Popular Matchings: Good, Bad, and Mixed

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Abstract

We consider the landscape of popular matchings in a bipartite graph G where every vertex has strict preferences over its neighbors. This is a very well-studied model in two-sided matching markets. A matching M is *popular* if it does not lose a head-to-head election against any matching, where each vertex casts a vote for the matching where it gets a better assignment. Roughly speaking, a popular matching is one such that there is no matching where more vertices are happier. The notion of popularity is more relaxed than *stability*: a classical notion studied for the last several decades. Popular matchings always exist in G since stable matchings always exist in a bipartite graph and every stable matching is popular.

Algorithmically speaking, the landscape of popular matching seems to have only a few bright spots. Every stable matching is a *min-size* popular matching and there are also simple linear time algorithms for computing a *max-size* popular matching and for the *popular edge* problem. All these algorithms reduce the popular matching problem to an appropriate question in stable matchings and solve the corresponding stable matching problem.

We now know NP-hardness results for many popular matching problems. These include the min-cost/max-weight popular matching problem and the problem of deciding if G admits a popular matching that is neither a min-size nor a max-size popular matching. For non-bipartite graphs, it is NP-hard to even decide if a popular matching exists or not.

A *mixed* matching is a probability distribution or a lottery over matchings. A popular mixed matching is one that never loses a head-to-head election against any mixed matching. As an allocation mechanism, a popular mixed matching has several nice properties. Moreover, finding a max-weight or min-cost popular mixed matching in G is easy (by solving a linear program). Interestingly, there is always an optimal popular mixed matching Π with a simple structure: $\Pi = \{(M_0, \frac{1}{2}), (M_1, \frac{1}{2})\}$ where M_0 and M_1 are matchings in G. Popular mixed matchings always exist in non-bipartite graphs as well and can be computed in polynomial time.

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