

# Dynamic Traffic Models in Transportation Science

Edited by

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## Abstract

Traffic assignment models are crucial for traffic planners to be able to predict traffic distributions, especially, in light of possible changes of the infrastructure, e.g., road constructions, traffic light controls, etc. The starting point of the seminar was the observation that there is a trend in the transportation community (science as well as industry) to base such predictions on complex computer-based simulations that are capable of resolving many elements of a real transportation system. On the other hand, within the past few years, the theory of dynamic traffic assignments in terms of equilibrium existence and equilibrium computation has not matured to the point matching the model complexity inherent in simulations. In view of the above, this interdisciplinary seminar brought together leading scientists in the areas *traffic simulations*, *algorithmic game theory* and *dynamic traffic assignment* as well as people from industry with strong scientific background who identified possible ways to bridge the described gap.

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## 1 Executive Summary

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Traffic assignment models play an important role for traffic planners to predict traffic distributions, especially, in light of possible changes of the infrastructure, e.g., road constructions, traffic light controls, speed limits, tolls, etc. The prevailing *mathematical* approaches used in the transportation science literature to predict such distributions can be roughly classified into static traffic assignment models based on aggregated static multi-commodity flow formulations and dynamic traffic assignment (DTA) models based on the methodology of flows over time. While static models have seen several decades of development and practical



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use, they abstract away too many important details and, thus, become less attractive. On the other hand, dynamic models are known to be notoriously hard to analyze in terms of existence, uniqueness and computability of dynamic equilibria.

In light of the prevailing computational difficulties for realistic-sized networks, the systematic optimization of such networks (e.g., by designing the network infrastructure, link tolls, or traffic light controls) becomes even more challenging as the resulting mathematical programs with equilibrium constraints contain already in the lower level presumably “hard” optimization-, complementarity- or variational inequality problems; not to speak of the resulting optimization problem for the first level.

On the other hand, there is a trend in the transportation science community to use *large-scale computer-based microsimulations* for predicting traffic distributions. The striking advantage of microscopic simulations over DTA models is that the latter usually ignores the feedback of changing network conditions on user behavior dimensions such as flexible departure time choice, mode choice, activity schedule choice, and such. Current simulation tools integrate all these dimensions and many more. The increasing model complexity, however, is by far not matched by the existing theory of dynamic traffic assignments.

The seminar brought together leading researchers from three different communities – Simulations (SIM), Dynamic Traffic Assignment (DTA) and Algorithmic Game Theory (AGT). This year's seminar was centered around three topics:

- *Horizontal queueing models.* Most of the static traffic assignment models assume that queues can occur, but do not take up any physical space. In order to make the current models more realistic one should assume that queues might affect traffic on other nearby road segments, thus, include possible spill-back effects.
- *Oligopolistic competition.* With the rise of autonomous vehicles new routing decisions need to be made. As a novel aspect, individual vehicles might be interested in selfishly optimizing their routes, but cooperate with other vehicles using the same software in order to decrease the average journey time.
- *Risk-averse travelers.* Current static traffic models often assume that each player is rational, and has the sole purpose of minimizing travel time or distance. However, the exact travel time of many routes might be uncertain at the moment of departure. Hence, travelers might stick to a more predictable route and might be unwilling to explore possibly better alternatives.

Again, the seminar was a big success both in terms of stimulating new and very fruitful collaborations. We got enthusiastic feedback from many participants which is also reflected in the survey conducted by Dagstuhl.

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### 3 Overview of Talks

#### 3.1 Equilibrium Computation in Atomic Splittable Routing Games with Convex Cost Functions

*Umang Bhaskar (TIFR Mumbai, IN)*

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We present polynomial-time algorithms as well as hardness results for equilibrium computation in atomic splittable routing games, for the case of general convex cost functions. These games model traffic in freight transportation, market oligopolies, data networks, and various other applications. An atomic splittable routing game is played on a network where the edges have traffic-dependent cost functions, and player strategies correspond to flows in the network. A player can thus split its traffic arbitrarily among different paths. While many properties of equilibria in these games have been studied, efficient algorithms for equilibrium computation are known for only two cases: if cost functions are affine, or if players are symmetric. Neither of these conditions is met in most practical applications. We present two algorithms for routing games with general convex cost functions on parallel links. The first algorithm is exponential in the number of players, while the second is exponential in the number of edges; thus if either of these is small, we get a polynomial-time algorithm. These are the first algorithms for these games with convex cost functions. We further give evidence that in general networks equilibrium computation may be hard, showing that given input  $C$  it is NP-hard to decide if there exists an equilibrium where every player has cost at most  $C$ .

#### 3.2 Travel behavior variability and congestion feedback in iterated transport simulations

*Gunnar Flötteröd (KTH – Royal Institute of Technology – Stockholm, SE)*

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Most transport micro-simulations rely on stochastic travel behavior models. Their stochasticity may be a meaningful modeling feature given expected (i.e. converged) network conditions. During simulation transients, however, their variability may be amplified by network congestion feedback and lead to convergence problems. Means to control travel behavior variability throughout the simulation process are hence considered.

### 3.3 The Price of Stability of Weighted Congestion Games

*Martin Gairing (University of Liverpool, GB)*

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**Joint work of** George Christodoulou, Martin Gairing, Yiannis Giannakopoulos, Paul Spirakis  
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**URL** <http://arxiv.org/abs/1802.09952>

We give exponential lower bounds on the Price of Stability (PoS) of weighted congestion games with polynomial cost functions. Our lower bound asymptotically matches the the upper bound on the Price of Anarchy upper bound. We further show that the PoS remains exponential even for singleton games. More generally, we also provide a lower bound on the PoS of approximate Nash equilibria. All our lower bounds extend to network congestion and hold for mixed and correlated equilibria as well.

On the positive side, we give a general upper bound on the PoS of approximate Nash equilibria, which is sensitive to the range  $W$  of the player weights. We do this by explicitly constructing a novel approximate potential function, based on Faulhaber’s formula, that generalizes Rosenthal’s potential in a continuous, analytic way. From the general theorem, we deduce two interesting corollaries. First, we derive the existence of an approximate pure Nash equilibrium with PoS at most  $(d + 3)/2$ ; the equilibrium’s approximation parameter ranges from  $O(1)$  to  $d + 1$  in a smooth way with respect to  $W$ . Secondly, we show that for unweighted congestion games, the PoS of  $\alpha$ -approximate Nash equilibria is at most  $(d + 1)/\alpha$ .

### 3.4 Great Tolls: How to Induce Optimal Flows under Strategic Link Operators

*Cristóbal Guzmán (Pontifical Catholic University of Chile, CL)*

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**Joint work of** José Correa, Cristóbal Guzmán, Thanasis Lianas, Evdokia Nikolova, Marc Schröder

Network pricing games provide a framework for modeling real-world settings with two types of strategic agents: owners (operators) of the network and users of the network. Owners of the network post a price for usage of the link they own so as to attract users and maximize profit; users of the network select routes based on price and level of use by other users. We point out that an equilibrium in these games may not exist, may not be unique and may induce an arbitrarily inefficient network performance.

Our main result is to observe that a slight regulation on the network owners market solves all three issues above. Specifically, if the authority could set appropriate caps (upper bounds) on the tolls (prices) operators can charge then: the game among the link operators has a unique and strong Nash equilibrium and the users’ game results in a Wardrop equilibrium that achieves the optimal total delay. We call any price vector with these properties a great set of tolls. As a secondary objective, we want to compute great tolls that minimize total users’ payments and provide a linear program that does this. We obtain multiplicative approximation results compared to the optimal total users’ payments for arbitrary networks with polynomial latencies of bounded degree, while in the single-commodity case we obtain a bound that only depends on the topology of the network. Lastly, we show how the same mechanism of setting appropriate caps on the allowable prices extends to the model of elastic demands.

### 3.5 Efficient Black-Box Reductions for Separable Cost Sharing

Anja Huber (*Universität Augsburg, DE*)

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Joint work of Tobias Harks, Martin Hoefer, Anja Huber, Manuel Surek

In cost sharing games with delays, a set of agents jointly allocates a finite subset of resources. Each resource has a fixed cost that has to be shared by the players, and each agent has a non-shareable player-specific delay for each resource. A prominent example is uncapacitated facility location (UFL), where facilities need to be opened (at a shareable cost) and clients want to connect to opened facilities. Each client pays a cost share and his non-shareable physical connection cost. Given any profile of subsets allocated by the agents, a separable cost sharing protocol determines cost shares that satisfy budget balance on every resource and separability over the resources. Moreover, a separable protocol guarantees existence of pure Nash equilibria in the induced strategic game for the agents.

In this talk, we study separable cost sharing protocols in several general combinatorial domains. We provide black-box reductions to reduce the design of a separable cost-sharing protocol to the design of an approximation algorithm for the underlying cost minimization problem. In this way, we obtain new separable cost-sharing protocols in games based on arbitrary player-specific matroids, single-source connection games without delays, and connection games on  $n$ -series-parallel graphs with delays. All these reductions are efficiently computable – given an initial allocation profile, we obtain a cheaper profile and separable cost shares turning the profile into a pure Nash equilibrium. Hence, in these domains any approximation algorithm can be used to obtain a separable cost sharing protocol with a price of stability bounded by the approximation factor.

### 3.6 Computing all Wardrop Equilibria parametrized by the Flow Demand

Max Klimm (*HU Berlin, DE*)

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Joint work of Max Klimm, Philipp Warode

We develop an algorithm that computes for a given undirected or directed network with flow-dependent piece-wise linear edge cost functions all Wardrop equilibria as a function of the flow demand. Our algorithm is based on Katzenelson's homotopy method for electrical networks. The algorithm uses a bijection between vertex potentials and flow excess vectors that is piecewise linear in the potential space and where each linear segment can be interpreted as an augmenting flow in a residual network. The algorithm iteratively increases the excess of one or more vertex pairs until the bijection reaches a point of non-differentiability. Then, the next linear region is chosen in a Simplex-like pivot step and the algorithm proceeds. We first show that this algorithm correctly computes all Wardrop equilibria in undirected single-commodity networks along the chosen path of excess vectors. We then adapt our algorithm to also work for discontinuous cost functions which allows to model directed edges and/or edge capacities. Our algorithm is output-polynomial in non-degenerate instances where the solution curve never hits a point where the cost function of more than one edge becomes non-differentiable. For degenerate instances we still obtain an output-polynomial algorithm computing the linear segments of the bijection by a convex program. The latter technique also allows to handle multiple commodities.

### 3.7 Microscopic simulation of taxicabs and autonomous vehicles with MATSim

*Kai Nagel (TU Berlin, DE)*

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From a transportation planning perspective, autonomous vehicles (AVs) can be seen as a mode of transport: Rather than, say, walking to the car and then driving, or walking to the pt stop, waiting, and then boarding, one would request an AV, wait for it, then board, etc. This can be simulated microscopically, where microscopic means that there are as many synthetic avatars as there are persons and vehicles in reality. One interesting problem is real-time dispatch. I will show results from simulations that reach up to synthetically replacing one million privately owned cars in Berlin by a fleet of 100'000 AVs, and then derive some policy recommendations based on these simulations. These include results for making that fleet electric, and thus (locally) free of emissions.

The presentation uses results obtained with and by Michał Maciejewski, Joschka Bischoff and Gregor Leich.

### 3.8 Equilibria in the fluid queueing model

*Neil Olver (VU University of Amsterdam, NL)*

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**Joint work of** Roberto Cominetti, Omar Larré, José Correa, Neil Olver

**Main reference** Roberto Cominetti, José R. Correa, Omar Larré: “Dynamic Equilibria in Fluid Queueing Networks”, CoRR, Vol. abs/1401.6914v1, 2014

**URL** <https://arxiv.org/abs/1401.6914>

I'll discuss the fluid queueing model, introduced by Vickrey in '69. It is probably the simplest model that plausibly captures the notion of time-varying flows. Although the model is quite simple, our current theoretical understanding of equilibrium behaviour in this model is rather limited, and many fundamental questions remain open. I'll survey a few aspects, such as a structural characterization by Koch and Skutella, and quite general existence and uniqueness results by Cominetti, Correa and Larré. In the second part of the talk I'll discuss a recent result (joint work with Roberto Cominetti and Jose Correa) where we resolve one simple-sounding question: do queue lengths remain bounded in the equilibria under natural necessary conditions?

### 3.9 Optimization and Simulation

*Carolina Osorio (MIT – Cambridge, US)*

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Simulation-based dynamic network models have the potential to provide a detailed (e.g., disaggregate) description of demand and of supply. Nonetheless, unlike analytical models, they are computationally inefficient to evaluate and their use to address transportation optimization problems is a challenge. In this talk we present simulation-based optimization

algorithms that enable the direct and efficient use of simulation-based dynamic network models for optimization. The main idea is to embed within the algorithms information from the analytical network models. The latter provide analytical problem-specific structural information, which enables the design of computationally efficient algorithms. We present case studies for high-dimensional intricate (e.g., non-convex) optimization problems, such as OD calibration, congestion pricing and signal control. We present results for large-scale networks, including Berlin, Singapore and Manhattan.

### 3.10 (Approximate) Equilibrium Computation for Games

*Rahul Savani (University of Liverpool, GB)*

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This talk will give an overview of equilibrium computation for games. We will cover different representations of games, exact and approximate solutions concepts, and the important algorithmic approaches and complexity-theoretic results. We will place the material in the context of traffic models where possible, and will mention some related open questions.

### 3.11 When is selfish routing bad? The price of anarchy in light and heavy traffic

*Marco Scarsini (LUISS Guido Carli – Rome, IT)*

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**Joint work of** Riccardo Colini-Baldeschi, Roberto Cominetti, Panayotis Mertikopoulos, Marco Scarsini

This paper examines the behavior of the price of anarchy as a function of the traffic inflow in nonatomic congestion games with multiple origin-destination (O/D) pairs. Empirical studies in real-world networks show that the price of anarchy is close to 1 in both light and heavy traffic, thus raising the question: can these observations be justified theoretically? We first show that this is not always the case: the price of anarchy may remain a positive distance away from 1 for all values of the traffic inflow, even in simple three-link networks with a single O/D pair and smooth, convex costs. On the other hand, for a large class of cost functions (including all polynomials), the price of anarchy does converge to 1 in both heavy and light traffic, and irrespective of the network topology and the number of O/D pairs in the network. We also examine the rate of convergence of the price of anarchy, and we show that it follows a power law whose degree can be computed explicitly when the network's cost functions are polynomials.

### 3.12 Earliest Arrival Transshipments in Networks With Multiple Sinks

*Miriam Schlöter (TU Berlin, DE)*

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Joint work of Miriam Schlöter, Martin Skutella

We study a classical flow over time problem that captures the essence of evacuation planning: Given a network with capacities and transit times on the arcs and sources/sinks with supplies/demands, an earliest arrival transshipment (EAT) sends the supplies from the sources to the sinks such that the amount of flow which has reached the sinks is maximized for every point in time simultaneously. So far, a lot of effort has been put into the development of algorithms for computing earliest arrival transshipments in dynamic networks with only a single sink, as in such networks earliest arrival transshipments do always exist. Regarding earliest arrival transshipments in networks with multiple sinks not much is known, aside from the fact that EATs do in general not exist in general networks. In particular, there is no PSPACE algorithm known for computing EATs in case of existence and also the complexity of deciding whether an earliest arrival transshipment solving a given transshipment problem in a multiple sink network does exist is still unknown.

This talk concentrates on our latest results regarding EATs in networks with multiple sinks: In particular, we formulate the first PSPACE algorithm that decides whether a given tight earliest arrival transshipment problem in a general network has solution and computes the EAT in case of existence. We show that in case of existence an earliest arrival transshipment can essentially be determined as a convex combination of special flows over time while minimizing a suitably defined submodular function. The solution we achieve has additionally the nice structural property of being a generalized temporally repeated flow. Additionally, we settle the complexity question by showing that in multiple sink networks it is NP-hard to decide whether an earliest arrival transshipment solving a given problem does exist.

### 3.13 Selfish Network Creation with Wardrop Followers

*Daniel Schmand (RWTH Aachen, DE)*

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Joint work of Daniel Schmand, Marc Schröder, Alexander Skopalik

We study the following network design game. The game is set in two stages. In the first stage some players, called providers, aim to maximize their profit individually by investing in bandwidth on edges of a given graph. The investment yields a new graph on which Wardrop followers, called users, travel from their source to their sink through the network. The cost for any user on an edge follows market principles and is dependent on the demand for that edge and the supplied bandwidth. The profit of the providers depends on the total utilization of their edges, the current price for their edges and the bandwidth. We analyze the existence, uniqueness and efficiency of Nash Equilibria in the described game.

### 3.14 Network Congestion Games are Robust to Variable Demand

*Marc Schröder (RWTH Aachen, DE)*

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**Joint work of** José Correa, Ruben Hoeksma, Marc Schröder

**URL** <https://www.dii.uchile.cl/~jcorrea/papers/Conferences/CHS2017.pdf>

Network congestion games have provided a fertile ground for the algorithmic game theory community. Indeed, many of the pioneering works on bounding the efficiency of equilibria use this framework as their starting point. In recent years, there has been an increased interest in studying randomness in this context though the efforts have been mostly devoted to understanding what happens when link latencies are subject to random shocks. Although this is an important practical consideration, it is not the only source of randomness in network congestion games. Another important source is the inherent variability of the demand that most practical networks suffer from. Therefore in this paper we look at the basic non-atomic network congestion game with the additional feature that demand is random. Our main result in this paper is that under a very natural equilibrium notion, in which the basic behavioral assumption is that users evaluate their expected cost according to the demand they experience in the system, the price of anarchy of the game is actually the same as that in the deterministic demand game. Moreover, the result can be extended to the more general class of smooth games.

### 3.15 Computing Efficient Nash Equilibria in Congestion Games

*Guido Schäfer (CWI – Amsterdam, NL)*

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**Joint work of** Pieter Kleer, Guido Schäfer

Congestion games constitute an important class of games which capture many applications in network routing, resource allocation and scheduling. Rosenthal (1973) established the existence of pure Nash equilibria in congestion games by exhibiting an exact potential function whose local minima coincide with the set of pure Nash equilibria of the game. We investigate structural properties which allow us to efficiently compute global minima of Rosenthal's potential function. To this aim, we use a polyhedral description to represent the strategy sets of the players and identify two general properties which are sufficient for our result to go through. In addition, we show that the resulting Nash equilibria provide attractive social cost approximation guarantees. Our contributions thus provide an efficient algorithm to find an approximately optimal Nash equilibrium for a large class of polytopal congestion games. Joint work with Pieter Kleer (CWI).

### 3.16 Simple, distributed, and powerful – improving local search for distributed resource allocation and equilibrium computation

Alexander Skopalik (Universität Paderborn, DE)

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Joint work of Vipin Ravindran Vijayalakshmi, Alexander Skopalik

Congestion games constitute an important class of games to model resource allocation by different users such as in traffic networks. We study the approximation ratio of local optima in these games. However, we allow for that the cost functions used during the local search procedure may be different from the overall objective function. Our analysis exhibits an interesting method to choose these cost functions to obtain a number of different results:

1. As computing an exact or even approximate pure Nash equilibrium is in general PLS-complete, Caragiannis et al. [FOCS 2011] presented a polynomial-time algorithm that computes  $(2 + \epsilon)$ -approximate pure Nash equilibria for games with linear cost functions and further results for polynomial cost functions. We show that this factor can be improved to  $1.67 + \epsilon$  by a seemingly simple modification of their algorithm using our technique.
2. Bilo and Vinci [EC 2016] presented an algorithm to compute load depended taxes that improve the price of anarchy e.g. for linear game from 2.5 to 2. Our methods yield slightly weaker results, e.g., 2.1 for linear games. However, our tax functions are locally computable and independent of the actual instance of the game. Their algorithm is a centralized algorithm and sensitive to changes of the instance such as e.g. the number of players.
3. Computing an optimal allocation in congestion games is NP-hard. The best known centralized approximation algorithm is due to Makarychev and Srividenko [FOCS14]. Again, our technique does not quite match these bounds but offers a modified local search procedure as a much simpler alternative which can easily be implemented in a distributed fashion.

### 3.17 Multiplicative Pacing Equilibria in Auction Markets

Nicolás Stier-Moses (Facebook – Menlo Park, US)

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Joint work of Vincent Conitzer, Christian Kroer, Eric Sodomka, Nicolás Stier-Moses

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URL <http://arxiv.org/abs/1706.07151>

Budgets play a significant role in real-world sequential auction markets such as those implemented by Internet companies. To maximize the value provided to auction participants, spending is smoothed across auctions so budgets are used for the best opportunities. Motivated by a mechanism used in practice by several companies, this paper considers a smoothing procedure that relies on *pacing multipliers*: on behalf of each bidder, the auction market applies a factor between 0 and 1 that uniformly scales the bids across all auctions. Reinterpreting this process as a game between bidders, we introduce the notion of *pacing equilibrium*, and prove that they are always guaranteed to exist. We demonstrate through examples that a market can have multiple pacing equilibria with large variations in several

natural objectives. Although we show that computing either a social-welfare-maximizing or a revenue-maximizing pacing equilibrium is NP-hard, we present a mixed-integer program (MIP) that can be used to find equilibria optimizing several relevant objectives. We use the MIP to provide evidence that: (1) equilibrium multiplicity occurs very rarely across several families of random instances, (2) static MIP solutions can be used to improve the outcomes achieved by a dynamic pacing algorithm with instances based on a real-world auction market, and (3) for our instances, bidders do not have an incentive to misreport bids or budgets provided there are enough participants in the auction.

### 3.18 Queues in the cyclically time-expanded network model

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We present extensions of the cyclically time-expanded network model for traffic signal optimization to cope with queues. The model has already shown its value for simultaneous traffic assignment and traffic signal optimization by computing competitive solutions. However, due to the lack of a proper first in-first out property and a limited storage capacity, several traffic situations were inaccurately modeled. In this talk, we present two constructions to handle overload, spill-back, and queues. We have a special focus on applications, namely the optimization of signals for exceptional mega events and (passive) transit signal priority, i.e., accelerating public transport by optimizing signal settings.

### 3.19 Non-separable costs and their impact on (a class of) user equilibrium algorithms

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This presentation discusses convergence properties of a class of algorithms for computing user equilibrium (UE) flows on large networks with multiple origins and destinations. The class derives from the formulation of the UE problem as a fixed point between a demand and a supply submodel. A general structure of this class of algorithms is presented. Typically, the more efficient algorithms decompose the UE problem into several sub-problems, usually defined in terms of origins and/or destinations that use common link subsets. In time-dependent problems, another decomposition dimension is the departure time interval. UE is achieved by iteratively solving a (fully or partially converged) step of each sub-problem. This involves adding or removing links from the sub-problem, deciding direction and size of the step (=swap) towards sub-problem-UE, and updating the link costs as a consequence of this swap. It is shown how efficient convergence of the combined sub-problems depends strongly (i) on the sequence of solving the sub-problems, and (ii) on whether the link cost updates are computed simultaneously after deciding swaps for all sub-problems or sequentially after each sub-problem. Moreover, non-separability of link costs (e.g. due to intersection or spillback interactions, or in time-dependent flows by definition) affects strongly whether a chosen sequence for solving sub-problems and updating link costs converges efficiently, if at all.

This is illustrated on an algorithmic solution for the Vickrey bottleneck model and a simple 2-route dynamic UE problem. These insights hint at improved solution sequences for these types of problems, which is ongoing work.

### 3.20 Effects of fixed-time vs. traffic-adaptive signal control on the total travel time in the user equilibrium in agent-based transport simulations

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**Joint work of** Nico Kühnel, Kai Nagel, Theresa Thunig

**URL** <https://svn.vsp.tu-berlin.de/repos/public-svn/publications/vspwp/2018/18-02/KuehnelThunigNagel2018LaemmerImpl.pdf>

When travelers choose a route to their destination, they also take signal green times into account: When a signal delays their travel time significantly, they will try out a different route on the next day. With a system-wide optimization of fixed-time signal plans, one can benefit from this user reaction. Fixed-time signals can intentionally make certain routes unattractive to force the user equilibrium (UE) towards the system optimum (SO) route pattern, i.e. to improve total travel time. In contrast, traffic-adaptive signals that control traffic based on local sensor data are usually de-centralized and not able to control system-wide effects. They aim to locally minimize delay at intersections which supports the UE routes. This can reduce waiting times and, thus, total travel time, but does not necessarily force the SO.

In this talk we will compare the effects of fixed-time and traffic-adaptive signals in the transport simulation MATSim. At first, the small network of Braess' paradox is analyzed. It is shown that in contrast to fixed-time signals, local delay-minimizing signals are not able to force the SO. Still, they are able to react to unexpected demand changes and improve waiting times, which will be shown in the second part of the talk with a real-world application of the city of Cottbus, Germany.

### 3.21 Oligopolistic Competitive Packet Routing

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**Joint work of** Laura Vargas Koch, Björn Tauer, Britta Peis, Veerle Timmermans

We study a game-theoretic variant of packet routing. Oligopolistic competitive packet routing games model situations in which traffic is routed in discrete units through a communication network over time. In contrast to classical packet routing, we are lacking a central authority to decide on an oblivious routing protocol. Instead, selfish acting decision makers (“players”) control a certain amount of traffic each, which needs to be sent as fast as possible from a player-specific origin to a player-specific destination through a commonly used network. The network is represented by a directed graph, each edge of which being endowed with a transit time, as well as a capacity bounding the number of traffic units entering an edge simultaneously. Additionally, a priority policy on the set of players is publicly known with respect to which conflicts at intersections are resolved. We prove the existence of a pure

Nash equilibrium and show that it can be constructed by sequentially computing an integral earliest arrival flow for each player. Moreover, we derive several tight bounds on the price of anarchy and the price of stability in single source games.

### 3.22 Competitive Packet Routing With Edge Priorities

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**Joint work of** Robert Scheffler, Martin Stehler, Laura Vargas Koch

**Main reference** Robert Scheffler, Martin Stehler, Laura Vargas Koch: “Nash equilibria in routing games with edge priorities”, CoRR, Vol. abs/1803.00865, 2018.

**URL** <http://arxiv.org/abs/1803.00865>

We present a game-theoretic variant of packet routing. In contrast to classical packet routing, we are lacking a central authority. Instead, selfish acting players route from a source to a sink and try to reach the sink as fast as possible. The network is represented by a directed graph, each edge of which being endowed with a transit time, as well as a capacity bounding the number of traffic units entering an edge simultaneously. A very natural way to motivate competitive packet routing arises from traffic. Here it makes sense to define priorities on the edges (main roads are preferred over smaller roads). This means if more player want to enter an edge than it is possible, the players coming from the main road are preferred. We present an algorithm computing a certain class of equilibria in these games, mistrustful equilibria and we analyze their properties. Further we examine the connection to earliest arrival flows.

## 4 Open problems

### 4.1 The Inefficiency of Wardrop Routing with Uncertain Demand

*Daniel Schmand (RWTH Aachen, DE), Anja Huber (Universität Augsburg, DE), and Veerle Timmermans (Maastricht University, NL)*

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We discussed open problems related to uncertainty of the demand in selfish routing games. We consider a model of a Wardrop-game in which the overall demand in the system is drawn from a probability distribution. We seek to understand the expected inefficiency of Wardrop equilibria in the system. An easy example shows that current lower bounds on the price of anarchy (i.e.  $4/3$  for linear functions) are not robust to changes in the overall demand. That is, we could already show that the price of anarchy decreases in the famous Pigou example. Further analysis of the inefficiency of Wardrop equilibria in games with uncertain demand is a very interesting research topic and remains open.

## 4.2 Stochastic Atomic Congestion Games

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We consider an atomic network congestion game in which each player is drawn independently with a given probability to play the game. Roughgarden (2015) and Correa, Hoeksma and Schroder (2017) already showed that the upper bound on the price of anarchy of these games is the same as for the deterministic game in which the set of players is fixed and known. It is however open whether this upper bound is tight, and/or can be characterized in terms of the probability of playing. Another potential direction would be to see whether atomic congestion games with independent, but identically distributed random weights can be shown to be potential games.

## 4.3 Complexity of Mixed Equilibria in Potential Games

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We discussed the complexity theoretic characterization of mixed Nash equilibria in Potential or Congestion Games. It is known that this problem is in CLS (and in the intersection of PPAD and PLS) but it is not known whether they are hard for this class. We also considered an alternative complexity class EOPL and discussed possible approaches for polynomial time algorithms for some special cases such as e.g. two player network congestion games. This problem is particularly interesting as it is not known whether finding a pure Equilibrium is PLS-hard. Only for so called restricted network games that has been shown. Although we were not able to solve the original problem, our discussion resulted in interesting new insights and observations.

## 4.4 Alternatives to Wardrop equilibrium and Convergence of iterated transport simulations

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Although Wardrop Equilibrium (WE) is a famous reference model for transportation, many researchers in transportation science focus instead on models that represent heterogeneity/unknown factors using probability distributions. This gives rise to a different model, Stochastic User Equilibrium. In a sense SUE generalises WE, since in its most general form SUE admits both continuous and discrete probability distributions for the random error terms in travel

utilities, including the case of probability mass 1 at a zero error (i.e. WE). As SUE is used relatively widely in transportation science, and (it seems) hardly at all in algorithmic game theory, it seems an area of potential future investigation for linking the two communities, especially in terms of studying how results for Nash-type games might extend to these alternative models.

The SUE mechanism for path choice is also attractive for use in non-equilibrium situations, in particular iterated simulations, since conditional path choice is unique and dispersed, but nevertheless can lead to computational problems. In this session we considered some recent development in utilising bounded forms of the logit choice model in an SUE context, as opposed to the unbounded models typically used. This has important implications in that for unbounded models all routes are used, whereas for bounded models we obtain something analogous to WE in a division between used and unused routes. The idea we pursued was to utilise such bounded models in iterated simulations, where the bounding could be systematically varied over the course of the simulation – so in an early stage we allow greater variance in order to generate a working set of routes that are sufficient to serve the demand, but in later stages would systematically reduce the variance so as to reduce the exploration and instead focus on re-balancing between routes. Some initial calculations have suggested a potential functional form to describe how the bound might be systematically altered across iterations.

In our proposal for open problems we also discussed two other issues that we believed to merit further exploration. The first was the problem of non-separable cost/latency functions, as might occur (for example) at an uncontrolled priority intersection, when the delay to a minor turning movement is mainly controlled by the flow of a different (major/priority) flow. The second was the issue of what were referred to as *anti-congestion games*, where the cost/latency of an alternative may be decreasing in the flow on that alternative (as may occur due to social/imitation/mass effects, or due to economies of scale).

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