

New Horizons in Parameterized Complexity

Edited by

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 19041 “New Horizons in Parameterized Complexity”.

Parameterized Complexity is celebrating its 30th birthday in 2019. In these three decades, there has been tremendous progress in developing the area. The central vision of Parameterized Complexity through all these years has been to provide the algorithmic and complexity-theoretic toolkit for studying multivariate algorithmics in different disciplines and subfields of Computer Science. These tools are universal as they did not only help in the development of the core of Parameterized Complexity, but also led to its success in other subfields of Computer Science such as Approximation Algorithms, Computational Social Choice, Computational Geometry, problems solvable in P (polynomial time), to name a few.

In the last few years, we have witnessed several exciting developments of new parameterized techniques and tools in the following subfields of Computer Science and Optimization: Mathematical Programming, Computational Linear Algebra, Computational Counting, Derandomization, and Approximation Algorithms. The main objective of the seminar was to initiate the discussion on which of the recent domain-specific algorithms and complexity advances can become useful in other domains.

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Edited in cooperation with Roohani Sharma

1 Executive Summary

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In 2019 the parameterized complexity (PC) community is celebrating two round dates: 30 years since the appearance of the paper of Abrahamson, Ellis, Fellows, and Mata in FOCS 1989, which can be considered as the starting point of PC, and 20 years since the appearance of the influential book of Downey and Fellows “*Parameterized Complexity*”.



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Editors: Fedor V. Fomin, Dániel Marx, Saket Saurabh, and Meirav Zehavi



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In these three decades, there has been tremendous progress in developing the area. The central vision of Parameterized Complexity through all these years has been to provide the algorithmic and complexity-theoretic toolkit for studying multivariate algorithmics in different disciplines and subfields of Computer Science. To achieve this vision, several algorithmic and complexity theoretic tools such as polynomial time preprocessing, aka kernelization, color-coding, graph-decompositions, parameterized integer programming, iterative compression, or lower bounds methods based on assumptions stronger than $P=NP$ have been developed. These tools are universal as they did not only help in the development of the core of Parameterized Complexity, but also led to its success in other subfields of Computer Science such as Approximation Algorithms, Computational Social Choice, Computational Geometry, problems solvable in P (polynomial time) to name a few.

All cross-discipline developments result in flow of ideas and methods in both directions. In the last few years, we have witnessed several exciting developments of new parameterized techniques and tools in the following subfields of Computer Science and Optimization: Mathematical Programming, Computational Linear Algebra, Computational Counting, Derandomization, and Approximation Algorithms. A natural question is whether these domain-centric methods and tools are universal. That is, can they permeate boundaries of subfields and be employed wherever Parameterized Complexity approach can be used? The main objective of the seminar was to initiate the discussion on which of the recent domain-specific algorithms and complexity advances can become useful in other domains.

The seminar collected 46 participants from 18 countries. The participants presented their recent results in 26 invited and contributed talks. Open problems were discussed in open problem and discussion sessions.

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3 Overview of Talks

3.1 Polynomial Kernel for Interval Vertex Deletion

Akanksha Agrawal (*Hungarian Academy of Sciences – Budapest, HU*)

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Joint work of Akanksha Agrawal, Daniel Lokshantov, Pranabendu Misra, Saket Saurabh, Meirav Zehavi

Given a graph G and an integer k , the Interval Vertex Deletion (IVD) problem asks whether there exists a vertex subset S of size at most k , such that $G-S$ is an interval graph. The existence of a polynomial kernel for IVD remained a well-known open problem in Parameterized Complexity. In this talk we look at a sketch of polynomial kernel for the problem (with the parameter being the solution size). Over the course of talk, we will mainly focus on a kernel for IVD, when parameterized by the vertex cover number. The ideas in discussed in the above kernel is one of the key ingredients in our kernel for IVD, when parameterized by the solution size.

3.2 FPT inspired Approximation Algorithms

Henning Fernau (*Universität Trier, DE*)

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Approximation algorithms predate parameterized algorithms by quite some time. Therefore, several algorithmic ideas have been transferred from approximation to FPT. However, there are also opportunities to translate typical FPT ideas into algorithmic ideas for approximation. We will showcase this by looking at data reductions. One of the nice features that come with using approximative data reductions is that the approximation algorithm can monitor itself during execution, thereby proving that the actual approximation ratio is (possibly far) better than the typical worst-case analysis would show. We will also present experimental results that prove that this approach could work very well in practice.

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3.3 On the Parameterized Complexity of Graph Modification to First-Order Logic Properties

Petr A. Golovach (*University of Bergen, NO*)

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Joint work of Petr A. Golovach, Fedor V. Fomin, Dimitrios M. Thilikos

Main reference Fedor V. Fomin, Petr A. Golovach, Dimitrios M. Thilikos: “On the Parameterized Complexity of Graph Modification to First-Order Logic Properties”, CoRR, Vol. abs/1805.04375, 2018.

URL <https://arxiv.org/abs/1805.04375>

We establish new connections between parameterized/kernelization complexity of graph modification problems and expressibility in logic. For a first-order logic formula φ , we consider

the problem of deciding whether an input graph can be modified by removing/adding at most k vertices/edges such that the resulting modification has the property expressible by φ . We provide sufficient and necessary conditions on the structure of the prefix of φ specifying when the corresponding graph modification problem is fixed-parameter tractable (parameterized by k) and when it admits a polynomial kernel.

3.4 Parameterized Resiliency Problems via ILP

Gregory Gutin (Royal Holloway, University of London, GB)

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Joint work of Jason Crampton, Gregory Gutin, Martin Koutecký, Remi Watrigant

Main reference Jason Crampton, Gregory Z. Gutin, Rémi Watrigant: “An Approach to Parameterized Resiliency Problems Using Integer Linear Programming”, CoRR, Vol. abs/1605.08738, 2016.

URL <https://arxiv.org/abs/1605.08738>

We introduce an extension of decision problems called resiliency problems. In resiliency problems, the goal is to decide whether an instance remains positive after any (appropriately defined) perturbation has been applied to it. To tackle these kinds of problems, some of which might be of practical interest, we introduce a notion of resiliency for Integer Linear Programs (ILP) and show how to use a result of Eisenbrand and Shmonin (Math. Oper. Res., 2008) on Parametric Linear Programming to prove that ILP Resiliency is fixed-parameter tractable (FPT) under a certain parameterization.

To demonstrate the utility of our result, we consider natural resiliency versions of several concrete problems, and prove that they are FPT under natural parameterizations. Our first results concern a four-variate problem which generalizes the Disjoint Set Cover problem and which is of interest in access control. We obtain a complete parameterized complexity classification for every possible combination of the parameters. Then, we introduce and study a resiliency version of the Closest String problem, for which we extend an FPT result of Gramm et al. (Algorithmica, 2003). We also consider problems in the fields of scheduling and social choice. We believe that many other problems can be tackled by our framework.

3.5 0/1/all CSPs, Half-Integral A-path Packing, and Linear-Time FPT Algorithms

Yoichi Iwata (National Institute of Informatics – Tokyo, JP)

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Joint work of Yoichi Iwata, Yutaro Yamaguchi, Yuichi Yoshida

Main reference Yoichi Iwata, Yutaro Yamaguchi, Yuichi Yoshida: “0/1/All CSPs, Half-Integral A-Path Packing, and Linear-Time FPT Algorithms”, in Proc. of the 59th IEEE Annual Symposium on Foundations of Computer Science, FOCS 2018, Paris, France, October 7-9, 2018, pp. 462–473, IEEE Computer Society, 2018.

URL <https://doi.org/10.1109/FOCS.2018.00051>

0/1/all CSPs can be solved in linear time by a simple DFS called a unit propagation. We consider an optimization variant of the CSPs where the objective is to delete the minimum subset of variables to make the given instance satisfiable. When the instance is unsatisfiable, the unit propagation finds a walk leading to a contradiction, and the size of the maximum half-integral packing of such walks gives a lower bound on the solution size. We provide an

$O(km)$ -time algorithm for computing the maximum half-integral packing, where k is the size of the packing and m is the number of constraints, and we show that a branch-and-bound method using this lower bound can solve the problem in linear FPT time. We also discuss several other applications.

3.6 Computing the Chromatic Number Using Graph Decompositions via Matrix Rank

Bart Jansen (TU Eindhoven, NL)

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Joint work of Bart M. P. Jansen, Jesper Nederlof

Main reference Bart M. P. Jansen, Jesper Nederlof: “Computing the Chromatic Number Using Graph Decompositions via Matrix Rank”, in Proc. of the 26th Annual European Symposium on Algorithms, ESA 2018, August 20-22, 2018, Helsinki, Finland, LIPIcs, Vol. 112, pp. 47:1–47:15, Schloss Dagstuhl – Leibniz-Zentrum fuer Informatik, 2018.

URL <http://dx.doi.org/10.4230/LIPIcs.ESA.2018.47>

Computing the smallest number q such that the vertices of a given graph can be properly q -colored is one of the oldest and most fundamental problems in combinatorial optimization. The q -Coloring problem has been studied intensively using the framework of parameterized algorithms, resulting in a very good understanding of the best-possible algorithms for several parameterizations based on the structure of the graph. For example, algorithms are known to solve the problem on graphs of treewidth tw in time $O^*(q^{tw})$, while a running time of $O^*((q - \epsilon)^{tw})$ is impossible assuming the Strong Exponential Time Hypothesis (SETH). While there is an abundance of work for parameterizations based on decompositions of the graph by vertex separators, almost nothing is known about parameterizations based on edge separators. We fill this gap by studying q -Coloring parameterized by cutwidth, and parameterized by pathwidth in bounded-degree graphs. Our research uncovers interesting new ways to exploit small edge separators.

We present two algorithms for q -Coloring parameterized by cutwidth $cutw$: a deterministic one that runs in time $O^*(2^{\omega \cdot cutw})$, where ω is the matrix multiplication constant, and a randomized one with runtime $O^*(2^{cutw})$. In sharp contrast to earlier work, the running time is independent of q . The dependence on cutwidth is optimal: we prove that even 3-Coloring cannot be solved in $O^*((2 - \epsilon)^{cutw})$ time assuming SETH. Our algorithms rely on a new rank bound for a matrix that describes compatible colorings. Combined with a simple communication protocol for evaluating a product of two polynomials, this also yields an $O^*((\lfloor d/2 \rfloor + 1)^{pw})$ time randomized algorithm for q -Coloring on graphs of pathwidth pw and maximum degree d . Such a runtime was first obtained by Bjorklund, but only for graphs with few proper colorings. We also prove that this result is optimal in the sense that no $O^*((\lfloor d/2 \rfloor + 1 - \epsilon)^{pw})$ -time algorithm exists assuming SETH.

3.7 N-fold IP: FPT algorithm and applications

Martin Koutecký (Technion – Haifa, IL)

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Joint work of Dušan Knop, Martin Koutecký, Asaf Levin, Shmuel Onn

Main reference Martin Koutecký, Asaf Levin, Shmuel Onn: “A Parameterized Strongly Polynomial Algorithm for Block Structured Integer Programs”, in Proc. of the 45th International Colloquium on Automata, Languages, and Programming, ICALP 2018, July 9-13, 2018, Prague, Czech Republic, LIPIcs, Vol. 107, pp. 85:1–85:14, Schloss Dagstuhl – Leibniz-Zentrum fuer Informatik, 2018.

URL <http://dx.doi.org/10.4230/LIPIcs.ICALP.2018.85>

Integer Linear Programming is a fundamental optimization problem. Basic FPT results about ILP have been shown in the 80’s by Papadimitriou and Lenstra, and Lenstra’s algorithm has been applied extensively in parameterized complexity since 2003. A new class of IPs of variable dimension called n -fold IPs has been extensively studied since the 2000’s, culminating in an FPT algorithm in 2013, which has been used in parameterized complexity for the first time in 2016. Since then, several important applications as well as extensions and improvements of this algorithm have been found.

In this talk I will define n -fold IPs, briefly overview the FPT algorithm solving it, and then focus on two classes of application. The first class concerns CLOSEST STRING-type problems and BRIBERY-type problems, for which the application of n -fold IP has led to the first single-exponential algorithms. The second class concerns problems from scheduling, where n -fold IP is the only known technique which yields FPT results for several fundamental problems such as minimization of sum of weighted completion times, or makespan minimization when machines have many different speeds. Finally, I will point out what I believe to be the two most important open problems in the area.

3.8 Parameterized Inapproximability: A (Semi-)Survey

Pasin Manurangsi (University of California – Berkeley, US)

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Joint work of Parinya Chalermsook, Marek Cygan, Karthik C. S., Guy Kortsarz, Bundit Laekhanukit, Pasin Manurangsi, Danupon Nanongkai, Luca Trevisan

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URL <https://doi.org/10.1109/FOCS.2017.74>

In this talk, I will survey some of the recent results on parameterized inapproximability, with focus on the total inapproximability of k -Dominating Set and k -Clique.

3.9 Decompositions of Unit Disk Graphs and Algorithmic Applications

Meirav Zehavi (Ben Gurion University – Beer Sheva, IL)

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URL <https://doi.org/10.1137/1.9781611975482.64>

In this talk, I will discuss decompositions of unit disk graphs with applications in the design of subexponential and exponential time parameterized algorithms.

3.10 Hitting Long Directed Cycles is Fixed-Parameter Tractable

Matthias Mnich (Universität Bonn, DE)

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Joint work of Alexander Göke, Dániel Marx, Matthias Mnich

The Directed Feedback Vertex Set (DFVS) problem takes as input a directed graph G and seeks a minimum-size vertex set S that hits all cycles in G ; this is one of Karp’s 21 NP-complete problems. Resolving the parameterized complexity status of the DFVS problem was a long-standing open problem until Chen et al. (STOC 2008, *J.ACM* 2008) showed its fixed-parameter tractability via a $4^k k! n^{O(1)}$ -time algorithm, where $k = |S|$. We give consider the wide generalization of the DFVS problem where we want to intersect/long/ directed cycles: find a minimum-size set S of arcs or vertices such that every simple directed cycle of $G - S$ has length at most ℓ . Our main result is an algorithm which solves this problem in time $2^{O(\ell^{3k^3 \log k} + k^{6 \log k \log \ell})} n^{O(1)}$. Our algorithm therefore provides an exact version of the Erdős-Pósa property for long cycles in directed graphs, which was recently proved by Kreutzer and Kawarabayashi [STOC 2015].

3.11 New Algorithms for Planar Subgraph Isomorphism

Jesper Nederlof (TU Eindhoven, NL)

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Joint work of Jesper Nederlof

Main reference Jesper Nederlof: “Detecting and Counting Small Patterns in Planar Graphs”, *CoRR*, arXiv:1904.11285v1 [cs.DS], 2019.

URL <https://arxiv.org/abs/1904.11285v1>

We present sub exponential time algorithms for finding and counting (induced) patterns in planar graphs.

3.12 Integer Programming in Parameter-Tractable Strongly-Polynomial Time

Shmuel Onn (Technion – Haifa, IL)

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Main reference Martin Koutecký, Asaf Levin, Shmuel Onn: “A Parameterized Strongly Polynomial Algorithm for Block Structured Integer Programs”, in Proc. of the 45th International Colloquium on Automata, Languages, and Programming, ICALP 2018, July 9-13, 2018, Prague, Czech Republic, LIPIcs, Vol. 107, pp. 85:1–85:14, Schloss Dagstuhl – Leibniz-Zentrum fuer Informatik, 2018.

URL <http://dx.doi.org/10.4230/LIPIcs.ICALP.2018.85>

Integer programming has been a powerful tool in classical combinatorial optimization due to its broad modeling power.

We establish a new fundamental FPT result on integer programming and hope it will provide a new tool that may allow to establish new FPT results for a variety of combinatorial optimization problems. We will be happy to learn of any such progress that may occur.

The result, which extends, improves, unifies and simplifies many results of the last decade, is the following.

Theorem: Integer programming can be solved in fixed parameter-tractable strongly-polynomial time $f(a,d)\text{poly}(n)$, for some polynomial of the number n of variables, and some function f of the maximum absolute value a of any entry of the matrix A defining the integer program and the minimum d between the treedepth of A and the treedepth of its transpose.

The slides of the talk are available below and on my homepage.

3.13 Approximation Schemes for Low-Rank Binary Matrix Approximation Problems

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Joint work of Fedor V. Fomin, Petr A. Golovach, Daniel Lokshtanov, Fahad Panolan, Saket Saurabh

Main reference Fedor V. Fomin, Petr A. Golovach, Daniel Lokshtanov, Fahad Panolan, Saket Saurabh: “Approximation Schemes for Low-Rank Binary Matrix Approximation Problems”, CoRR, Vol. abs/1807.07156, 2018.

URL <https://arxiv.org/abs/1807.07156>

We provide a randomized linear time approximation scheme for a generic problem about clustering of binary vectors subject to additional constraints. The new constrained clustering problem encompasses a number of problems and by solving it, we obtain the first linear time-approximation schemes for a number of well-studied fundamental problems concerning clustering of binary vectors and low-rank approximation of binary matrices. Our algorithm runs in time $f(k, \epsilon) \cdot n \cdot m$, where f is some computable function, k is the number of clusters, n is the number of binary vectors in the input and m is the dimension of these vectors.

3.14 On Subexponential Parameterized Algorithms for Steiner Tree and Directed Subset TSP on Planar Graphs

Marcin Pilipczuk (University of Warsaw, PL)

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Joint work of Dániel Marx, Marcin Pilipczuk, Michał Pilipczuk

Main reference Dániel Marx, Marcin Pilipczuk, Michał Pilipczuk: “On Subexponential Parameterized Algorithms for Steiner Tree and Directed Subset TSP on Planar Graphs”, in Proc. of the 59th IEEE Annual Symposium on Foundations of Computer Science, FOCS 2018, Paris, France, October 7-9, 2018, pp. 474–484, IEEE Computer Society, 2018.

URL <https://doi.org/10.1109/FOCS.2018.00052>

There are numerous examples of the so-called “square root phenomenon” in the field of parameterized algorithms: many of the most fundamental graph problems, parameterized by some natural parameter k , become significantly simpler when restricted to planar graphs and in particular the best possible running time is exponential in $\mathcal{O}(\sqrt{k})$ instead of $\mathcal{O}(k)$ (modulo standard complexity assumptions). We consider two classic optimization problems parameterized by the number of terminals. The STEINER TREE problem asks for a minimum-weight subtree connecting a given set of terminals T in an edge-weighted graph. In the SUBSET TRAVELING SALESMAN problem we are asked to visit all the terminals T by a minimum-weight closed walk. We investigate the parameterized complexity of these problems in planar graphs, where the number $k = |T|$ of terminals is regarded as the parameter. Our results are the following:

- SUBSET TSP can be solved in time $2^{\mathcal{O}(\sqrt{k} \log k)} \cdot n^{\mathcal{O}(1)}$ even on edge-weighted directed planar graphs. This improves upon the algorithm of Klein and Marx [SODA 2014] with the same running time that worked only on undirected planar graphs with polynomially large integer weights.
- Assuming the Exponential-Time Hypothesis, STEINER TREE on undirected planar graphs cannot be solved in time $2^{o(k)} \cdot n^{\mathcal{O}(1)}$, even in the unit-weight setting. This lower bound makes STEINER TREE the first “genuinely planar” problem (i.e., where the input is only planar graph with a set of distinguished terminals) for which we can show that the square root phenomenon does not appear.
- STEINER TREE can be solved in time $n^{\mathcal{O}(\sqrt{k})} \cdot W$ on undirected planar graphs with maximum edge weight W . Note that this result is incomparable to the fact that the problem is known to be solvable in time $2^k \cdot n^{\mathcal{O}(1)}$ even in general graphs.

A direct corollary of the combination of our results for STEINER TREE is that this problem does not admit a parameter-preserving polynomial kernel on planar graphs unless ETH fails.

3.15 Hitting minors on bounded treewidth graphs

Ignasi Sau Valls (CNRS – Montpellier, FR)

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Joint work of Julien Baste, Ignasi Sau, Dimitrios M. Thilikos

Main reference Julien Baste, Ignasi Sau, Dimitrios M. Thilikos: “Hitting minors on bounded treewidth graphs”, CoRR, Vol. abs/1704.07284, 2017.

URL <https://arxiv.org/abs/1704.07284>

For a fixed collection of graphs \mathcal{F} , the \mathcal{F} -M-Deletion problem consists in, given a graph G and an integer k , decide whether there exists $S \subseteq V(G)$ with $|S| \leq k$ such that $G \setminus S$

does not contain any of the graphs in F as a minor. We are interested in its parameterized complexity when the parameter is the treewidth of G , denoted by tw . Our objective is to determine, for a fixed F , the smallest function f_F such that F-M-Deletion can be solved in time $f_F(tw) \cdot n^{O(1)}$ on n -vertex graphs. We prove that $f_F(tw) = 2^{2^{O(tw \cdot \log tw)}}$ for every collection F , that $f_F(tw) = 2^{O(tw \cdot \log tw)}$ if all the graphs in F are connected and at least one of them is planar, and that $f_F(tw) = 2^{O(tw)}$ if in addition the input graph G is planar or embedded in a surface. When F contains a single connected planar graph H , we obtain a tight dichotomy about the asymptotic complexity of H-M-Deletion. Namely, we prove that $f_H(tw) = 2^{\theta(tw)}$ if H is a minor of the banner (that is, the graph consisting of a C_4 plus a pendent edge) that is different from P_5 , and that $f_H(tw) = 2^{\theta(tw \cdot \log tw)}$ otherwise. All the lower bounds hold under the ETH. We also consider the version of the problem where the graphs in F are forbidden as topological minors, and prove similar results, except that, in the algorithms, instead of requiring F to contain a planar graph, we need it to contain a subcubic planar graph. We also prove that, for this problem, $f_{K_{1,i}}(tw) = 2^{\theta(tw)}$ for every $i \geq 1$, while for the minor version it holds that $f_{K_{1,i}}(tw) = 2^{\theta(tw \cdot \log tw)}$ for every $i \geq 4$.

3.16 On a polynomial kernel for Directed Feedback Vertex Set

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In the Directed Feedback Vertex Set (DFVS), the input is a directed graph D and an integer k , and the question is to determine whether there exists a set of vertices of D of size at most k whose removal makes the digraph acyclic. The problem concerning the existence of a polynomial kernel for DFVS is an interesting and challenging open problem in the field of parameterized complexity. We take a step towards answering this question by giving a polynomial kernel for DFVS with an enriched parameter. In particular, we study DFVS parameterized by the solution size (k) and the size of a treewidth- η modulator of the underlying undirected graph of D (say ℓ). In particular, we give a kernel for DFVS of size $(k + \ell)^{O(1)}$. This result also generalizes the result by Bergougnoux et al. that gives a polynomial kernel for DFVS parameterized by the feedback vertex set of the underlying undirected graph of D . As a corollary, our result implies a polynomial kernel for DFVS on instances that are $k^{O(1)}$ vertices away from having bounded treewidth.

4 Open Problems

4.1 Shortest Three Disjoint Path

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Decide the complexity of the Shortest Three Disjoint Paths problem: Given an undirected unweighted graph $G = (V, E)$ and three pairs of distinct terminal vertices (s_1, t_1) , (s_2, t_2) , and $(s_3, t_3) \in V \times V$, find three pairwise vertex disjoint paths connecting s_i with t_i for $i = 1, 2, 3$ in G , respectively, of minimum total length (the number of edges in the three paths). Already the restriction to planar graphs of maximum degree three is open. For Shortest *Two* Disjoint Paths, a randomized polynomial time algorithm is known [1], and in the planar maximum degree three case there is a deterministic polynomial time algorithm that also counts the number of solutions [2]. For Shortest Three Disjoint Paths in planar graphs a deterministic polynomial time algorithm is known when all terminals lie on the same face [3].

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4.2 Counting forests with few components

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Suppose G is an undirected graph on n vertices. Kirchhoff’s Matrix-tree Theorem expresses the number of spanning trees in G as the determinant of an $(n - 1) \times (n - 1)$ matrix, thus providing a polynomial-time algorithm for counting (exactly) the spanning trees in G . Building on this, Liu and Chow [2] gave a method to count the number of $(k + 1)$ -component (spanning) forests in G , for any $k \geq 0$. Their recursive procedure is polynomial-time for any fixed k , but the exponent grows with k . In modern terminology, they showed that counting $(k + 1)$ -component forests is in XP. My question is **whether counting $(k + 1)$ -component forests is in FPT**.

The general form of this open problem is: Start with a set of structures having some property Π , for example, Π might be the property of being a spanning tree of a graph G . Assume that there is a polynomial-time algorithm for counting structures with property Π . Now perturb the property Π to Π' , and consider the derived problem of counting structures with property Π' . Introduce a parameter k to measure the extent of the perturbation, for example, k might be the number of “missing edges” in a spanning tree. It is natural to ask whether this perturbed counting problem is in FPT or XP, or is $\#W[1]$ -hard, etc., regarded

as a problem parameterized by k . An example of a solved problem of this type is counting matchings in a planar graph with $2k$ uncovered vertices or “monomers”, which was shown to be $\#W[1]$ -hard by Curticapean [1].

Returning to spanning trees, it is natural to perturb the structures in the opposite direction and consider trees with “excess edges”, in other words, connected spanning subgraphs with $n + k - 1$ edges. (The parameterization is chosen so that spanning trees correspond again to $k = 0$). Surprisingly, despite the obvious similarity to counting forests (indeed the problems are dual in matroid theoretic terms), it is not even known whether the counting problem is in XP. Note that there is no contradiction here: the class of graphic matroids is not closed under taking duals.

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4.3 Makespan Minimization on Identical Machines ($P||C_{\max}$) by #job types

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In the Makespan Minimization on Identical Machines ($P||C_{\max}$) by #job types, we are given m identical machines, τ types of jobs, $n_j \in \mathbb{N}$ jobs of type $j \in [\tau]$, where each job of type $j \in [\tau]$ has processing time $p_j \in \mathbb{N}$, and the question is to find a schedule minimizing the makespan C_{\max} , i.e., the time when the last job finishes.

Goemans and Rothvoss [1] have shown that the problem is solvable in time roughly $\mathcal{O}^*((\log p_{\max})^{2^\tau})$, where $p_{\max} = \max_j p_j$. With respect to τ this is an XP algorithm, or an FPT algorithm if p_{\max} has size polynomial in the encoding length of the instance. It remains open **whether the problem is FPT or $W[1]$ -hard**.

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4.4 Stochastic bounding box

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Let P be a set of n points in \mathbb{R}^d and assume that each point p of P has a number $\pi(p) \in (0, 1]$ associated to it. We construct a random subset R of P where we include each point p of P with probability $\pi(p)$, where the decision for each point is made independently. We want

to compute the expected volume of the minimum axis-parallel box that contains R . In the plane this can be done in $O(n \log n)$ time [1], assuming that each arithmetic operation takes constant time. Using the 2-dimensional case as base case, one can solve the problem in $O(n^{d-1} \log n)$ time for each $d \geq 3$. The non-stochastic version can be solved in $O(dn)$ trivially. **Is the problem $W[1]$ -hard or FPT when parameterized by the dimension d ?**

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4.5 Tight bound for the number of multibudgeted important separators

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Consider a directed graph G with distinguished source $s \in V(G)$, sink $t \in V(G)$, and a partition $E(G) = E_1 \uplus E_2 \uplus \dots \uplus E_\ell$ of the arc set. A set $C \subseteq E(G)$ is an $s-t$ separator if there is no path from s to t in $G - C$. An $s-t$ separator C is a *minimal $s-t$ separator* if no proper subset of C is an $s-t$ separator. A minimal $s-t$ separator D *dominates* a minimal $s-t$ separator C if every vertex reachable from s in $G - C$ is also reachable from s in $G - D$ and for every $i \in [\ell]$ we have $|C \cap E_i| \geq |D \cap E_i|$. A minimal $s-t$ separator D is *important* if no other minimal $s-t$ separator dominates it. The classic result asserts that for the single-budget case $\ell = 1$ there are at most 4^k important separators of size at most k [3, 1].

In our recent IPEC'18 paper [2] we show a generalization of this result for multibudgeted case with a bound of $2^{\mathcal{O}(k^2 \log k)}$ for the number of multibudgeted important separators of size at most k . However, the best known lower bound is $2^{\Omega(k \log k)}$ attained via the following simple construction. Let $\ell = k$ and let G consist of k paths $(P_j)_{j=1}^k$; each path P_j starts in s , ends in t , and consists of k edges $(e_j^i)_{i=1}^k$ in this order such that $e_j^i \in E_i$ for every $i, j \in [k]$. Then any minimal $s-t$ separator in G is a multibudgeted important separator. **Please close the gap.**

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4.6 Three disjoint paths that are each shortest paths

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Let G be an edge-weighted graph and let $(s_i, t_i)_{i=1}^k$ be terminal pairs in G . The task is to find paths $(P_i)_{i=1}^k$ such that each P_i is a shortest path in G from s_i to t_i and the paths P_i are pairwise vertex-disjoint. The problem has been introduced by Eilam-Tzoref [2] who showed that it is polynomial-time solvable for $k = 2$ in undirected graphs with strictly positive edge weights. Bérczi and Kobayashi [1] generalized this result to $k = 2$ in directed graphs with strictly positive edge weights and later two independent groups [3, 4] showed also a generalization to $k = 2$ in undirected graphs with nonnegative edge weights. However, the **three terminal pair case remains widely open**, even in undirected unweighted graphs.

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4.7 k -exchange TSP parameterized by $k + d$

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In the k -exchange TSP problem, given as input an undirected graph $G = (V, E)$ with maximum degree d , a TSP tour $C \subseteq E$, and an integer k , the goal is to decide if there is a shorter TSP tour C' with $|C \setminus C'| \leq k$?

Local search with the above k -exchange neighborhoods is widely used in heuristic TSP solvers. When parameterized by k only, Marx [2] proved W[1]-hardness. In order to make the local search practical, state-of-the-art local search solvers use the following two heuristics.

1. Sparsify the graph by picking top- d *important* incident edges for each vertex. For example, LKH [1] uses α -*nearness* as the importance measure and reduces the degrees to $d = 5$.
2. Focus on *sequential moves*. A k -exchange move C' is called sequential if the symmetric difference $C \Delta C'$ forms a simple cycle. When the maximum degree is d , the exhaustive search for sequential k -moves runs in $d^{O(k)}n$ time.

If **k -exchange TSP parameterized by $k + d$** is W[1]-hard, we can justify focusing on the sequential moves, and if it is FPT, we may have practical improvements.

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4.8 Count k -Walks

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Given a graph G with n vertices and m edges, and a number k , the goal is to compute the number of all k -step walks in G . If A is the adjacency matrix of G , this number is just the sum of all entries in A^k . It can thus be computed in time $O(\log k \cdot n^\omega)$ where ω is the matrix multiplication constant. It can also be computed in time $O(k(n+m))$. The open question is: **can the problem be computed in time $o(k) \cdot (n+m)$ or even $O(n+m)$?** Or would this violate some complexity hypothesis?

4.9 Shortest Vector Problem (SVP) in ℓ_1 Norm

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Definition: Given a lattice $\mathcal{L} \subseteq \mathbb{Z}^n$ (specified by its basis), determine whether there exists a nonzero vector $\mathbf{x} \in \mathcal{L}$ whose ℓ_1 norm is at most k (i.e. $\sum_{i \in [n]} |x_i| \leq k$). Here k is our parameter. *Relevant Literature:* The non-parameterized version of the problem is very well-studied. In particular, the NP-hardness (under randomized reductions) was proved by Ajtai [1]. Later, it was shown to be hard to approximate to factor of $(2-\varepsilon)$ by Micciancio [6]. Subsequently, the factor was dramatically improved to $2^{(\log n)^{0.5-\varepsilon}}$ by Regev and Rosen [7] and then to $2^{(\log n)^{1-o(1)}}$ by Haviv and Regev [4].

The issue in adapting these proofs is that the aforementioned reductions inherently produce non-integral lattices. In particular, the lattices in [1] and [6] are irrational, whereas the lattices from [7, 4] comes from norm embeddings from ℓ_2 to ℓ_1 which, if discretize, does not result in any valuable parameter anymore.

In recent works [2, 3], it was shown that SVP in ℓ_p is $W[1]$ -hard (under randomized reductions) for all $p > 1$, but the proof fails for $p = 1$. The approach taken there was adapted from the work of Khot [5], which fails for $p = 1$ due to technical reasons. (Note that this is true even for the non-parameterized case.)

It would be nice if the **parameterized hardness of SVP** in ℓ_1 norm can be established. On the other hand, any **non-trivial positive results** would be interesting. For instance, even an **$f(k)$ -FPT-approximation** (for some function f) would be very nice, since this would also lead to a **$g(k)$ -FPT-approximation** (for some function g) for SVP in ℓ_2 norm as well.

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4.10 Polynomial kernel for Bicolored P_3 -Deletion

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In the BICOLORED P_3 -DELETION problem, given a graph $G = (V, E)$, where E is partitioned into a set E_r of red edges and a set E_b of blue edges, and an integer $k \in \mathbb{N}$, the goal is to decide whether we can delete at most k edges from G such that the remaining graph contains no bicolored P_3 as induced subgraph? Here a *bicolored P_3* is a path on three vertices with one blue and one red edge. The open question is : **Does Bicolored P_3 -Deletion parameterized by k admit a polynomial-size problem kernel?**

The reference for this is [1].

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4.11 Metric TSP with Deadlines

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In the METRIC TSP WITH DEADLINES problem, we are given an instance of METRIC TSP with n cities, out of which a subset of $k \ll n$ cities is distinguished. Additionally, an integer D is provided as input. The goal is to find a minimum-cost tour that visits all distinguished cities before the deadline D , or concludes that no such tour exists.

Böckenhauer et al. [1] showed that this problem admits a 2.5-approximation in time $f(k) \cdot n^{O(1)}$, and also proved a lower bound of 2 on the approximability of the problem by fixed-parameter algorithms under the assumption that $P \neq NP$. The open problem is to **close the gap between 2 and 2.5 on the fixed-parameter approximability of this problem.**

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4.12 Optimization over Degree Sequences

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We consider *Optimization over Degree Sequences* problem defined below. Given functions $f_1, \dots, f_n : \{0, 1, \dots, n\} \rightarrow \mathbb{R}$, find a graph G on $[n]$ maximizing $\sum_{i=1}^n f_i(d_i(G))$, where $d_i(G)$ is the degree of vertex i in G .

We know quite a little about the complexity of this problem. If all the functions are the same $f_1 = \dots = f_n = g_1$ then we can solve it in polynomial time [2]. What is the **complexity if each f_i equals one of two given functions g_1, g_2** ? What is the **(parameterized) complexity if each f_i equals one of k given functions g_1, \dots, g_k** ? What is the **(parameterized) complexity for restricted classes of functions (e.g., convex, concave, k -piecewise linear)**?

The problem can obviously be generalized to r -uniform hypergraphs. However, then some severe restrictions on the functions should be applied, since already the following decision problem is NP-complete: given d_1, \dots, d_n , is there a 3-uniform hypergraph H with $d_i(H) = d_i$ for all i ? (The NP-completeness of this is shown in [2] solving a long open problem from [1].)

We will be happy to learn of any progress on this problem that you may come up with.

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4.13 Dynamic Cluster Editing

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We consider the following “dynamic version” of a well-studied graph-based data clustering problem.

Dynamic Cluster Completion with Edge-Based Distance: In this problem the input is an undirected graph G and a cluster graph¹ G_c over the same vertex set, and two nonnegative integers, a budget k and a distance bound d . The question is to decide if there exists a cluster graph G'_c with $E(G) \subseteq E(G'_c)$ such that

- $|E(G) \oplus E(G'_c)| \leq k$ and
- $|E(G_c) \oplus E(G'_c)| \leq d$?

Herein, \oplus denotes the symmetric difference between two sets. This is a simple (still NP-hard) version of DYNAMIC CLUSTER EDITING (where adding and deleting edges from G is allowed in order to generate a cluster graph) restricted to edge additions.

¹ That is, a disjoint union of cliques.

It is open whether **Dynamic Cluster Completion with Edge-Based Distance is fixed-parameter tractable when parameterized by k** . Notably, in the conference version of Luo et al. [1] at FSTTCS 2018 DYNAMIC CLUSTER COMPLETION WITH EDGE-BASED DISTANCE was erroneously claimed to be fixed-parameter tractable for parameter k ; the proof was flawed.

DYNAMIC CLUSTER COMPLETION WITH EDGE-BASED DISTANCE is known to be fixed-parameter tractable when parameterized by d and it has a polynomial kernel when parameterized by $k + d$ [1]. Refer to Luo et al. [1] for motivation in terms of compromise clustering, local search, and target cluster graphs. Luo et al. study several variants of DYNAMIC CLUSTER EDITING, listing some further open problems.

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4.14 Resolution

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The *incidence graph* of a CNF formula F is the bipartite graph between the clauses and the variables of F , where a clause C and a variable x are adjacent if x appears negated or unnegated in C . The *primal graph* of F has as vertices the variables of F , two variables are adjacent if they appear together negated or unnegated in a clause of F . By *resolution* we can obtain from clauses $C \vee x$ and $D \vee \neg x$ the clause $C \vee D$. A *resolution refutation of F of size t* is a sequence C_1, \dots, C_t of clauses such that C_t is the empty clause and for each $i \in [t]$, either $C_i \in F$ or C_i can be obtained from C_j and C_ℓ for some $1 \leq j, \ell < i$. The open question is: **Do unsatisfiable CNF formulas have FPT-sized resolution refutations, parameterized by the treewidth of the incidence graph?**

Known results: (i) CNF formulas have FPT-sized resolution refutations, parameterized by the pathwidth of the incidence graph [1], and (ii) parameterized by the treewidth of the primal graph [3]. (iii) If F is a 3CNF formula, then it has an FPT-sized resolution refutation parameterized by the treewidth k of the incidence graph, since the treewidth of the primal graph is then at most $3k + 2$ [2]. (iv) One can transform in polynomial time a CNF formula F whose incidence graph has treewidth k into an equisatisfiable 3CNF formula F' whose primal graph has treewidth at most $3k + 3$ [4]; if F' is unsatisfiable it has an FPT-sized resolution refutation by (iii) but contains additional variables that were not in F .

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