Report from Dagstuhl Seminar 18422

Shape Analysis: Euclidean, Discrete and Algebraic Geometric Methods

Edited by

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Abstract

In computer vision, geometric processing and image analysis, the notation of a shape of a 3-D object has been studied either by an embedded manifold for the continuous stetting or as a collection of a discrete set of marker positions on the manifold. Within the last years, there have been many rapid developments in the field of shape representation, shape correspondence and shape manipulation with technical applications ranging from laser-range scanners to 3-D printing. Classic shape analysis tools from differential geometry have a fresh influence in the field, often powered by modern methods from optimization and numerical computing. At the same time, discrete geometric methods and related techniques such as from mathematical morphology have evolved significantly. Moreover, techniques like deep learning gained a significant influence in the development of corresponding methods and tools. New developments from tropical geometry have a high potential for use in shape analysis.

The topics in our seminar addressed the sketched challenges and developments that will be useful for shape analysis. Especially we aimed to discuss the possibilities of combining fields like tropical geometry with more classical techniques as for instance from mathematical morphology. We discussed possibilities of applying machine learning methods in this context and considered recent advances from more classical fields like differential geometry and partial differential equations that can be useful for setting up and understanding shape analysis methods in all of these approaches.

This seminar brought together 26 researchers from North America and Europe engaged in recent and upcoming developments in shape analysis who view these challenges from different perspectives and who together discussed the pressing open problems and novel solutions to them.

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Edited in cooperation with Robert Dachsel

1 Executive Summary

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Dagstuhl seminar 18422 Shape Analysis: *Euclidean, Discrete and Algebraic Geometric Methods* took place October 14–19, 2018. 26 researchers from North America and Europe discussed state-of-the-art, current challenges, and promising future research directions in the areas of 2-D and 3-D shape analysis from a cross-disciplinary point of view. Participants included international experts from the fields of continuous-scale shape analysis, discrete shape analysis, tropical geometry and numerical computing. The seminar consisted of an opening and getting to know session and 26 scientific presentation sessions. Furthermore, there was time for extensive discussions both between the talks and in the evenings.

The topics in our seminar addressed the sketched challenges and developments that will be useful for shape analysis. Especially we aimed to discuss the possibilities of combining fields like tropical geometry with more classical techniques as for instance from mathematical morphology. We discussed possibilities of applying machine learning methods in this context and considered recent advances from more classical fields like differential geometry and partial differential equations that can be useful for setting up and understanding shape analysis methods in all of these approaches.

The purpose of this seminar was to address these challenges with the latest tools related to geometric, algorithmic and numerical concepts. To do so, we brought together researchers working on shape analysis topics from different perspectives. The purpose in bringing together researchers from those different communities sharing substantial interest in shape analysis was to explore the benefits of a cross-disciplinary point of view.

Promising new ways to combine the latest techniques from these different fields were identified during in-depth discussions in small groups. Some especially promising research directions in the areas of deep learning, mathematical morphology, shape from shading, modelling deformable shapes, and tropical geometry were discussed in small groups between the talks and in the evenings.

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3 Overview of Talks

3.1 Tropical geometry, Optimal Control and Mean-payoff Games

Marianne Akian (Ecole Polytechnique – Palaiseau, FR)

Tropical algebra can be seen as the limit of a deformation of usual agebra or linked with usual algebra by using the notion of valuation. This allows one to define tropical geometry and in particular tropical convex sets. Tropical polyhedra can be seen as the set of solutions of a finite number of two sided tropical inequalities or equalities. In a work with Gaubert and Guterman (2012), we proved that the above external representation of tropical polyhedra is equivalent to the notion of mean-payoff zero-sum games. Then, the polyhedra is nonempty if and only if there exists a winning position for the maximization player. We then show how this result can be combined with more recent results of Grigoriev and Podolskii (2014) and Allamigeon, Gaubert and Skomra (2018) to obtain such characterizations for polynomial equations, and for stochastic games respectively.

3.2 Heat kernel semigroups and morphological semigroups on ultrametric spaces for hierarchical data processing

Jesús Angulo (Ecole des Mines de Paris, FR)

Joint work of Jesús Angulo, Santiago Velasco-Forero

Ultrametric spaces are the natural mathematical structure to deal with data embedded into a hierarchical representation. This kind of representations is ubiquitous in morphological image processing, from pyramids of nested partitions to more abstract dendograms from minimum spanning trees. This talk presents a formal study of morphological operators for functions defined on ultrametric spaces. First, the notion of ultrametric structuring function is introduced. Then, using as basic ingredient the convolution in (max,min)-algebra, the multi-scale ultrametric dilation and erosion are defined and their semigroup properties are stated. It is proved in particular that they are idempotent operators and consequently they are algebraically ultrametric closing and opening too. Some preliminary examples illustrate the behavior and practical interest of ultrametric dilations/erosions.

3.3 Descriptors of Q-convexity: Theory and Applications (Part 2)

Péter Balázs (University of Szeged, HU)

Motivated by a special reconstruction problem in binary tomography, we start out from a one-dimensional convexity measure and generalize it to two dimensions using the concept of Q-convexity. We show a way how to normalize this measure and study the performance of this descriptor in several image classification problems. Our results are comparable and in some cases even better than those achieved by other methods based on convexity descriptors. While these latter ones take 180 directions into consideration, our measure uses just two directions in the 2D plane.

3.4 Extending Classic Photometric Stereo

Michael Breuß (BTU Cottbus, DE)

In this talk the results on recent work in the area of photometric stereo for non-Lambertian surfaces is presented. The photometric stereo (PS) problem is a classic problem in computer vision. Given several input images, taken from the same point of view but under different lighting conditions, the task is to compute the 3D shape of objects depicted in the photographed scene. The crucial modeling steps to solve this inverse problems are the camera model that yields how the 3D world is mapped to a 2D image, and the light reflectance model that accounts for the light reflectance of objects in a scene. The classic simple setting features Lambertian i.e. completely diffuse reflectance and an orthographic camera for easy mathematical modeling, respectively. The talk shows how a perspective geometry changes the mathematical setting. Moreover complex non-Lambertian reflectance models like the Blinn-Phong model known from computer graphics are addressed for the PS problem.

3.5 On Geometric Centers of Point Constellations

Alfred M. Bruckstein (Technion – Haifa, IL)

Joint work of Alfred M. Bruckstein, Yael Yankelevsky

We present a new result on the characterization of Steiner Centers for planar point constellations. The Steiner Center is the location that minimizes the excess area of coverage of the convex hull of the planar point constellation by the superposition of the disks with diameters defined by the variable point and all points in the given constellation. Proving the higher dimensional equivalent of this characterization remains an open problem.

3.6 Descriptors of Q-convexity: Theory and Applications (Part 1)

Sara Brunetti (University of Siena, IT)

In this talk, we propose a new idea to design a measure for shape descriptors based on the geometrical concept of Quadrant-convexity. We may derive a “quantitative” representation which provides a measure of concavity, and permits to define complex relations like enlacement and interlacement between objects. In addition, we discuss a different approach based on the notion of salient points and Quadrant convex hull giving rise to a “qualitative” representation of the image. The derived shape descriptors have the following features: 1)their values range from 0 to 1; 2)they are invariant by reflection and point symmetry; 3) they are scale-tolerant 4) their computation can be easily and efficiently implemented.

3.7 Geometry of Neural Networks with Piecewise Linear Activations

Vasileios Charisopoulos (Cornell University, US)

Deep neural networks are considered to be the state of the art in many image recognition challenges, such as object detection, segmentation, and other tasks, often surpassing human performance. However, despite their accessibility, their theoretical properties are still being investigated.

We will present an approach based on tropical mathematics to investigate one of these properties for neural networks with piecewise linear activations. Our approach relies on the duality between the vertices of Newton Polytopes and maximizers of corresponding polynomials in the (max, +) or tropical algebra. It enables us to recover bounds on the number of linear regions of neural network layers, which serve as a measure of their expressive power. We will attempt to demonstrate the aforementioned duality and the geometric objects involved starting from a much simpler learning model, mimicking the linear perceptron in the (max, +) setting.

3.8 A Feature Descriptor based on Time Evolution Equations

Robert Dachsel (BTU Cottbus, DE)

A major task in non-rigid shape analysis is to retrieve correspondences between two almost isometric 3D objects. For the descriptor based approach, a simplified shape representation, called feature descriptor which is invariant under almost isometric transformations is required. A successful class of feature descriptors employ the spectral decomposition of the Laplace-Beltrami operator. Important examples are the heat kernel signature and the wave kernel signature, being expressed as a series expression of the Laplace-Beltrami's low frequency eigenfunctions and eigenvalues. This choice makes the descriptors robust to high frequent noise but particularly vulnerable to global distortions of a shape. In this talk we tackle this problem and explore an alternative solution strategy. We model physically phenomena on shapes as carried by the heat, Schrödinger and wave equation. Therefore we chose a direct discretization using a combined finite volume and time integration approach. This approach leads to a local acting feature descriptor class being able to handle global distortions of a shape. By a detailed evaluation at hand of a standard shape data set we demonstrate that our approach may yield significant improvements over state of the art methods for finding correct shape correspondences.

3.9 Data representation and topology-based analysis for data visualization

Leila De Floriani (University of Maryland – College Park, US)

Joint work of Leila De Floriani, Riccardo Fellegara, Federico Iuricich

This talk provides an overview on our research on data representation and topology-based approaches to scientific data visualization. We discuss first a new compact and modular data structure for simplicial complexes in arbitrary dimensions, the Stellar tree, and then topology-based techniques for data transformation based on topological tools. Specifically, we discuss discrete Morse theory, a combinatorial counterpart of smooth Morse theory, and its application in visualization as a segmentation tool as well as in computing persistent homology for shape analysis.

3.10 Geometry of facial shapes and machine learning methods for synthesis of expressive multimodal avatars

Panagiotis Filntisis (National Technical University of Athens, GR)

We explore the geometry of faces via statistical modeling and deformable shape models such as Active Appearance Models (AAMs), for modeling facial emotional expressions and movements. We also investigate how machine/deep learning methods can be coupled with AAMs in order to synthesize smooth trajectories of different facial shapes that occur during emotional speech.

To do this, we use both established machine learning methods such as hidden Markov models as well as "deeper" models (i.e., Deep Neural Networks) and show how deep models can better capture the complexity of facial shapes and the large variations of emotional speech and expressions.

The resulting machine learning models can be used to synthesize expressive multimodal faces (avatars-talking heads) that can also adapt their shape to target emotions using a small amount of data, as well as interpolate different facial shapes to create more complex shapes.

3.11 Towards the Analysis of Multivariate Data based on discrete Morse Theory

Federico Iuricich (Clemson University, US)

Due to the advances in sensor technology, we are in need of new tools for analyzing shapes of constantly increasing complexity. Topological Data Analysis is an active research area providing robust and data-agnostic descriptors of data. The most widely used tool in TDA is persistent homology which provides a way to compute topological features in a multi-scale manner and summarizes them in a persistence diagram. Persistence diagrams can be converted in descriptors suitable for learning tasks like shape classification or retrieval.

Multi-parameter Persistent Homology (MPH), is a generalization of persistent homology devoted to the investigation of multivariate data, i.e. data provided with multiple functions. Currently, computing MPH is very challenging even with data of modest size and scalable algorithms are needed for extracting this information efficiently. In this talk, we will present a new approach to Multidimensional Persistent Homology computation, inspired by Discrete Morse theory, based on reducing the input complex through the definition of a discrete gradient field. Experimental results will be presented showing the efficiency of our algorithm. Moreover, we will discuss future applications to scientific data analysis and visualization.

3.12 Computing and regularizing medial axes in 3D

Tao Ju (Washington University – St. Louis, US)

Medial axes is a classical concept in computational geometry and has been the basis of most of today's skeletal shape descriptors. In this talk, I will present our recent work in addressing two major roadblocks in using medial axes for 3D shape analysis: the difficulty in computing the medial axes of general 3D shapes, and the sensitivity of the medial axes to noise. First, I will describe a novel sampling-based algorithm for computing 3D medial axes that is numerically robust, simple to implement, and theoretically sound. Second, I will present a medial axes regularization method that is guided by a novel significance measure in 3D and capable of producing a family of skeletons that are descriptive and robust to noise. Finally, I will briefly discuss some applications of skeletons in biomedicine.

3.13 Geometric problems in isogeometric analysis

Bert Jüttler (Johannes Kepler Universität Linz, AT)

Part I: Isogeometric Segmentation of Free-form Solids
Part II: Arc Fibrations of Planar Domains

The two parts of this talk reported on the authors' recent work on selected geometric problems in isogeometric analysis (IGA). The framework of IGA, which was introduced in 2005 by T.J.R. Hughes et al., combines the mathematical technologies of numerical simulation and geometric design. The interaction of these two fields leads to the formulation of many new and challenging problems, two of which have been addressed in this talk.

The first part was devoted to the segmentation problem. Given a three-dimensional solid object in boundary representation, find a segmentation into topological cuboids, which admit a parameterization by tensor-product B-splines. We presented the isogeometric segmentation pipeline, which is a semi-automatic process that creates such a segmentation. The process, which is based on a recursive splitting algorithm, relies on a cost-function, which measures the quality of a splitting step. This part of the talk was based on joint work with M. Kapl, D. M. Nguyen, Q. Pan and M. Pauley.

The second part discussed a special instance of a parameterization. More precisely, we investigated the parameterization of planar domains by families of circular arcs. It is well known that star-shaped domains possess particularly simple polar parameterizations, which are formed by the line segments that connect a suitably chosen center with the points on the domain's boundary. The polar parameterization is valid (i.e., regular everywhere except for the center) if the center is located in the kernel of the domain. In the case of a domain with a smooth free-form boundary curve, the kernel is a convex polygon which is formed by (some of) the boundary's inflection tangents. These parameterizations possess numerous applications, most recently also including domain parameterization in isogeometric analysis.

Since the class of star-shaped domains is quite limited, we propose to increase the flexibility of the underlying polar parameterizations by considering circular arcs that connect the center with the points on the domain's boundary. Parameterizations that are regular everywhere except at the center are said to form an arc fibration of a planar domain. We analyze the existence of an arc fibration with a given center and present an algorithm that computes it in the affirmative case. In addition, we explore the arc fibration kernel of a domain, which contains the suitable center points. This part of the talk is joint work with M.-S. Kim and S. Maroscheck.

3.14 Statistical learning under group actions, with applications to cryo-electron microscopy

Joe Kileel (Princeton University, US)

In many problems in computer vision, robotics and image/signal processing, we wish to recover latent variables from observations suffering unknown shifts or rotations. One example is cryo-electron microscopy (cryo-EM), recognized by the Nobel Prize 2017 in Chemistry. Here the challenge is to estimate the 3D structure of a protein from many, very noisy 2D images taken at unknown viewing directions.

In this talk, I will place cryo-EM reconstruction inside a framework for statistical learning under noisy group actions. I will prove a tight relation between the sample complexity for statistical learning under noisy group actions and the invariant theory of the underlying symmetry group. On the algebra side, this motivates apparently new questions in invariant theory – to which we offer partial algorithmic answers in general. As for the cryo-EM case, we will give a novel ab initio 3D reconstruction algorithm, which is both sample and computationally efficient – at least under model assumptions.

I will review needed background information along the way. Overall, I hope to convey the flavor of some algebraic-geometric methods (still under development) in image analysis.

3.15 Invariant Representations of Shapes and Forms: Self Functional Maps

Ron Kimmel (Technion – Haifa, IL)

A classical approach for surface classification is to find a compact algebraic representation for each surface that would be similar for objects within the same class and preserve dissimilarities between classes. We introduce self functional maps as a novel surface representation that satisfies these properties, translating the geometric problem of surface classification into an algebraic form of classifying matrices. The proposed map transforms a given surface into a universal isometry invariant form defined by a unique matrix. The suggested representation is realized by applying the functional maps framework to map the surface into itself. The key idea is to use two different metric spaces of the same surface for which the functional map serves as a signature. As an example we suggest the regular and the scale invariant surface laplacian operators to construct two families of eigenfunctions. The result is a matrix that encodes the interaction between the eigenfunctions resulted from two different Riemannian manifolds of the same surface. Using this representation, geometric shape similarity is converted into algebraic distances between matrices. In contrast to geometry understanding there is the emerging field of deep learning. Learning systems are rapidly dominating the areas of audio, textual, and visual analysis. Recent efforts to convert these successes over to geometry processing indicate that encoding geometric intuition into modeling, training, and testing is a non-trivial task. It appears as if approaches based on geometric understanding are orthogonal to those of data-heavy computational learning. We propose to unify these two methodologies by computationally learning geometric representations and invariants and thereby take a small step towards a new perspective on geometry processing.

3.16 Duality of Convolution Operators: A Tool for Shape Analysis?

Christer Oscar Kiselman (Uppsala University, SE)

Duality is a term which represents a collection of ideas where two sets of mathematical objects confront each other. An important example is the dual of a normed space, where the linear forms on the space operate on its points. A most successful duality is that between the space $\mathcal{D}(\Omega)$ of test functions (smooth functions of compact support) and its dual space $\mathcal{D}'(\Omega)$ of distributions. The distributions that are not given by a locally integrable function live like ghosts in the dark, perceptible only through their actions on test functions.

The dual of a family of functions or sets reflects some but typically not all properties of the family. As an example, the supporting function of a set gives faithful information on the closed convex hull of the set, but forgets about any holes or cavities.

Mathematical morphology can be quite helpful in providing guiding concepts and ideas in the study of discrete convolution operators and many other topics, like discrete optimization. Here we apply these ideas to the duality of convolution operators.

It all goes back to Galois, two centuries ago. I put it under the common roof of lower and upper inverses of mappings between preordered sets.

3.17 Divergence-Free Shape Interpolation and Correspondence

Zorah Lähner (TU München, DE)

Joint work of Zorah Lähner, Marvin Eisenberger, Daniel Cremers

We present a novel method to model and calculate deformation fields between shapes embedded in $\mathbb{R}^D$. Our framework combines naturally interpolating the two input shapes and calculating correspondences at the same time. The key idea is to compute a divergence-free deformation field represented in a coarse-to-fine basis using the Karhunen-Loève expansion. The advantages are that there is no need to discretize the embedding space and the deformation is volume-preserving. Furthermore, the optimization is done on downsampled versions of the shapes but the morphing can be applied to any resolution without a heavy increase in complexity. We show results for shape correspondence, registration, inter- and extrapolation on the TOSCA and FAUST data sets.

3.18 Mathematical Morphology and Tropical Geometry

Petros Maragos (National Technical University of Athens, GR)

Mathematical Morphology and Tropical Geometry share the same max/min-plus scalar arithmetic and max/min-plus matrix algebra. In this Dagstuhl Seminar on shape analysis, we summarize their common ideas and algebraic structure, extend both of them using weighted lattices, and outline applications on geometry, dynamical systems, control and optimization, multimodal signal processing and machine learning. We begin our presentation by summarizing elementary max/min-plus morphological operators such as Minkowski shape and image operators and providing elementary concepts from tropical geometry. Further, we show how vision scale-space PDEs such as the Gaussian scale-space through Maslov Dequantization become morphological dilation/erosion scale-space PDEs. Then, we extend the underlying max-plus algebra to a max-* algebra where matrix operations and signal convolutions are generalized using a (max,*) arithmetic with an arbitrary binary operation "*" that distributes over max. This theory is based on complete weighted lattices and allows for both finite- and infinite-dimensional spaces. We outline applications of dynamical systems and geometry on weighted lattices including min-plus systems for distance transforms, max-product systems for tracking salient events in multimodal videos, max-fuzzy-norms systems, and max/min-plus perceptrons for machine learning. Finally, we outline the optimal solution of systems of max-* equations using weighted lattice adjunctions and projections, possibly with sparsity constraints, and show how it applies to optimal regression for fitting max-* tropical curves on data.

3.19 Towards polarisation in the wild: when the illumination makes the difference

Roberto Mecca (Italian Institute of Technology – Genova, IT)

In this talk I present recent results regarding the shape from polarisation problem where a new differential formulation has been derived. It consists of a homogeneous linear partial differential equation that provides the level-set of the surface for the diffuse polarisation phenomena. This new formulation allows to derive geometrical hints of the 3D shape for specular polarisation as well since the shift in the phase angle brings the level-set orthogonally to be shifted with respect to the diffuse case. The interchangeability of the proposed differential model between diffuse and specular reflection allows a step forward for studying the mixed polarisation problem.

The ability to extract level-set from dielectric objects using polarimetric images has been tested using real world data with different type of illuminations. The capability of this method to discern diffuse from specular reflections shows the importance of the illumination.

3.20 Geo'metric' Learning: Deep Isometric Manifold Learning Using Sparse Geodesic Sampling

Gautam Pai (Technion – Haifa, IL)

We explore a fully unsupervised deep learning approach for computing distance-preserving maps that generate low-dimensional embeddings for a certain class of manifolds. We use the Siamese configuration to train a neural network to solve the problem of least squares multidimensional scaling for generating maps that approximately preserve geodesic distances. By training with only a few landmarks, we show a significantly improved local and non-local generalization of the isometric mapping as compared to analogous non-parametric counterparts. Importantly, the combination of a deep-learning framework with a multidimensional scaling objective enables a numerical analysis of network architectures to aid in understanding their representation power. This provides a geometric perspective to the generalizability of deep learning.

3.21 Morphological Adjustments for Provably Robust Machine Learning

Frank R. Schmidt (Robert Bosch GmbH – Stuttgart, DE)

In recent years, deep learning has become the state of the art in machine learning for image classification. Trained on a specific dataset, it can usually generalize to images from the same dataset that were not observed during training. Nonetheless, it suffers severely from an artefact that is known as “adversarial perturbation”, i.e., small perturbation of correctly classified images will be classified incorrectly. As long as adversarial examples are possible, deep networks cannot be used for products that are expected to make autonomous decisions in critical situations.

We will show how one can certify those elements of a dataset that do not suffer from these adversarial examples. To this end, each data point is augmented via a morphological operation. The resulting set is tracked through a network. In order to certify that no element of this set is misclassified, one has to solve an ILP. We show that a specific relaxation leads to reliable certificates and that this certification approach can be used in order to make deep networks more robust.

3.22 Principal geodesic analysis in shell space

William Smith (University of York, GB)

Joint work of William Smith, B. Heeren, C. Zhang, M. Rumpf

Important sources of shape variability, such as articulated motion of body models or soft tissue dynamics, are highly nonlinear and are usually superposed on top of rigid body motion which must be factored out. We propose a novel, nonlinear, rigid body motion invariant Principal Geodesic Analysis (PGA) that allows us to analyse this variability, compress large variations based on statistical shape analysis and fit a model to measurements. For given input shape data sets we show how to compute a low dimensional approximating submanifold on the space of discrete shells, making our approach a hybrid between a physical and statistical model. General discrete shells can be projected onto the submanifold and sparsely represented by a small set of coefficients. We demonstrate two specific applications: model-constrained mesh editing and reconstruction of a dense animated mesh from sparse motion capture markers using the statistical knowledge as a prior

3.23 Analysis of Planar Ornaments Within and Beyond Symmetry Groups

Sibel Tari (Middle East Technical University – Ankara, TR)

The scientific study of the ornaments constructed by repeating a base motif is the study of the repetition structure, i.e. symmetry. Planar case where only four geometric operations (translation, rotation, mirror reflection and glide reflection) are permitted has been thoroughly studied; the repetition structure in planar case could take one of the seventeen forms. In this talk, using examples from Islamic art and Escher, we argue that the artistic side of the problem is as interesting as the mechanical repetition structure, hence requires scientific inquiry. Specifically, intriguing color permutations, clever choices of asymmetric interlocking forms, hyper-symmetries, symmetry breaking ideas, all that come with the artistic freedom, make patterns appear more symmetric than they really are. After going through examples, we suggest a change of perspective when analyzing ornaments and present custom designed ornament dataset.

3.24 Differential approaches to Shape-from-X Problems

Silvia Tozza (Sapienza University of Rome, IT)

In this talk I will give a small overview of differential approaches used to solve problems belonging to the Shape-from-X class, limiting the presentation to the orthographic projection case. More in details, I will start from the classical orthographic Shape-from-Shading (SfS) problem with a single input image, explaining why it is an ill-posed problem and which are the possible directions to arrive to a well-posed problem. Hence, I will move to the Photometric stereo SfS problem firstly with two images plus boundary conditions and secondly with three input images in order to avoid the need of boundary conditions. Finally, I will talk about the Shape-from-Polarisation problem by showing how combine shading and polarisation information in order to directly reconstruct the surface height. We are able to linearise the constraints involved, arriving to a new unified PDE formulation of several proposed methods with different nice properties (e.g. albedo invariant or phase angle invariant).

3.25 Medians and related measures for multidimensional data

Martin Welk (UMIT – Hall in Tirol, AT)

Median filtering is a simple and robust procedure for e.g. denoising and aggregating 1D real data. Various generalisations to higher-dimensional data space have been proposed which differ in their theoretical properties such as degree of equivariance w.r.t. different transformation groups, behaviour when the data aggregate near hyperplanes etc., but differ also in their computational expense. In this talk important multidimensional median concepts are juxtaposed: the L^1 median, the transformation–retransformation L^1 median, Oja's simplex median, the half-space median and the convex-hull-stripping median. They are compared regarding their use in filtering images, in filtering geometric data in $\mathbb{R}^d$ and on Riemannian manifolds. Relations to PDE filters for images are discussed. Based on experimental evidence an interesting relation between the convex-hull-stripping median and a PDE well-known in shape analysis is shown.

3.26 Estimation of Laminar Coordinate Systems between Two Surfaces

Laurent Younes (Johns Hopkins University – Baltimore, US)

Given two disjoint open surfaces, we discuss the problem of estimating a parametrization $(x, y, z) \to F(x, y, z)$ of an open subset of the three-dimensional space such that the set $[z = 0]$ corresponds to one of the surfaces, $[z = 1]$ to the other one, and for each t, $(x, y) \to F(x, y, t)$ is an embedding onto the surface $[z = t]$, with the additional constraint that the derivative of F with respect to t is always normal to this surface.

The resulting construction is a natural way to build a foliation between the two surfaces for which the length of the transverse lines provides a good definition of thickness of the considered volume. It is designed with the objective of analyzing cortical volumes, for which a robust definition of thickness is essential for the characterization of degenerative diseases. Our estimation algorithm uses a version of the large deformation diffeomorphic metric mapping between surfaces in which normality constraints of the velocity field are enforced.

The estimated coordinate system can also be used to estimate "cortical layers" as laminar segmentations of the inter-surface region that satisfy a local equi-volumetric condition called Bok's hypothesis. Experimental results on human and feline brains will be presented as illustrations.

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