dan Suciu (University of Washington – Seattle, US)

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Joint work of Mahmoud Abo Khamis, Hung Q. Ngo, Dan Suciu
Main reference Mahmoud Abo Khamis, Hung Q. Ngo, Dan Suciu: “What Do Shannon-type Inequalities, Submodular Width, and Disjunctive Datalog Have to Do with One Another?”, in Proc. of the 36th ACM SIGMOD-SIGACT-SIGAI Symposium on Principles of Database Systems, PODS 2017, Chicago, IL, USA, May 14-19, 2017, pp. 429–444, ACM, 2017.
URL <http://dx.doi.org/10.1145/3034786.3056105>

We survey the following topics connecting finite model theory and algorithms: tree decomposition of conjunctive queries, upper bounds for full conjunctive queries, worst case optimal algorithms for conjunctive queries, and the submodular width and associated algorithms for Boolean conjunctive queries.

3.18 The Fluted Fragment Revisited

Lidia Tendera (University of Opole, PL)

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Joint work of I. Pratt-Hartmann, W. Szostak, L. Tendera
Main reference Ian Pratt-Hartmann, Wiesław Szostak, Lidia Tendera: “Quine’s Fluted Fragment is Non-Elementary”, in Proc. of the 25th EACSL Annual Conference on Computer Science Logic, CSL 2016, August 29 - September 1, 2016, Marseille, France, LIPIcs, Vol. 62, pp. 39:1–39:21, Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2016.
URL <https://doi.org/10.4230/LIPIcs.CSL.2016.39>

We revisit the fluted fragment (FL) of first-order logic, originally identified by Quine, in which the order of quantification of variables matches their order of appearance as arguments to predicates. Fluted formulas arise naturally as first-order translations of quantified English sentences in which no quantifier-rescoping occurs. Also Boolean modal logic maps, under the standard first-order translations, to FL. However, even the two-variable restriction of FL is not contained in other decidable fragments of first-order logic identified by considering the standard translation of modal logic (e.g. guarded fragment, unary negation fragment or guarded negation fragment).

In [P-HST 16] we have shown that the satisfiability problem for this fragment has non-elementary complexity; more precisely, we consider, for all m greater than 1, the intersection of the fluted fragment and the m -variable fragment of first-order logic. We showed that this subfragment forces $(m/2)$ -tuply exponentially large models, and that its satisfiability problem is $(m/2)$ -NExpTime-hard. We rounded off showing that the m -variable fluted fragment has the m -tuply exponential model property, and that its satisfiability problem is in m -NExpTime.

We are interested in seeing if there are other useful applications of FL or of its restrictions to some bounded number of variables (greater than two).

3.19 On Monadic Transitive Closure Logic MTC

Wolfgang Thomas (RWTH Aachen, DE)

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Monadic transitive closure Logic MTC is the extension of first-order logic by the construct that allows to pass from a formula $F(x, y)$ to $F^*(x, y)$, expressing that a path from x to y exists where each step is accordance with F . Parameters are also admitted; then one proceeds from $F(x, y, z_1, \dots, z_n)$ to $F^*(x, y, z_1, \dots, z_n)$. MTC is a very natural logic for expressing reachability properties. We survey results on the expressive power of MTC over the domains of finite words, finite ranked trees, and (labelled or unlabelled) finite two-dimensional grids. Over words, precisely the regular languages are definable in MTC (and it is open whether in general a nesting of the MTC-operator is necessary). Over trees, MTC is located strictly between tree-walking automata and standard tree automata (as shown in results of Bojanczyk, Colcombet, Segoufin, ten Cate, and others). Over grids the power of MTC is not yet clear; we are lacking a good method to establish non-definability results for MTC. Such a method is needed – for example – to show that there are sets of unlabelled grids that are definable in existential monadic second-order logic EMSO but not in MTC; an example suggested by N. Schweikardt is the set of unlabelled grids of dimensions $n \times 2^n$. The definability in EMSO is clear by expressing a 0-1-labelling of the grid columns that represents a binary counter from 0 to $(2^n) - 1$.

3.20 Sparsity and Stability

Szymon Torunczyk (University of Warsaw, PL)

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Joint work of Sebastian Siebertz, Michał Pilipczuk

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URL <http://arxiv.org/abs/1705.09336>

I will talk about some recent developments in the study of the connections between nowhere-denseness (introduced by Nesetril and Ossona de Mendez), uniform quasi-wideness (introduced by Dawar) and stability (developed by Shelah).

The slides are available at <https://www.mimuw.edu.pl/~szymtor/talks/dag17/stability.html>.

3.21 Descriptive complexity of arithmetic circuit classes

Heribert Vollmer (Leibniz Universität Hannover, DE)

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Joint work of Arnaud Durand, Anselm Haak, Juha Kontinen, Heribert Vollmer

We study the class $\#AC^0$ of functions computed by constant-depth polynomial-size arithmetic circuits of unbounded fan-in addition and multiplication gates. Inspired by Immerman’s characterization of the Boolean class AC^0 , we develop a model-theoretic characterization of

$\#AC^0$, which can be interpreted as follows: Functions in $\#AC^0$ are exactly those functions counting winning strategies in first-order model checking games.

Extending this, we introduce a new framework for a descriptive complexity approach to arithmetic computations. We define a hierarchy of classes based on the idea of counting assignments to free function variables in first-order formulas. We completely determine the inclusion structure and show that $\#P$ and $\#AC^0$ appear as classes of this hierarchy. In this way, we unconditionally place $\#AC^0$ properly in a strict hierarchy of arithmetic classes within $\#P$. We determine which classes in the hierarchy are feasible.

3.22 Dynamic Complexity: What we can do

Nils Vortmeier (TU Dortmund, DE)

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Joint work of Samir Datta, Raghav Kulkarni, Anish Mukherjee, Thomas Schwentick, Nils Vortmeier, Thomas Zeume

Main reference Samir Datta, Anish Mukherjee, Thomas Schwentick, Nils Vortmeier, Thomas Zeume: “A Strategy for Dynamic Programs: Start over and Muddle Through”, in Proc. of the 44th International Colloquium on Automata, Languages, and Programming, ICALP 2017, July 10-14, 2017, Warsaw, Poland, LIPIcs, Vol. 80, pp. 98:1–98:14, Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2017.

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URL http://dx.doi.org/10.1007/978-3-662-47666-6_13

DynFO, as defined by Patnaik and Immerman, is the class of queries that can be dynamically maintained using first-order logic to update the query result and possibly further auxiliary relations, whenever the input changes. Recently, several interesting maintainability results were obtained. For example, it was shown that the following queries are in DynFO:

- Reachability in arbitrary directed graphs
- Undirected reachability under first-order defined insertions
- MSO-definable queries on graphs with bounded treewidth

In this talk, I will give an overview of these developments. The talk is complemented by the talk by Thomas Zeume on lower bounds in dynamic complexity.

3.23 Hanf Locality and Invariant Elementary Definability

Scott Weinstein (University of Pennsylvania, US)

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Joint work of Steven Lindell, Henry Towsner, Scott Weinstein

We introduce some notions of invariant elementary definability which extend the notions of first-order order-invariant definability, and, more generally, definability invariant with respect to arbitrary numerical relations. In particular, we study invariance with respect to

expansions which depend not only on (an ordering of) the universe of a structure, but also on the particular relations which determine the structure; we call such expansions presentations of a structure. We establish two locality results in this context. The first is an extension of the original Hanf Locality Theorem to boolean queries which are invariantly definable over classes of locally finite structures with respect to elementary, neighborhood-bounded presentations. The second is a non-uniform version of the Fagin-Stockmeyer-Vardi Hanf Threshold Locality Theorem to boolean queries which are invariantly definable over classes of bounded degree structures with respect to elementary, neighborhood-bounded, local presentations.

3.24 Symmetric Circuits for Rank Logic

Gregory Wilsenach (University of Cambridge, GB)

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Joint work of Anuj Dawar, Gregory Wilsenach

Anderson and Dawar (2014) showed that fixed-point logic with counting (FPC) can be characterised by uniform families of polynomial-size symmetric circuits. We give a similar characterisation for fixed-point logic with rank (FPR) by means of symmetric circuits including rank gates. This analysis requires a significant extension of previous methods to deal with gates computing Boolean functions which are not themselves symmetric. In particular, we show that the support theorem of Anderson and Dawar can be extended to circuits whose gates satisfy a property that we term “matrix-symmetry”.

3.25 Dynamic Complexity: What we cannot do

Thomas Zeume (TU Dortmund, DE)

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Joint work of Thomas Schwentick, Nils Vortmeier, and Thomas Zeume

Main reference Thomas Zeume: “The dynamic descriptive complexity of k -clique”, *Inf. Comput.*, Vol. 256, pp. 9–22, 2017.

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In the dynamic descriptive complexity framework of Patnaik and Immerman, the result of a database query is updated by logical formulas after the insertion or deletion of tuples. The formulas may use additional auxiliary relations that need to be updated as well.

In this talk I will discuss the current status of proving lower bounds in this dynamic context. The talk complements the talk by Nils Vortmeier on recent upper bounds in dynamic complexity.

4 Open problems

4.1 An open problem on gaps for vertex cover

Albert Atserias (UPC – Barcelona, ES)

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In short, the open problem is this: Find pairs of graphs with the same numbers of vertices, one of which does not have vertex covers that are significantly smaller than the set of all vertices, the second of which does have vertex covers of size about half the number vertices, and yet the two graphs are indistinguishable by k -variable first-order logic with counting quantifiers for large values of k .

Problem Statement

Prove (or disprove) that, for every positive integer k and every positive real ϵ there exist infinitely many positive integers n , and pairs of n -vertex graphs G_n and H_n , that satisfy the following conditions:

1. every vertex cover of G_n has size at least $(1 - \epsilon)n$,
2. some vertex cover of H_n has size at most $(1/2 + \epsilon)n$, and
3. $G_n \equiv_{C^k} H_n$, where \equiv_{C^k} denotes indistinguishability by k -variable counting logic C^k .

A vertex cover in a graph is a set of vertices that touches all the edges. Counting logic is the smallest class of formulas that contains the atomic formulas and is closed under negation, disjunction, and counting quantifiers of the type $\exists^{\geq t} x$ for some positive integer t . The meaning of $\exists^{\geq t} x$ is that there are at least t many witnesses for x . In the k -variable fragment of counting logic, denoted by C^k , all variables are in x_1, \dots, x_k (and can be reused).

Background

For $k \leq 2$ the problem is solved [3], but the problem is open even for $k = 3$. A variant of the question asks for the fastest growing integer function $k(n)$ for which for every ϵ there exist infinitely many n , and n -vertex graphs G_n and H_n , that satisfy all three conditions with the third replaced by $G_n \equiv_{C^{k(n)}} H_n$. If C^k is replaced by standard k -variable first-order logic L^k , then the problem is solved for any $k = o(\log n)$ [3].

Back to C^k , the problem was first raised in [4, 5, 2] as a way of proving that, on n -vertex graphs, the Sherali-Adams hierarchy of LP relaxations requires $\Omega(k)$ many levels to be able to improve over the 2-approximation factor that is achieved by the standard LP relaxation of vertex cover. This specific consequence is known to be true for any $k = n^{o(1)}$ [6]. The work in [7] shows that the solution to the problem as stated would imply that any fixed number of levels of the Lasserre hierarchy of SDP relaxations fails to improve over the 2-approximation factor. Unlike the Sherali-Adams hierarchy, this limitation of the Lasserre hierarchy is an open problem even for fixed but small numbers of levels.

The PCP Theorem implies that the size of the minimum vertex cover is NP-hard to approximate within some constant $c > 1$ [1], which has been shown to be at least 1.36 [8], and the Unique Games Conjecture implies the problem is NP-hard to approximate within any constant that is smaller than 2 [9].

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4.2 Some open problems regarding (quasi-)monotone arithmetic circuits for expressing positive integers

Kousha Etessami (University of Edinburgh, GB)

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A “hard” problem

[Allender, Bürgisser, Kjeldgaard-Pedersen, Miltersen, 2006]:

PosSLP: Given an *arithmetic circuit* (Straight Line Program) with gates $\{+, *, -\}$, with integer inputs, decide whether the output is > 0 .

PosSLP basically captures all of polynomial time in the unit-cost arithmetic RAM model of computation.

► **Theorem 1** ([ABKM’06]). *There is a P-time (Turing) reduction from the **square-root sum problem** to PosSLP, and PosSLP can be decided in the Counting Hierarchy: $P^{PP^{PP^{PP}}}$.*

(Nothing better is known as an upper bound or lower bound for PosSLP.)

Some open problems

Question: Can we obtain better complexity bounds for PosSLP?

Here is a very basic approach: Given a $\{+, *, -\}$ -circuit, C , guess a monotone $\{+, *\}$ -circuit, C' , as a “witness of positivity”, and verify $\text{val}(C - C') = 0$ in co-RP, where $\text{val}(C)$ is the value of the output of the circuit. (Checking equality to 0 is ACIT-equivalent ([ABKM'06]). ACIT is a variant of polynomial identity testing, where the polynomial is represented by a circuit, and ACIT is decidable in co-RP.) For $a \in \mathbb{N}$, let $\tau(a)$ denote size of smallest $\{+, *, -\}$ -circuit expressing a . Let $\tau_+(a)$ denote size of smallest monotone $\{+, *\}$ -circuit expressing a .

► **Conjecture 2** (“ τ vs. τ_+ Conjecture”). *This approach does not work. In other words there exists a family $\langle a_n \rangle_{n \in \mathbb{N}}$ of positive integers, such that $\tau(a_n) \in O(n)$, but such that for some fixed constant $c > 0$ $\tau_+(a_n) \in 2^{\Omega(n^c)}$.*

► **Remark (Valiant'79).** proved an exponential lower bound separating the power of monotone vs. non-monotone circuits for expressing families of monotone polynomials (namely, the perfect matching polynomials for planar graphs). This does not imply any lower bounds in the integer setting (at least not in any direct way).

The current state of knowledge for Conjecture 1 is abismal: [Saranurak-Jindal'12] show there is an infinite family of positive numbers (namely $a_n := 2^{2^n} - 1$), such that $\tau_+(a_n) \geq \tau(a_n) + 1$.

It seems “obvious” that much better lower bounds should be possible for “harder” families of numbers. (But not for $a_n = 2^{2^n} - 1$ itself: we easily have $\tau_+(a_n) \leq 2n + 1$ for these.) In fact (joint work with [P. Sinclair, undergraduate thesis project, U. of Edinburgh, 2016]): if some seemingly plausible number-theoretic conjectures hold, then one can (very slightly) improve on the additive $\tau_+(a_n) \geq \tau(a_n) + 1$ lower bound by [Saranurak-Jindal'12], making it $\tau_+(a_n) \geq \tau(a_n) + 3$. (But of course this is nowhere near what we need to establish Conjecture 1. At this point even a super-linear lower bound would be a mini-breakthrough.)

A potentially better approach

Definition: call an arithmetic circuit, C , *quasi-monotone* if it consists of some arbitrary $\{+, *, -\}$ -subcircuits, $C_i, i = 1, \dots, k$, whose *squares* $(C_i)^2$ are the only inputs to a monotone $\{+, *\}$ -circuit, C' , whose output gate gives the output value of the entire circuit C .

Note: these circuits generalize both monotone circuits and sums of squares (S.O.S.).

Better approach for attempting to improve the complexity of PosSLP: Given a $\{+, *, -\}$ -circuit, C , guess a pair of quasi-monotone circuits C' and C'' as a “witness of positivity” for C , & verify the equality $\text{val}((C'' + 1) * C - C') = 0$, which can be done in co-RP.

Here is a **very optimistic** conjecture:

► **Conjecture 3** (“Very effective Positivstellensatz for integers”). *This works: there exists a polynomial $p(x)$, such that for any $a \in \mathbb{N}$ with $\tau(a) = n$, there exist quasi-monotone circuits C'_a and C''_a , such that $\text{size}(C'_a) \leq p(n)$ and $\text{size}(C''_a) \leq p(n)$, and such that*

$$a = \frac{\text{val}(C'_a)}{\text{val}(C''_a + 1)}.$$

If true, this conjecture would of course imply that $\text{PosSLP} \in \mathbf{MA}$.

This would be a big improvement over the current best known upper bound, which is in the counting hierarchy.

(I first presented these conjectures at a workshop in Princeton, in honor of Yannakakis’s 60th birthday, in 2013. I thought there would be a lot more progress by now.)

4.3 Limit laws for random expansions of product structures

Erich Grädel (RWTH Aachen, DE)

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Joint work of Faried Abu Zaid, Erich Grädel, Anuj Dawar, Matthias Hoelzel, Wied Pakusa

Let \mathfrak{A} be a finite structure with a finite (not necessarily relational) vocabulary σ , and let τ be another finite relational vocabulary with $\sigma \cap \tau = \emptyset$. For each n , let \mathfrak{A}^n be the n -fold product of \mathfrak{A} , defined in the usual way. We consider the probability spaces $S_\tau^n(\mathfrak{A})$ consisting of all $\sigma \cup \tau$ -expansions of \mathfrak{A}^n , with the uniform probability distribution.

For every sentence $\psi \in \text{FO}(\sigma \cup \tau)$, let $\mu_n(\psi)$ denote the probability that a randomly chosen structure $\mathfrak{B} \in S_\tau^n$ is a model of ψ .

Problem: Give a complete classification of those finite structures \mathfrak{A} for which the following limit law holds: For every finite relational vocabulary τ and for every sentence $\psi \in \text{FO}(\sigma \cup \tau)$ there exists a (dyadic rational) number q such that

$$\lim_{n \rightarrow \infty} \mu_n(\psi) = q.$$

Partial answers. We have proved in [1] that such a limit law holds for $\mathfrak{A} = (\mathbb{Z}_p, +, 0)$, for any prime p , not just for FO but also for $L_{\infty\omega}^\omega$. The proof generalizes the classical techniques, based on extension axioms, for proving the 0-1 law for FO and $L_{\infty\omega}^\omega$ on random graphs and random finite relational structures. As a consequence (and this was the motivation for considering this problem) it follows that the summation problem for Abelian groups is not definable in LFP, see [1]. I then conjectured that such a limit law might in fact hold for *any* finite structure \mathfrak{A} .

However, this fails dramatically: Mathias Hoelzel (unpublished) has proved that no such limit law holds even in the very simple case where $\mathfrak{A} = (\{0, 1\}, \leq)$. Indeed, based on Kaufmann's proof of the nonconvergence law for monadic second-order logic on random finite structures [2], one can show that there is a first-order formula $\phi(x, y)$ of a vocabulary $\{\leq\} \cup \tau$, that almost surely defines a linear order on $S_\tau^n(\mathfrak{A})$, as n goes to ∞ . From this one easily obtains a sentence ψ which, for growing n , almost surely expresses that n is even, and which hence has no asymptotic probability on the spaces $S_\tau^n(\mathfrak{A})$.

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4.4 Counting/Decision for FO

Dan Suciu (University of Washington – Seattle, US)

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We propose an open problem on probabilistic inference over symmetric structures.

4.5 Intermediate size lower bounds for set-containment join in the relational algebra with aggregates

Jan Van den Bussche (Hasselt University, BE)

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The relational algebra with aggregates extends first-order logic with arithmetic functions and aggregate operators [1]. In the standard relational algebra, without arithmetic or aggregates (but order comparisons are allowed), it is known that every expression for testing nonemptiness of the set-containment join must produce intermediate results of quadratic size [2]. It is an open question whether this also holds for the relational algebra with aggregates.

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4.6 Some Questions about Invariant Definability

Scott Weinstein (University of Pennsylvania, US)

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In my talk at the Dagstuhl Seminar on “Finite and Algorithmic Model Theory,” entitled *Hanf Locality and Elementary Invariant Definability* (based on work joint with Steven Lindell and Henry Towsner), I posed some questions about the strength of invariant definability over restricted classes of structures. Let \mathcal{F}^d be the collection of finite structures over some fixed finite relational signature τ whose Gaifman graphs are of degree $< d \leq \omega$, (so \mathcal{F}^ω is the collection of all finite τ -structures) and let \mathcal{K}^ω be the collection of all structures whose Gaifman graphs are *locally finite*, that is, the degree of every node in the Gaifman graph of the structure is finite. A *local order* on a structure A is a ternary relation $\prec(x, y, z)$ on the universe $|A|$ of A such that for every $a \in |A|$, the binary relation $\prec(a, y, z)$ linearly orders the neighbors of a in the Gaifman graph of A . For a given class of τ -structures \mathcal{C} , we write $\mathbf{FO}[\mathcal{C}]$ for the collection of boolean queries over \mathcal{C} that are defined by first-order τ -sentences, $\mathbf{FO}(<)[\mathcal{C}]$ for the collection of boolean queries over \mathcal{C} that are defined by first-order $\tau \cup \{<\}$ -sentences that are order-invariant over \mathcal{C} , and $\mathbf{FO}(\prec)[\mathcal{C}]$ for the collection of boolean queries over \mathcal{C} that are defined by first-order $\tau \cup \{\prec\}$ -sentences that are local-order-invariant over \mathcal{C} . We posed the following questions.

1. For all $d < \omega$, $\mathbf{FO}(<)[\mathcal{F}^d] = \mathbf{FO}[\mathcal{F}^d]$?
(It is well-known that $\mathbf{FO}(<)[\mathcal{F}^\omega] \neq \mathbf{FO}[\mathcal{F}^\omega]$.)
2. For all $d \leq \omega$, $\mathbf{FO}(\prec)[\mathcal{F}^d] = \mathbf{FO}[\mathcal{F}^d]$?
(Of course, it may be that the answer is different for $d < \omega$ and for ω , as some anticipate is the case for question 1.)
3. $\mathbf{FO}(<)[\mathcal{K}^\omega] = \mathbf{FO}[\mathcal{K}^\omega]$?
4. $\mathbf{FO}(\prec)[\mathcal{K}^\omega] = \mathbf{FO}[\mathcal{K}^\omega]$?

Participants

- Faried Abu Zaid
TU Ilmenau, DE
- Albert Atserias
UPC – Barcelona, ES
- Christoph Berkholz
HU Berlin, DE
- Dietmar Berwanger
ENS – Cachan, FR
- Mikolaj Bojanczyk
University of Warsaw, PL
- Thomas Colcombet
University Paris-Diderot, FR
- Claire David
University Paris-Est –
Marne-la-Vallée, FR
- Anuj Dawar
University of Cambridge, GB
- Arnaud Durand
Paris Diderot University, FR
- Kousha Etesami
University of Edinburgh, GB
- Ronald Fagin
IBM Almaden Center –
San Jose, US
- Diego Figueira
University of Bordeaux, FR
- Jakub Gajarsky
TU Berlin, DE
- Erich Grädel
RWTH Aachen, DE
- Etienne Grandjean
Caen University, FR
- Martin Grohe
RWTH Aachen, DE
- Lauri Hella
University of Tampere, FI
- Sandra Kiefer
RWTH Aachen, DE
- Phokion G. Kolaitis
University of California –
Santa Cruz, US
- Manfred Kufleitner
Universität Stuttgart, DE
- Dietrich Kuske
TU Ilmenau, DE
- Carsten Lutz
Universität Bremen, DE
- James F. Lynch
Clarkson University –
Potsdam, US
- Jerzy Marcinkowski
University of Wrocław, PL
- Joanna Ochremiak
University Paris-Diderot, FR
- Martin Otto
TU Darmstadt, DE
- Wied Pakusa
University of Oxford, GB
- Svenja Schalthöfer
RWTH Aachen, DE
- Thomas Schwentick
TU Dortmund, DE
- Luc Segoufin
IINRIA & ENS Ulm, FR
- Sebastian Siebertz
University of Warsaw, PL
- Dan Suciu
University of Washington –
Seattle, US
- Lidia Tendera
University of Opole, PL
- Wolfgang Thomas
RWTH Aachen, DE
- Szymon Torunczyk
University of Warsaw, PL
- Jouko Väänänen
University of Amsterdam
- Jan Van den Bussche
Hasselt University, BE
- Victor Vianu
University of California –
San Diego, US
- Heribert Vollmer
Leibniz Universität
Hannover, DE
- Nils Vortmeier
TU Dortmund, DE
- Scott Weinstein
University of Pennsylvania, US
- Gregory Wilsenach
University of Cambridge, GB
- Thomas Zeume
TU Dortmund, DE

