

# Brief Announcement: Integrating Temporal Information to Spatial Information in a Neural Circuit

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## Abstract

In this paper, we consider networks of deterministic spiking neurons, firing synchronously at discrete times. We consider the problem of translating temporal information into spatial information in such networks, an important task that is carried out by actual brains. Specifically, we define two problems: “First Consecutive Spikes Counting” and “Total Spikes Counting”, which model temporal-coding and rate-coding aspects of temporal-to-spatial translation respectively. Assuming an upper bound of  $T$  on the length of the temporal input signal, we design two networks that solve two problems, each using  $O(\log T)$  neurons and terminating in time  $T + 1$ . We also prove that these bounds are tight.

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**Introduction.** Algorithms in the brain are inherently distributed. Although each neuron has relatively simple dynamics, as a distributed system, a network of neurons exhibits strong computational power. Recently, there has been increasing interest in using biologically plausible spiking neuronal dynamics to solve different computational problems [4, 1]. In this paper, we consider a network of spiking neurons with a deterministic synchronous firing rule operating in discrete time, similar to that of Lynch, et al. [4], in order to simplify the analysis and focus on the computational principles.

One of the most important questions in neuroscience is how humans integrate information over time. Sensory inputs such as visual and auditory stimulus are inherently temporal; however, brains are able to integrate the temporal information to a single concept, such as a moving object in a visual scene, or a word in a sentence. There are two kinds of neuronal codings: rate coding and temporal coding. Rate coding is a neural coding scheme assuming that most of the information is coded in the firing rate of the neurons. It is most commonly seen in muscle in which the higher firing rates of motor neurons correspond to higher intensity in muscle contraction. On the other hand, rate coding cannot be the only neural coding brains employ. A fly is known to react to new stimuli and change its direction of flight within 30-40 ms. There is simply not enough time for neurons to compute the firing rate which is the average of spike counts over an interval. Therefore, neuroscientists have proposed the idea of temporal coding, assuming the information is coded in the temporal firing patterns. One of the popular temporal codings is the “first-to-spike” coding. It has been shown that the timing of the first spike encodes most information of an image in retinal cells [2].



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We propose two toy problems to model how brains extract information from different codings. “First Consecutive Spikes Counting” (FCSC) counts the first consecutive interval of spikes, which is equivalent to counting the distance between the first two spikes, a prevalent temporal coding scheme in sensory cortex. “Total Spikes Counting” (TSC) counts the number of the spikes over an arbitrary interval, which is an example of rate coding.

In this paper, we design two networks that solve the above two problems in time  $T + 1$  with  $O(\log T)$  neurons. We also show that our time bounds are tight. We remark that although our problems are biologically inspired, the optimal solutions we propose are not biologically plausible. Our networks are not noise tolerant, whereas the actual neuronal dynamics are highly noisy. However, we hope that our results can demonstrate an important computational principle: unstable intermediate states can carry temporal information and then converge to a stable representation efficiently.

In other work on spiking neural networks, Hitron and Parter [3] tackled a similar problem to our TSC problem. Our results differ in three ways. First, our network has time bound  $T + 1$  while theirs is  $T + O(\log T)$ . Second, we provide a time lower bound result and show that our time bound is optimal. Third, they additionally consider an approximate version of the problem while we focus on the exact version of the problem.

**Model.** We consider a Spiking Neural Network (SNN) model with deterministic synchronous firing at discrete times. Our network structure consists of a directed graph  $(V, E)$  with bias,  $b : V \rightarrow \mathbb{R}$  and weight function,  $w : E \rightarrow \mathbb{R}$ . In this paper, we fix input neuron  $x \in V$  and  $m$  output neurons  $\{y_i\}_{0 \leq i < m} \subset V$ . The dynamics of neuron  $z \in V$  at each time step  $t \geq 1$  is governed by

$$z^{(t)} = \Theta\left(\sum_{y \in P_z} w_{yz} y^{(t-1)} - b_z\right).$$

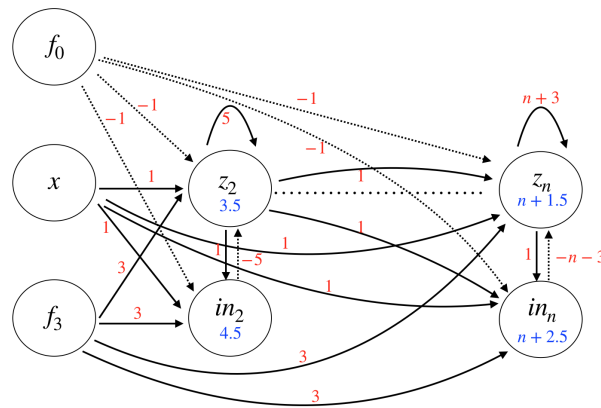
where  $z^{(t)}$  is the indicator function of neuron  $z$  firing at time  $t$ .  $P_z$  is the set of presynaptic neurons of  $z$ , and  $\Theta$  is a nonlinearity. Here we take  $\Theta$  as the Heaviside function given by  $\Theta(x) = 1$  if  $x > 0$  and 0 otherwise. At  $t = 0$ , we let  $z^{(0)} = 0$  if  $z$  is not an input neurons.

**The First Consecutive Spikes Counting(T) (FCSC(T)).** Given an input neuron  $x$  and the max input length  $T$ , we consider any input firing sequence such that for all  $t \geq T$ ,  $x^{(t)} = 0$ . Define  $L_x$  to be the length of the first consecutive spikes interval. Then we say a network of neurons solves FCSC(T) in time  $t'$  with  $m'$  neurons if there exists an injective function  $F : \{0, \dots, T\} \rightarrow \{0, 1\}^m$  which serves as an encoding of the count such that for all  $x$  and for all  $t, t' \geq t'$  we have  $y^{(t)} = F(L_x)$  and the network has  $m'$  total neurons.

Intuitively, FCSC serves as a toy model for encoding distance between spikes, a prevalent temporal coding in sensory cortex. For mathematical convenience, we model the problem as counting the distance between non-spikes which is mathematically equivalent to counting the distance between spikes in our model.

**The Total Spikes Counting(T) (TSC(T)).** Given an input neuron  $x$  and the max input length  $T$ , we consider any input firing sequence such that for all  $t \geq T$ ,  $x^{(t)} = 0$ . Define  $L_x$  to be the total number of spikes in the sequence. Then we say a network of neurons solves TSC(T) in time  $t'$  with  $m'$  neurons if there exists an injective function  $F : \{0, \dots, n\} \rightarrow \{0, 1\}^m$  which serves as an encoding of the count such that for all  $x$  and for all  $t, t' \geq t'$  we have  $y^{(t)} = F(L_x)$  and the network has  $m'$  total neurons.

Intuitively, TSC serves as a toy model for rate coding implemented by spiking neural networks, because the network is able to extract the rate information by counting the number of spikes over arbitrary intervals.



■ **Figure 1** Modified Binary Counter Part of the TSC Network.

**Results.** Our contributions in the paper are to design networks that solve these two problems respectively with matching lower bounds in terms of the number of neurons.

► **Theorem 1.** *There exist networks with  $O(\log T)$  neurons that solve the FCSC( $T$ ) and TSC( $T$ ) problems, both in time  $T + 1$ .*

Our FCSC network does binary counting by precomputing simultaneous carrying and then captures the counter into persistent neurons. TSC network has further complications because it requires the network to count spikes regardless of intervening non-spikes. In particular, TSC presents an interesting difficulty: there are conflicting objectives between maintaining the count when no spike arrives and updating the count when a spike arrives. To overcome this difficulty, we allow the network to enter an unstable intermediate state which carries the information of the count. The intermediate state then converges to a stable state that represents the count after a computation step without inputs. To be concrete, our TSC network contains a modified binary counter (Figure 1) which does delayed simultaneous carrying but we need to handle the subtle behaviors of delay carefully in our design. Our time lower bound result shows that this delay is indeed necessary.

► **Theorem 2.** *There is no network with  $o(T)$  neurons that solves FCSC( $t$ ) problem in time  $t$  for all  $0 \leq t \leq T$ . The same holds for the TSC problem.*

Intuitively, the proof of the time lower bound uses the fact that if the network has to solve the problem without delay, then it must stabilize immediately at each time step. Therefore, the neurons that fire at the last round will continue firing. By injectivity of the representation, we can conclude that the network can at most count up to the network size.

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