Petri Net Reachability Problem

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Abstract

Petri nets, also known as vector addition systems, are a long established model of concurrency with extensive applications in modelling and analysis of hardware, software and database systems, as well as chemical, biological and business processes. The central algorithmic problem for Petri nets is reachability: whether from the given initial configuration there exists a sequence of valid execution steps that reaches the given final configuration. The complexity of the problem has remained unsettled since the 1960s, and it is one of the most prominent open questions in the theory of verification. In this presentation, we overview decidability and complexity results over the last fifty years about the Petri net reachability problem.

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1 Outline

The presentation focuses on the reachability problem for vector addition systems with states given as multi-dimensional weighted automata. Main results about 1) reachability in small dimensions, 2) boundedness problems, and 3) decidability and complexity results for the reachability problem in any dimension will be overviewed.

- For small dimensions, the complexity of the reachability problem depends on the dimension and on the way weights are encoded (in unary or in binary). In dimension one, the reachability problem can be easily shown to be NL-complete when weights are written in unary thanks to a hill-cutting argument (the same argument that applies on pushdown automata). When updates are given in binary, this argument can only provide a complexity in between NP and PSPACE. Nevertheless, by using some additional arguments, the problem was proved to be NP-complete in [5]. The complexity of the reachability problem is also known in dimension two. Thanks to a precise analysis of the algorithm introduced in [12], the problem was proved to be PSPACE-complete in [1] for binary updates. This last result was extended later in [4] to show that the problem is NL-complete for unary updates. In dimension three the complexity of the reachability problem is nowadays open whatever the encoding of the weights.
- The Karp and Miller algorithm introduced in [6] is central for deciding the reachability problem in general dimension. It provides a way for computing the maximal value of a bounded counter. Notice that even if this value can be Ackermannian [16], deciding the boundedness of a counter is known to be EXPSPACE-complete [3] by extending the Rackoff's proof [17].

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- The reachability problem was first proved to be decidable by Mayr [14, 15] in 1981. This proof was improved later by Kosaraju [7] and Lambert [8] by introducing an algorithm that decompose the set of executions. We call this decomposition the KLM decomposition (following the initials of the contributed authors). In [10] we proved that the KLM decomposition can be interpreted as ideal decompositions for a natural well-quasi order on the set of executions. In that paper we also provided a cubic-Ackermaninan complexity upper-bound of the reachability problem; the very first complexity upper-bound for the reachability problem. This bound was recently improved in [11], by proving that there exists a KLM decomposition algorithm that works in time primitive recursive in fixed dimension, and at most Ackermannian in general. Concerning lower-bounds, recently in [2], the complexity of the problem was shown to be TOWER-hard, improving the best-known EXPSPACE complexity lower-bound given by Lipton [13] in 1976. Nowadays, the exact complexity of the reachability problem is still open between TOWER and ACKERMANN. In order to close that problem, either we need to improve the recent TOWER lower bound, or we need to design an algorithm improving the ACKERMANN upper bound. The very simple algorithm introduced in [9], based on Presburger inductive invariant seems to be a good candidate for that late direction.

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