A Manual Comparison of Convex Hull Algorithms

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Abstract

We have verified experimentally that there is at least one point set on which Andrew's algorithm (based on Graham's scan) to compute the convex hull of a set of points in the plane is significantly faster than a brute-force approach, thus supporting existing theoretical analysis with practical evidence. Specifically, we determined that executing Andrew's algorithm on the point set

$$P = \{(1,4),(2,8),(3,10),(4,1),(5,7),(6,3),(7,9),(8,5),(9,2),(10,6)\}$$

takes 41 minutes and 18 seconds; the brute-force approach takes 3 hours, 49 minutes, and 5 seconds.

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1 Introduction

Computational efficiency lies at the heart of the research area of computational geometry, and more broadly at that of algorithm design. As we explain in every single paper, it is of vital importance, since data sets are growing ever larger, and advancing hardware cannot compete with the pitiless mathematics of asymptotic behaviour. The same growth in both data size and computation speed is also making the notion of efficiency more abstract, and more difficult to explain to inhabitants of a world in which instant computation is the norm.

In an attempt to make the concept of efficiency more tangible by scaling both the size of the data set and the speed of the computation to something more human, we report on a manual execution of two different algorithms to compute the convex hull of the set

$$P = \{(1,4),(2,8),(3,10),(4,1),(5,7),(6,3),(7,9),(8,5),(9,2),(10,6)\} \quad \text{(Figure 1)}$$

of ten points in $\mathbb{R}^2$. The execution has been recorded and timed.

Figure 1 Left: the set P of 10 points in the plane. Right: the convex hull of P.

The experiment is inspired by the introductory chapter of the well-known computational geometry textbook of de Berg *et al.* [2], which uses the convex hull problem to illustrate the value of efficient algorithms by comparing a brute-force algorithm which runs in $O(n^3)$ time to the $O(n \log n)$ time algorithm by Andrew [1], which is in turn based on Graham's scan algorithm [3].

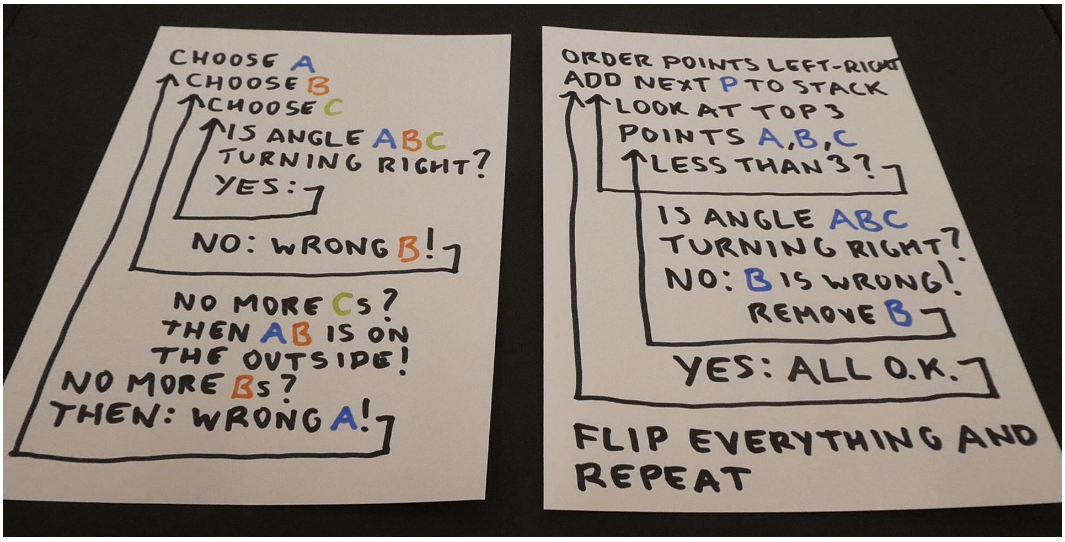

Figure 2 Pseudocode of the two algorithms. Left: a cubic-time brute-force algorithm. Right: Adrew's adaptation of Graham's scan algorithm.

2 Implementation

We chose two different algorithms to test: a fairly straightforward cubic-time algorithm, and the algorithm of Andrew [1]. We made some minor adaptations out of practical considerations. Pseudocode of the two algorithms is provided in Figure 2.

We implemented the algorithms using light cardboard points and semi-transparant tape for the edges (or potential edges) of the hull. Points are stored in standard postal envelopes, and geometric predicates are tested by (temporarily) embedding the points physically on a 30-centimeter unit grid and sticking tape edges onto the grid. As coordinates range from 1 to 10, this causes some fluctuation in the time required to perform an operation, depending on the walking distance to the respective point.

We expect there is additional bias in the results, due to several environmental factors. In particular, over the course of the execution of the brute-force algorithm, there is a measurable increase in the number of operations per minute.

3 Results

The execution of the brute-force algorithm took 3 hours, 49 minutes, and 5 seconds. The execution of Andrew's algorithm took 41 minutes and 18 seconds.

References

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