Hybrid SAT-Based Consistency Checking Algorithms for Simple Temporal Networks with Decisions

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Abstract

A Simple Temporal Network (STN) consists of time points modeling temporal events and constraints modeling the minimal and maximal temporal distance between them. A Simple Temporal Network with Decisions (STND) extends an STN by adding *decision time points* to model temporal plans with decisions. A decision time point is a special kind of time point that once executed allows for deciding a truth value for an associated Boolean proposition. Furthermore, STNDs label time points and constraints by conjunctions of literals saying for which *scenarios* (i.e., complete truth value assignments to the propositions) they are relevant. Thus, an STND models a family of STNs each obtained as a *projection* of the initial STND onto a *scenario*. An STND is consistent if there exists a consistent scenario (i.e., a scenario such that the corresponding STN projection is consistent). Recently, a hybrid SAT-based consistency checking algorithm (HSCC) was proposed to check the consistency of an STND. Unfortunately, that approach lacks experimental evaluation and does not allow for the synthesis of all consistent scenarios. In this paper, we propose an incremental HSCC algorithm for STNDs that (i) is faster than the previous one and (ii) allows for the synthesis of all consistent scenarios and related early execution schedules (offline temporal planning). Then, we carry out an experimental evaluation with KAPPA, a tool that we developed for STNDs. Finally, we prove that STNDs and disjunctive temporal networks (DTNs) are equivalent.

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Supplement Material `https://github.com/matteozavatteri/kappa`

1 Introduction

Context and motivations. Temporal networks are a possible formalism to model temporal plans and check the coherence of temporal constraints that impose lower and upper bounds on the temporal distance of the modeled events. A *Simple Temporal Network* (*STN*, [11]) is a formalism able to model an unconditional temporal plan in which all components (events

and their time distances) are under control. Temporal events are modeled as *time points* and their occurrence is modeled by executing the corresponding time points (i.e., by assigning them real values).

A temporal plan is *consistent* if we can schedule all the events, each at a specific time instant, such that all constraints are satisfied. If this is possible, a schedule can be synthesized for both offline planning (the plan is available before starting) or online planning (the plan is generated while executing). The difference between these two planning approaches is that in the former the consistency checking algorithm returns a solution, whereas in the latter the algorithm returns a *minimal network* to generate any solution.

However, STNs fail to model temporal plans where the occurrence of some events or the satisfaction of some constraints must happen only if some decision has been made. The decisions we are interested in have Boolean domain[1]. Thus, a temporal plan subject to decisions would ask us (not) to execute some time points or to satisfy some constraints depending on what decisions we have made.

A few proposals to handle decisions within the temporal network formalisms built on top of STNs have been put forth. For instance, *Drake* [9] provides an executive for temporal plans with choices based on *Labeled STNs* that do not specify decision points (in a node-sense), whereas *Temporal Plan Networks (TPNs, [21])* extend STNs with decision nodes and model decisions as outgoing edges from such nodes. *Simple Temporal Networks with Decisions* (*STNDs*, [4, 29]) extend STNs by adding decision points that can influence both the execution of time points and the satisfaction of constraints. Several (possibly different) STNs may arise when projecting an STND onto a scenario that models the complete interpretation of the decisions. For any scenario, we are only interested in executing the time points and satisfy the constraints *entailed* by it.

So far, only one hybrid SAT-based consistency checking algorithm (*HSCC*) has been devised to check the consistency of an STND [4]. If the algorithm finds a consistent scenario, i.e., a scenario for which the STN projection is consistent, then a solution (*scenario* plus *schedule*) exists. In this case, any schedule in the solution set involves the STN-projection corresponding to the related scenario.

This algorithm allows for an offline planning where all decisions are made before starting and the corresponding schedule is already known. However, this algorithm has never been implemented nor evaluated, and it does not allow for the synthesis of all consistent scenarios.

Contributions. Our contribution is three-fold. First, we provide `STND-HSCC2`, a novel HSCC algorithm for STNDs that (i) is faster than the existing algorithm as it rules out inconsistent scenarios as early as possible, and (ii) allows for the synthesis of all consistent scenarios and related early execution schedules (offline temporal planning). The algorithm is hybrid because a SAT-solver and a shortest path algorithm mutually influence each other (the output of the former becomes the input of the latter and vice versa continuously). Second, we discuss Kappa, a tool that we developed for STNDs for the experimental evaluation and in which, as a minor contribution, we adapted the previous algorithm to support the synthesis of all consistent scenarios. Third, we prove that STNDs are equivalent to disjunctive temporal networks (DTNs).

[1] This is not a restriction as every finite set $\{0, 1, 2, \ldots, n-1\} \subset \mathbb{N}$ of different discrete choices (i.e., decisions) can be represented in an equivalent binary notation by using $\lceil \log n \rceil$ bits where if the i^{th} bit is 1 (respectively, 0), then it means that the i^{th} decision holds (respectively, does not hold). Of course this binary representation may express more than n possibilities (e.g., to express $\{0, 1, 2\}$ we need $\lceil \log 3 \rceil = 2$ bits with which we can also express "3" when all bits are set). Again, this is not a problem if we add unsatisfiable constraints for all combinations representing numbers $\geq n$ (if any).

Organization. Section 2 provides background on STNs and STNDs, and the current consistency checking algorithm for STNDs. Section 3 discusses `STND-HSCC2`, a faster HSCC algorithm for STNDs. Section 4 discusses KAPPA and the related experimental evalutation. Section 5 proves that STNDs are equivalent to DTNs. Section 6 discusses related work. Section 7 sums up and discusses future work.

2 Background

A *Simple Temporal Network* (*STN*, [11]) is a pair $\langle \mathcal{T}, \mathcal{C} \rangle$, where $\mathcal{T} = \{X, \dots\}$ is a set of time points (continuous variables) and $\mathcal{C} = \{(Y - X \leq k), \dots\}$ is a set of constraints, for $X, Y \in \mathcal{T}$, $k \in \mathbb{R} \cup \pm\{\infty\}$.

An STN is *consistent* if there exists an assignment of real values to the time points such that all constraints are satisfied.

Given a consistent STN, a *schedule* is a function $S \colon \mathcal{T} \to \mathbb{R}$ assigning real values to time points such that if X is executed before Y, then $S(X) \leq S(Y)$. An *early execution* of an STN consists of finding a schedule executing the time points as soon as possible (e.g., by using the Floyd-Warshall algorithm [11]).

Given a set $\mathcal{P} = \{d, \dots\}$ of Boolean propositions, a *label* $\ell = \lambda_1 \dots \lambda_n$ is any finite conjunction of literals λ_i, where a literal is either d (positive literal) or $\neg d$ (negative literal), and we omit the $\wedge$ connective to ease reading. The *empty label* is denoted by $\boxdot$. The *label universe of* $\mathcal{P}$, denoted by $\mathcal{P}^*$, is the set of all possible (consistent) labels drawn from $\mathcal{P}$; e.g., if $\mathcal{P} = \{d, e\}$, then $\mathcal{P}^* = \{\boxdot, d, e, \neg d, \neg e, de, d\neg e, \neg de, \neg d\neg e\}$. Two labels ℓ_1, ℓ_2 are *consistent* if their conjunction $\ell_1\ell_2$ is satisfiable. A label ℓ_1 *entails* a label ℓ_2 (written $\ell_1 \Rightarrow \ell_2$) if all literals in ℓ_2 appear in ℓ_1 too (i.e., if ℓ_1 is more *specific* than ℓ_2). For instance, if $\ell_1 = d\neg e$ and $\ell_2 = d$, then ℓ_1 and ℓ_2 are consistent since $d\neg ed$ is satisfiable, and ℓ_1 entails ℓ_2 since $d\neg e \Rightarrow d$.

A *scenario* is a mapping $s \colon \mathcal{P} \to \{\top, \bot\}$ assigning a truth value to each $d \in \mathcal{P}$. A scenario *satisfies* a label ℓ (in symbols $s \models \ell$) if ℓ evaluates to true under the interpretation given by s (e.g., if $s(d) = \top$ and $s(e) = \bot$, then $s \models d\neg e$).

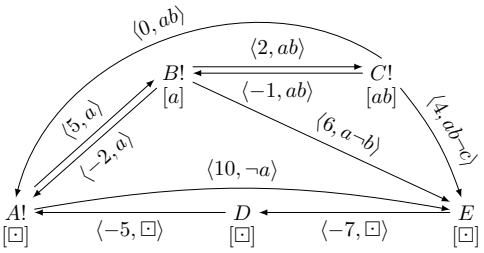
▶ **Definition 1.** *A* Simple Temporal Network with Decisions *(*STND*, [4, 29, 33]) is a tuple* $\mathcal{S} = \langle \mathcal{T}, \mathcal{DT}, \mathcal{P}, O, L, \mathcal{C} \rangle$*, where:*

- $\mathcal{T} = \{X, \dots, Z\}$ *is a finite set of time points.*
- $\mathcal{DT} \subseteq \mathcal{T} = \{D!, \dots, H!\}$ *is a finite set of decision time points.*
- $\mathcal{P} = \{d, \dots, g\}$ *is a finite set of Boolean propositions.*
- $O \colon \mathcal{P} \to \mathcal{DT}$ *is a bijection assigning a unique proposition to each decision time point* $D!$ *that controls the truth value assignment to* d *(*$O^{-1} \colon \mathcal{DT} \to \mathcal{P}$ *models the inverse).*
- $L \colon \mathcal{T} \to \mathcal{P}^*$ *is a function assigning labels to time points.*
- $\mathcal{C}$ *is a finite set of* labeled *constraints* $(Y - X \leq k, \ell)$ *where* $X, Y \in \mathcal{T}$, $k \in \mathbb{R} \cup \pm\{\infty\}$ *and* $\ell \in \mathcal{P}^*$.

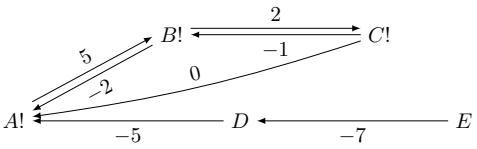
The STN-projection of an STND $\mathcal{S}$ *with respect to a scenario* s *(written* $\pi_s(\mathcal{S})$*) is an STN* $\langle \mathcal{T}_s, \mathcal{C}_s \rangle$ *built as follows:*

- $\mathcal{T}_s = \{X \mid X \in \mathcal{T} \wedge s \models L(X)\}$
- $\mathcal{C}_s = \{(Y - X \leq k) \mid (Y - X \leq k, \ell) \in \mathcal{C} \wedge s \models \ell\}$

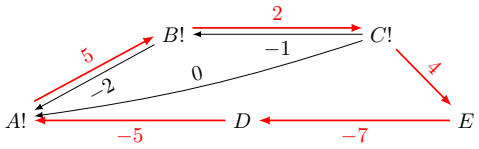
$\mathcal{S}$ *is consistent if there exists a scenario* s *such that* $\pi_s(\mathcal{S})$ *is consistent. A solution is a pair* $\langle s, S \rangle$*, where* s *is a scenario,* S *is a schedule with domain* $\mathcal{T}_s$*, and* $S(Y) - S(X) \leq k$ *holds for each* $(Y - X \leq k) \in \mathcal{C}_s$*. Checking consistency of STNDs is NP-complete [4].*



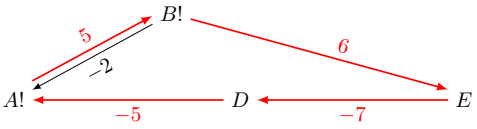
(a) Simple Temporal Network with Decisions.



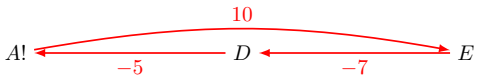
(b) STN-Projection onto $s(a) = s(b) = s(c) = \top$.



(c) STN-Projection onto $s(a) = s(b) = \top, s(c) = \bot$.



(d) STN-Projection onto $s(a) = \top, s(b) = \bot$, $s(c) \in \{\top, \bot\}$.



(e) STN-Projection onto $s(a) = \bot, s(b)$, $s(c) \in \{\top, \bot\}$.

Figure 1 An example of STND and related STN-projections (where thick red edges highlight negative cycles).

We represent an STND graphically by extending the distance graph of an STN into a labeled distance (multi)graph. The set of nodes still coincides with the set of time points, whereas each edge $X \to Y$ labeled by $\langle k, \ell \rangle$ represents $(Y - X \leq k, \ell) \in \mathcal{C}$. Time points' labels are shown below the nodes. Many $\langle k, \ell \rangle$ can be specified for the same $X \to Y$ provided their ℓ are different (if two labels are equal, we keep the smallest k). Figure 1a shows an example of STNDs, whereas Figures 1b–1e show its STN-projections.

A label ℓ labeling a time point or a constraint is *honest* if for each literal d or $\neg d$ in ℓ we have that $\ell \Rightarrow L(D!)$, where $D! = O(d)$ is the decision time point associated to d; ℓ is dishonest otherwise. For example, consider $L(C!) = ab$ in Figure 1a. $C!$ appears in a solution only if $s(a) = s(b) = \top$. However, deciding $\top$ for b implies that $B!$ appears in the solution too, which in turn requires that $A!$ appears (before $B!$) in the same solution with $\top$ decided for a. Thus, an honest ℓ containing b or $\neg b$ should also contain a. A label on a constraint is *coherent* if it is at least as expressive as the labels of the time points appearing in the constraint (i.e., ℓ contains all literals in the labels of the two connected time points).

▶ **Definition 2** (Well-definedness). *An STND is well-defined [17, 29, 33] if*

- $L(X) \Rightarrow L(O(d))$ *and* $(O(d) - X \leq 0) \in \mathcal{C}$ *for each* $X \in \mathcal{T}$ *and* $\{d, \neg d\} \in L(X)$*, and*
- $\ell \Rightarrow L(Y) \wedge L(X)$ *for each* $(Y - X \leq k, \ell) \in \mathcal{C}$*, and* $\ell \Rightarrow L(O(d))$ *for each literal* $\{d, \neg d\} \in \ell$*.*

Figure 1a is an example of well-defined STND modeling a temporal plan with 3 decisions ($A!$, $B!$ and $C!$) and two (instantaneous) activities (D and E). $A!$ is the first time point to execute. We execute $B!$ if and only if we decided $\top$ for a ($L(B!) = a$) and $C!$ if we further decided $\top$ for b too ($L(C!) = ab$). Instead, $A!$, D and E are always executed ($L(A!) = L(D) = L(C) = \boxdot$). We can execute $B!$ (if we decide so) after at least 2 ($B! \to A!$

Algorithm 1 `STND-HSCC1`($\mathcal{S}$).

Input: An STND $\mathcal{S} = \langle \mathcal{T}, \mathcal{DT}, \mathcal{P}, O, L, \mathcal{C} \rangle$.
Output: A solution $\langle s, S \rangle$ for $\mathcal{S}$ if $\mathcal{S}$ is consistent; "*inconsistent*" if $\mathcal{S}$ is inconsistent.
`STN-CC` is a consistency checking algorithm for STNs.

```
1 φ ← ⋀_{p∈P}(p ∨ ¬p)                                  ▷ Make every assignment possible
2 while true do
3     s ← SAT-SOLVE(φ)                                  ▷ Try to find a satisfying assignment s
4     if φ is unsatisfiable then
5         return inconsistent
6     ⟨T_s, C_s⟩ ← π_s(S)                               ▷ Project S onto s
7     if STN-CC(⟨T_s, C_s⟩) then
8         return ⟨s, S⟩                                 ▷ where S is an early schedule for ⟨T_s, C_s⟩
9     φ ← φ ∧ CUT-SCENARIO(S, ⟨T_s, C_s⟩)               ▷ Exclude this scenario
```

labeled by $\langle -2, a \rangle$) and within 5 time units since $A!$ was executed ($A! \to B!$ labeled by $\langle 5, a \rangle$). The same happens for $C!$ with respect to $B!$ (after 1 and within 2 time units since $B!$). Instead, we always execute D after minimum 5 time units since $A!$ ($D \to A!$ labeled by $\langle -5, \boxdot \rangle$) and E after 7 time units since D ($E \to D$ labeled by $\langle -7, \boxdot \rangle$). Furthermore, if we decide

- $\bot$ for a, then we must execute E within 10 time units since $A!$ ($A! \to E$ labeled by $\langle 10, \neg a \rangle$),
- $\top$ for a and $\bot$ for b, then we must execute E within 6 time units since $B!$ ($B! \to E$ labeled by $\langle 6, a\neg b \rangle$),
- $\top$ for both a and b and $\bot$ for c, then we must execute E within 4 time units since $C!$ ($C! \to E$ labeled by $\langle 4, ab\neg c \rangle$).

Therefore negative values on edges model delays, positive ones model deadlines.

To check the consistency of an STND, we can use the *hybrid SAT-based consistency checking (HSCC)* algorithm proposed in [4]. `STND-HSCC1`, specified in Algorithm 1, (`STND-CC` in [4]) maintains a formula φ specifying CNF clauses over propositions in $\mathcal{P}$. Initially, φ allows for all truth value assignments. In each round of the algorithm we ask the SAT solver for a truth value assignment making φ true. Such an assignment (if any) corresponds to a scenario s over which we can project the STND and check if the resulting STN is consistent ("SAT-solver influences directed weighted graph algorithm"). If so, we return this scenario and a valid schedule for the projected STN (i.e., a solution). Otherwise, we apply De Morgan's rules to the negation of the *relevant part* of the scenario containing the negative cycle (`CUT-SCENARIO` in Algorithm 2) and add the resulting clause to φ and go ahead with the next round ("directed weighted graph algorithm influences the SAT-solver"). This makes the approach hybrid. If φ has become unsatisfiable it means that all STN-projections are inconsistent and therefore the STND is inconsistent. Similar approaches are described in [23, 28].

An example of round for the network in Figure 1a is as follows. Suppose that `SAT-SOLVE`(φ) $= ab\neg c$ (i.e., $s(a) = s(b) = \top$ and $s(c) = \bot$). Since the STN $\pi_{ab\neg c}(\mathcal{S})$ is inconsistent (Figure 1c admits a negative cycle), we add to φ the clause $\neg(a \wedge b \wedge \neg c)$, which simplifies to $(\neg a \vee \neg b \vee c)$, to ask the SAT solver for a different truth value assignment excluding this projection (if any). Figure 1a is consistent if and only if $s(a) = s(b) = s(c) = \top$. A possible schedule for Figure 1b is $S(A!) = 0, S(B!) = 2, S(C!) = 3, S(D) = 5, S(E) = 12$. Any other combination leads to a projection containing a negative cycle (Figures 1c-1e).

Algorithm 2 `CUT-SCENARIO`$(\mathcal{S}, \langle \mathcal{T}_s, \mathcal{C}_s \rangle)$.

Input: An STND $\mathcal{S} = \langle \mathcal{T}, \mathcal{DT}, \mathcal{P}, O, L, \mathcal{C} \rangle$ and one of its STN-projections $\langle \mathcal{T}_s, \mathcal{C}_s \rangle$
Output: A clause ψ expressing the cut of the relevant part of the scenario.

1 $\psi \leftarrow \top$ ▷ ψ will contain the relevant part of s
2 **foreach** *constraint* $c \in \mathcal{C}_s$ **do**
3 $\quad \psi \leftarrow \psi \wedge \ell_c$ ▷ where ℓ_c is the corresponding label of c in $\mathcal{S}$
4 **return** `DeMorgan`$(\neg(\psi))$ ▷ the clause expressing the cut of ψ

3 A Faster HSCC Algorithm for STNDs

`STND-HSCC1` is correct [4], but it suffers from the limitation that projections are tested only when the SAT solver returns a complete truth value assignment. Consider Figure 1a and assume that the SAT solver starts on the formula $\varphi = (a \vee \neg a) \wedge (b \vee \neg b) \wedge (c \vee \neg c)$, which makes every truth value assignment possible. Suppose that in the search tree the SAT solver decides $\bot$ for a proposition d going down to the left and $\top$ going down to the right in the search tree, and assume that the order of visit is left then right. The first truth value assignment returned is $a = \bot$, $b = \bot$ and $c = \bot$ (corresponding to the scenario $s(a) = s(b) = s(c) = \bot$). Now `STND-HSCC1` would project the STND in Figure 1a onto s to obtain the STN shown in Figure 1e and eventually detect the negative cycle. However, the negative cycle could have been detected much earlier, say, when a was assigned $\bot$. Indeed, all projections of any scenario containing $s(a) = \bot$ boil down to Figure 1e (no matter which Boolean values are assigned to b and c). Therefore, a clever implementation of this algorithm calls for an early detection of negative cycles. Before proceeding with it, we must refine the concept of scenario and projection so that they support "unknown" propositions (i.e., propositions that have not been assigned a value yet).

▶ **Definition 3.** *A* scenario *is (now) a mapping* $s \colon \mathcal{P} \to \{\top, \bot, -\}$ *assigning either* true, false *or* unknown *to each proposition* $d \in \mathcal{P}$*. A scenario* s satisfies *a label* ℓ *if* ℓ *evaluates to true under the following interpretation given by* s*:*

1. $s \models \lambda$ *iff* $(\lambda = d \wedge s(d) = \top)$ *or* $(\lambda = \neg d \wedge s(d) = \bot)$,
2. $s \models \ell$ *iff* $s \models \lambda_1$ *and* $\ldots$ *and* $s \models \lambda_n$ *for* $\ell = \lambda_1 \ldots \lambda_n$.

Note that s *never satisfies a label containing a literal for which the corresponding proposition is unknown in* s*.*

The definition of STN projection remains the same as that given in Definition 1 but extended to the new definition of scenario. As a result, Figure 1e now becomes a representative also for any scenario s such that $s(a) = \bot$ and $s(b), s(c) \in \{\top, \bot, -\}$. Another example is Figure 1d, extending $s(c) \in \{\top, \bot\}$ to $s(c) \in \{\top, \bot -\}$. Now we have everything we need to hunt down inconsistent scenarios as early as possible.

`STND-HSCC2` (Algorithm 3) is a brand new algorithm to check the consistency of STNDs. It allows for the synthesis of a single or all scenarios admitting a consistent schedule for the corresponding STN-projection. `STND-HSCC2` still initializes a CNF formula φ making all truth value assignments possible. Then, it starts the SAT-solver and hooks a listener to the corresponding run. Such a listener is able to operate on φ by adding CNF clauses on the fly if needed and is triggered by two main events: *assume* and *solution found*.

An *assume* ($d = \top$ or $d = \bot$) event (Algorithm 3, lines 8-12) triggers an action of the listener to "look ahead" if the STN-projection obtained by projecting the STND onto the scenario built from the current truth value assignment and *extended* with this assumption

Algorithm 3 `STND-HSCC2`$(\mathcal{S}, all)$.

Input: An STND $\mathcal{S} = \langle \mathcal{T}, \mathcal{DT}, \mathcal{P}, O, L, \mathcal{C} \rangle$ and a Boolean value *all* meaning *all consistent scenarios* iff $all = \top$.
Output: A single or all scenarios s along with schedule(s) S for the projection $\pi_s(\mathcal{S})$ if $\mathcal{S}$ is consistent, "*inconsistent*" otherwise.

1 $\varphi \leftarrow \bigwedge_{p \in \mathcal{P}} (p \vee \neg p)$ ▷ Make every assignment possible
2 $Sols = \emptyset$ ▷ The set of (all) consistent scenarios (global variable)
3 Hook a *listener* to the run of the SAT solver and make it detect negative cycles as early as possible (**on** blocks (below) define the event-driven behavior).
4 `SAT-SOLVE`(φ)
5 **if** $Sols = \emptyset$ **then**
6 **return** *inconsistent*
7 **return** *Sols*
8 **on** *assume* $d = \top$ *or assume* $d = \bot$*:* ▷ Partial model
9 Build a scenario s from the current truth value assignment extended with $s(d) = \top$ or $s(d) = \bot$ (depending on the case)
10 $\langle \mathcal{T}_s, \mathcal{C}_s \rangle \leftarrow \pi_s(\mathcal{S})$ ▷ Get the STN projection
11 **if** *BellmanFord*$(\langle \mathcal{T}_s, \mathcal{C}_s \rangle)$ *detects a negative cycle* **then**
12 $\varphi \leftarrow \varphi \wedge$ `CUT-SCENARIO`$(\mathcal{S}, \langle \mathcal{T}_s, \mathcal{C}_s \rangle)$ ▷ Add clause
13 **on** *solution found:* ▷ Complete model
14 Build a scenario s from the current truth value assignment
15 $\langle \mathcal{T}_s, \mathcal{C}_s \rangle \leftarrow \pi_s(\mathcal{S})$ ▷ Get the STN projection
16 **if** *BellmanFord*$(\langle \mathcal{T}_s, \mathcal{C}_s \rangle)$ *does not detect any negative cycle* **then**
17 $Sols \leftarrow Sols \cup \{\langle s, S \rangle\}$ ▷ S is an early schedule for $\langle \mathcal{T}_s, \mathcal{C}_s \rangle$
18 **if** *BellmanFord*$(\langle \mathcal{T}_s, \mathcal{C}_s \rangle)$ *detects negative cycle or all is true* **then**
19 $\varphi \leftarrow \varphi \wedge$ `CUT-SCENARIO`$(\mathcal{S}, \langle \mathcal{T}_s, \mathcal{C}_s \rangle)$ ▷ Add clause

contains a negative cycle. If so, we extend φ by adding (on the fly) a CNF clause modeling the negation of the part of s containing a negative cycle in order to avoid getting the same scenario again. If the projection is consistent, `STND-HSCC2` does nothing and lets the run go.

A *solution found* event (Algorithm 3, lines 13–19) extends the behavior of the listener described for *assume* as follows. When triggered, the listener builds a scenario from the current truth value assignment (which does not need to be extended with anything else this time). Then, it checks if the corresponding STN-projection contains a negative cycle. If it does not, then it computes an (early) schedule S for the STN projected onto s and adds the pair $\langle s, S \rangle$ to the set of solutions. If it detects a negative cycle (or all consistent scenarios are sought), then it acts as for *assume* events.

Eventually, when the run of the SAT solver ends, either $Sols = \emptyset$ (and thus the starting STND is inconsistent), or *Sols* contains at least 1 solution (scenario-schedule).

Like `STND-HSCC1`, `STND-HSCC2` is sound and complete because it is based on a SAT-solver that allows us to iterate on all the models. Whenever we add a clause, we exclude a relevant part of a scenario that we do not want to get anymore. The sooner, the better.

Besides the SAT solver, all other internal sub-procedures (mostly, algorithms for directed weighted graphs) are well known to be sound and complete, and run in polynomial time.

4 Experimental Evaluation

We developed Kappa, a tool for STNDs that takes in input a specification of an STND and acts both as a solver and as a solution verifier. Kappa relies on SAT4J [3], a Java library compliant with the IPASIR interface that specifies how to interact with a SAT solver [1].

Kappa implements both `STND-HSCC1` and `STND-HSCC2`. We extended `STND-HSCC1` so that it allows for the synthesis of all consistent scenarios as well. In this way, we could carry out a more accurate experimental evaluation comparing the two algorithms when seeking single or all consistent scenarios.

We randomly generated 2200 STNDs partitioned in benchmark 11 sets, each one containing 100 consistent STNDs and 100 inconsistent STNDs. Regardless of the set, each STND has exactly 100 time points. The first set (`100TimePoints/10Decisions`) specifies 10 decision time points, the second set (`100TimePoints/11Decisions`) specifies 11 decision time points and so on, up to the eleventh one (`100TimePoints/20Decisions`) that specifies 20 decision time points. Each STND has a maximum number of constraints of $|\mathcal{T}| \times |\mathcal{DT}|$. Time points and constraints are randomly labeled so that the resulting STND is well defined. The weights on labeled edges range from -100 to 100. See the link "Supplement Material" before Section 1 to get Kappa and these benchmark sets.

We ran Kappa on these benchmark sets without imposing any limit to collect data (time and space) for both `STND-HSCC1` and `STND-HSCC2` when seeking a single or all consistent scenarios.

We graphically show the results in Figure 2, where x-axes always represent the number (#) of decision time points (i.e., the set under analysis) and y-axes represent either the average time elapsed or space consumed when analyzing the instances in that set.

Figure 2a shows the results of the analysis run on the sets containing consistent STNDs when seeking a single consistent scenario. The graph shows that `STND-HSCC2` is significantly faster than `STND-HSCC1` for STNDs specifying more than 16 decision time points. Figure 2b shows the results of the same analysis when seeking all consistent scenarios: despite a normal general worsening of performances (all consistent scenarios are sought and not just one) `STND-HSCC2` is faster than `STND-HSCC1` for STNDs specifying 20 decisions. Figure 2c shows the results of the analysis on the sets containing inconsistent STNDs. `STND-HSCC2` has no competitors here, whereas `STND-HSCC1` starts having a serious exponential blow up for STNDs specifying more than 14 decisions. Figure 2d shows the average space consumed when synthesizing all consistent scenarios. The curve grows exponentially according to the number of decision time points (recall that `STND-HSCC1` and `STND-HSCC2` return the same set of consistent scenarios in such an analysis, therefore we only show the data for `STND-HSCC2`).

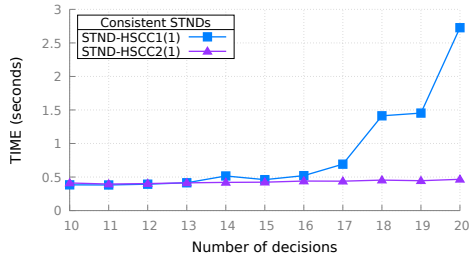
We verified all synthesized solutions. No constraint was violated.

5 Equivalence with Disjunctive Temporal Networks

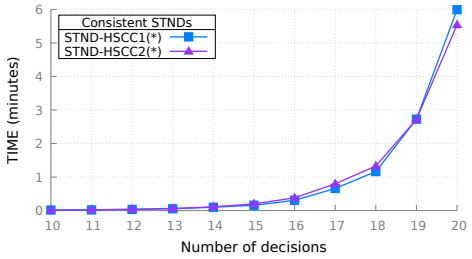
Disjunctive temporal networks (*DTNs*, [25]) allow for disjunctions of temporal constraints (i.e., alternatives) in a temporal problem and are a possible formalism to model the *disjunctive temporal problem* (*DTP*). For example, we might want that once an event modeled by a time point X happened, another event modeled by a time point Y happens *either* after 10 (seconds, minutes, hours, ...) *or* within 5. Such a constraint would look like

$$(X - Y \leq -10) \vee (Y - X \leq 5)$$

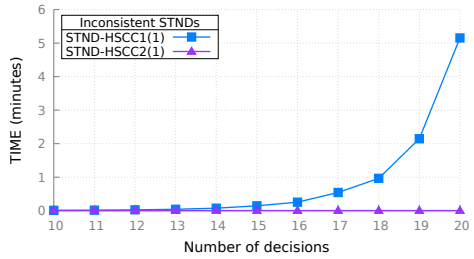
Any assignment of real values to X and Y satisfies the constraint if it satisfies (at least) one disjunct.



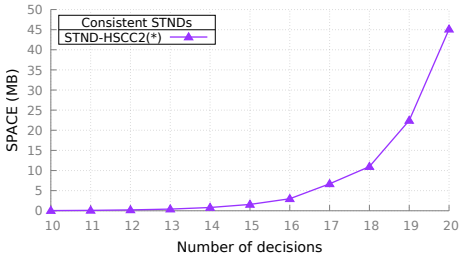
(a) Consistency checking time on consistent STNDs when seeking a single consistent scenario.



(b) Consistency checking time on consistent STNDs when seeking all consistent scenarios.



(c) Consistency checking time on inconsistent STNDs.



(d) Space consumed when synthesizing all consistent scenarios.

Figure 2 Experimental evaluation with KAPPA.

Differently from the initial proposal in [11], where disjunctions of intervals were allowed on the *same pairs of time points* only, the work we consider here is the one in [25], not having such a restriction.

▶ **Definition 4.** *A* disjunctive temporal network *(*DTN*) is a pair* $\langle \mathcal{T}, \mathcal{C} \rangle$*, where*

- $\mathcal{T}$ *is the usual finite set of time points, and*
- $\mathcal{C}$ *is a finite set of temporal constraints each one having the form*

$$\underbrace{(Y_1 - X_1 \leq k_1) \vee \cdots \vee (Y_n - X_n \leq k_n)}_{n \text{ disjuncts (atoms)}}$$

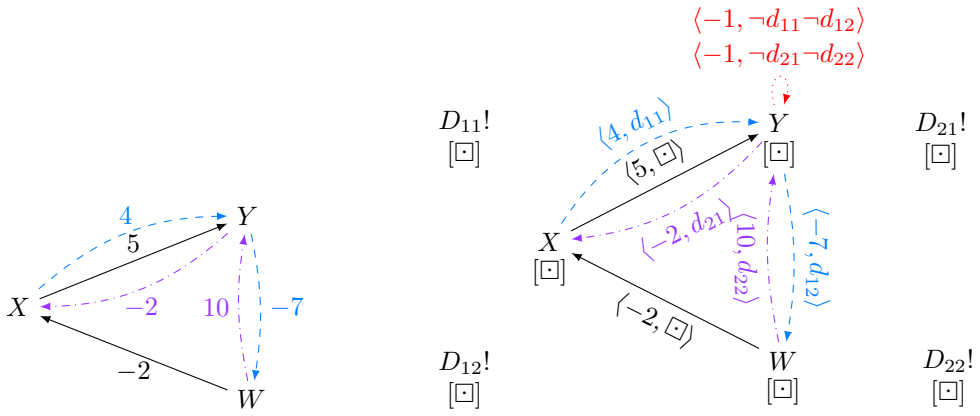
where $X_i, Y_i \in \mathcal{T}$ *and* $k_i \in \mathbb{R}$*. A temporal constraint is* non-disjunctive *if and only if it contains one disjunct,* disjunctive *otherwise.*

A DTN is consistent *if there exists an assignment of real values to all time points (i) always satisfying all non-disjunctive constraints and (ii) satisfying at least one disjunct for each disjunctive constraint.*

We write $D(i)$ to shorten the i^{th} disjunctive constraint and more specifically $D(i, j)$ to refer to the j^{th} disjunct of the i^{th} disjunctive constraint [25].

We represent a DTN graphically through a *colored multi graph*, where *black edges* model non-disjunctive constraints (i.e., those constraints that must always hold), whereas *colored edges* (different from black) model disjunctive constraints (i.e., those $D(i)$s for which at least one disjunct must hold). Each disjunctive constraint is assigned to a different color (we also use a unique line pattern for each color).

To give an example, consider the following DTN, whose corresponding colored multi graph is shown in Figure 3a.



(a) Example of DTN. **(b)** Corresponding STND.

Figure 3 Representing and encoding DTNs into STNDs.

- $\mathcal{T} = \{X, Y, W\}$
- $\mathcal{C} = \{\overbrace{(Y - X \leq 5), (X - W \leq -2)}^{\text{must always hold}}, \overbrace{\overbrace{(Y - X \leq 4)}^{D(1,1)} \vee \overbrace{(W - Y \leq -7)}^{D(1,2)}}^{D(1)},$
  $\underbrace{\underbrace{(X - Y \leq -2)}_{D(2,1)} \vee \underbrace{(Y - W \leq 10)}_{D(2,2)}}_{D(2)}\}$

The DTN in Figure 3a is consistent, and, for example, the assignment $X = 0$, $Y = 3$ and $W = 5$ satisfies:

- all non-disjunctive constraints (solid black edges),
- $D(1,1)$ but not $D(1,2)$ for the disjunctive constraints $D(1)$ (dashed blue edges),
- $D(2,1)$ and also $D(2,2)$ for the disjunctive constraint $D(2)$ (dashdotted purple edges).

We now proceed by proving that STNDs and DTNs are equivalent. We first give a strongly polynomial time encoding from DTNs to STNDs and then the vice versa (and we provide examples throughout this discussion).

5.1 Encoding DTNs into STNDs

We encode the DTN in Figure 3a into the corresponding STND in Figure 3b as follows.

We generate a "core" STND containing all time points and all non-disjunctive constraints of the starting DTN and labeling them by $\boxdot$, since all time points must always be assigned a value and all non-disjunctive constraints must always be satisfied.

For each disjunctive constraint $D(i)$ in the DTN, we add to the STND as many decision time points $D_{ij}!$ as the number of disjuncts $D(i,j)$. These decision time point are not constrained to any other time point in the STND (i.e., free to take any value). Any disjunct $D(i,j)$ in the DTN appears as a constraint in the STND labeled by d_{ij} (the proposition associated to $D_{ij}!$) so that when $d_{ij} = \top$, the disjunct of the DTN (labeled constraint in the STND) must hold and when $d_{ij} = \bot$ we are not obliged to satisfy it. Moreover, we impose that at least one disjunct $D(i,j)$ for any disjunctive constraint $D(i)$ must hold (otherwise, it would be possible to disable them all by setting all d_{ij} to $\bot$). We enforce this condition by adding a negative self loop labeled by $\neg d_{ij_1} \dots \neg d_{ij_n}$ on any time point of the STND.

In Figure 3b we added four decision time points $D_{11}!$, $D_{12}!$, $D_{21}!$ and $D_{22}!$ and the labeled constraints $X \to Y$ labeled by $\langle 4, d_{11}\rangle$, $Y \to W$ labeled by $\langle -7, d_{12}\rangle$, $Y \to X$ labeled by $\langle -2, d_{21}\rangle$ and $W \to Y$ labeled by $\langle 10, d_{22}\rangle$ to switch on and off the disjuncts $D(1,1)$, $D(1,2)$, $D(2,1)$ and $D(2,2)$ through the truth value assignments to d_{11}, d_{12}, d_{21} and d_{22}. Finally, we added two negative self loops $Y \to Y$ labeled by $\langle -1, \neg d_{11}\neg d_{12}\rangle$ and $\langle -1, \neg d_{21}\neg d_{22}\rangle$ to prevent a disjunctive constraint $D(i)$ from being excluded (red self loop at Y). Note that the "-1" is meaningless: *any negative number* (e.g., -3, -159 or $-\epsilon$) is fine for this purpose. Likewise, the choice of time point Y is meaningless too. Any time point would be fine for this purpose (e.g., $X \to X$ labeled by the same constraints). Negative self loops are the more intuitive way to enforce these conditions. However, nothing would have prevented us from creating cycles of negative sum with respect to these labels involving many time points.

To ease reading and understand "what goes where", we colored the STND in Figure 3b with the same colors of the DTN in Figure 3a and showed the added negative cycles in red.

This encoding is strongly polynomial. The number of time points in the STND is equal to the number of time points in the DTN plus as many decision time points as the number of disjuncts $D(i,j)$ contained in all disjunctive constraints $D(i)$ in the DTN. The number of constraints in the STND is equal to the the number of non-disjunctive constraints plus as many constraints as the number of disjuncts $D(i,j)$ contained in all disjunctive constraints $D(i)$ in the DTN plus as many constraints as the number of disjunctive constraints $D(i)$ to model negative loops.

Any consistent scenario in the STND says which disjuncts (at least one for each disjunctive constraint) are satisfied for the solution. If the STND is inconsistent, then so is the DTN.

5.2 Encoding STNDs into DTNs

We encode the STND in Figure 4a into the corresponding STND in Figure 4b as follows.

First of all, if the STND has labels on nodes we convert it to its streamlined version having only label on edges. The process of streamlining a temporal network was first discussed in [5] for CSTNs. However, that process works for STNDs as well (as consistency is basically entailed by controllability). Then, we generate a "core" DTN having the same set of time points of the STND (we drop all "!" from the names) and all constraints labeled by $\boxdot$ in the STND as non-disjunctive constraints in the DTN.

For each proposition d associated to a decision time point $D!$ in the STND, we add to the DTN a time point d and the disjunctive constraint

$$\underbrace{(d - D \leq 0)}_{\text{means } d = \bot} \vee \underbrace{(D - d \leq -1)}_{\text{means } d = \top}$$

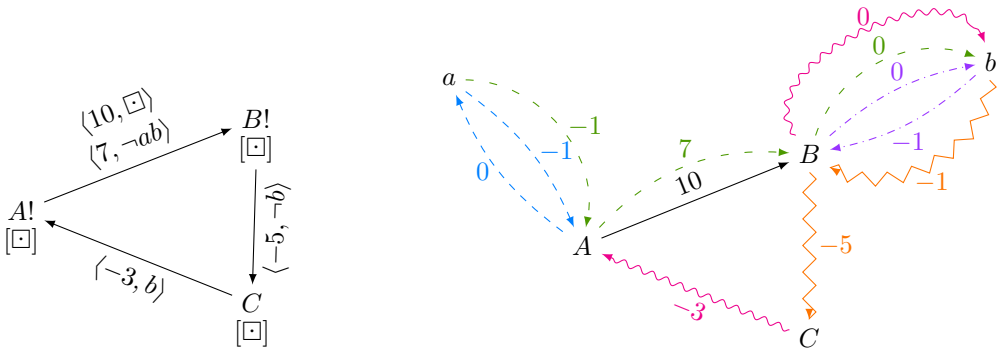
where the former says that d "occurs within" D, whereas the latter says that d occurs at least 1 after D (a way to to simulate a Boolean condition).

Now, every constraint $X \to Y$ labeled by $\langle k, d\neg e f \dots\rangle$ (in the STND) implies the following "meta constraint" in the DTN:

$$(\underbrace{D - d \leq -1}_{d} \wedge \underbrace{e - E \leq 0}_{\neg e} \wedge \underbrace{F - f \leq -1}_{f} \dots) \Rightarrow Y - X \leq k$$

which can be rewritten as

$$\neg(D - d \leq -1 \wedge e - E \leq 0 \wedge F - f \leq -1 \dots) \vee Y - X \leq k$$



(a) Example of (streamlined) STND. **(b)** Corresponding DTN.

Figure 4 Encoding STNDs into DTNs.

and finally simplified to

$$\underbrace{(d - D \leq 0)}_{\neg d} \vee \underbrace{(E - e \leq -1)}_{e} \vee \underbrace{(f - F \leq 0)}_{\neg f} \cdots \vee (Y - X \leq k)$$

Note that for any D and d (in the DTN) we define $\neg(D - d \leq -1)$ as $d - D \leq 0$ and $\neg(d - D \leq 0)$ as $D - d \leq -1$ since they are abstracting Boolean conditions only and we are not therefore interested in a specific numeric value. Therefore, for any labeled constraint in the STND we add such a disjunctive constraint to the DTN.

In Figure 4b we add two time points a and b and the following constraints:

- $B - A \leq 10$ (solid black edges),
- $D(1)\colon (a - A \leq 0) \vee (A - a \leq -1)$ (dashed blue edges),
- $D(2)\colon (b - B \leq 0) \vee (B - b \leq -1)$ (dashdotted purple edges),
- $D(3)\colon (A - a \leq -1) \vee (b - B \leq 0) \vee (B - A \leq 7)$ (loosely dashed green edges),
- $D(4)\colon (B - b \leq -1) \vee (C - B \leq -5)$ (zigzag orange edges),
- $D(5)\colon (b - B \leq 0) \vee (A - C \leq -3)$ (snake magenta edges).

We show the "colored" DTN graph in Figure 4b. Now, the DTN is consistent if and only if the STND is so. A solution of the DTN corresponds to a consistent scenario in the STND. The truth value assignment to the propositions in the STND depends on the real value assignments to the time points modeling those propositions in the DTN. For any proposition d in the STND, d is false iff in the DTN the time point d has a value not greater than $D!$ and d is true in the STND iff in the DTN the value of time point d is greater than $D!$ (the assignment to the other time points defines a schedule consistent for the scenario).

This encoding is strongly polynomial. The number of time points in the DTN is the same of that in the STND plus as many time points as the number of propositions in the STND. The number of constraints in the DTN is given by the number of unlabeled constraints in the STND (non-disjunctive constraints in the DTN), plus as many disjunctive constraints as the number of labeled constraints in the STND (whose labels are different from ⊡). Also, for any disjunctive constraint $D(i)$ in the DTN, the number of disjuncts of $D(i)$ is $n+1$ where n is the number of literals contained in the label of the corresponding constraint in the STND and the "+1" refers to the inequality.

6 Related Work

STNs [11] model fully-controllable plans but do not employ decisions. *Drake* [9] is an executive for temporal plans with choices modeled as Labeled STNs that extend STNs by labeling constraints with environments (set of instantiated discrete variables). There are no decision points and time points are unlabeled. During execution, choices are discriminated by generating conflicts according to the timing Drake decides to schedule some event. STNDs differ from Drake in their specification and in how decisions are made. STNDs rely on a more structured approach by using labels instead of environments, labeling time points to prevent them from being executed when some literal in the label is still unknown, enforcing well-defined properties and making decisions only upon the execution of the related decision time points. A *Disjunctive Temporal Network (DTN)* [25] extends an STN with disjunctive constraints. Any solution to a DTN must satisfy all non-disjunctive constraints (i.e., STN-constraints) and at least one disjunct for each disjunctive constraint. Labeled STNs and DTNs are equivalent [9]. DTNs and STNDs are equivalent too, therefore, Labeled STNs are equivalent to STNDs as well.

Temporal Plan Networks (TPNs) [21] extend STNs by adding decision nodes and symbolic constraints to model temporal plans with controllable choices modeled as outgoing edges from a decision node. Taking one of these outgoing edges means making a particular decision. Time points are not labeled and activities are modeled as pair of non-decision nodes (start,end). A symbolic constraint is either Ask(c) (is c true?) and Tell(c) (c is true!) where c a literal. Symbolic constraints may exclude activities from being executed. A plan is consistent if it satisfies both temporal and symbolic constraints. TPNs do not specify more than one temporal constraint on the same edge. Consistency is checked by visiting the nodes of the graph from start to end taking one edge (modeling a decision) at a time. If the resulting STN-projection is inconsistent, the algorithm backtracks to the last decision node that still has unexplored outgoing edges. STNDs label nodes and consistency is checked in a hybrid way.

A *Controllable Conditional Temporal Problem (CCTP)* [28] is an optimization problem for temporal plans with choices and thus incomparable with STNDs.

Pike [22] is an executive for temporal plans with both controllable and uncontrollable choices modeled as *Temporal Plan Networks with Uncertainty (TPNUs)*, which extend TPNs with uncontrollable choices. Pike adapts its controllable choices to the uncontrollable ones made by a human. STNDs do not have uncontrollable choices. *CCTPs with Uncertainty (CCTPUs)* [27] address temporal plans with controllable choices and uncontrollable durations, whereas in [18], TPNUs are extended to support uncontrollable durations (strong controllability only). In both works, relaxation techniques are used to restore controllability of an uncontrollable plan. STNDs do not have uncontrollable parts.

Several extensions of *STNs* address uncertain domains. For example, *Simple Temporal Networks with Uncertainty* (*STNUs*, [24]) add uncontrollable (but bounded) durations by means of a finite set of contingent links, whereas *Conditional Simple Temporal Networks* (*CSTNs*, [17]), and the Conditional Temporal Problem (*CTP*, [26]) considered formerly, extend STNs by turning the constraints conditional with uncontrollable truth value assignments observable upon the execution of some special kind of time points called observations. Finally, *Conditional Simple Temporal Networks with Uncertainty* (*CSTNUs*, [16, 15]) merge STNUs and CSTNs, whereas *Conditional Simple Temporal Networks with Uncertainty and Decisions* (*CSTNUDs*, [29, 33]) add conditional constraints with controllable truth value assignments decidable upon the execution of some special kind of time points called decision. CSTNUDs

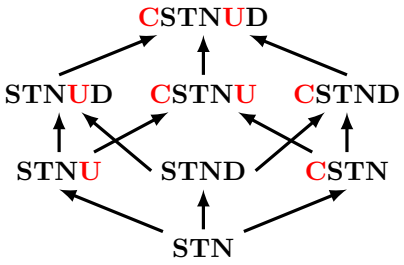


Figure 5 A hierarchy of simple temporal networks. Acronyms containing **D** (resp., **C**) mean that formalisms deal with controllable (resp., uncontrollable) conditionals, whereas those containing **U**, mean that formalisms deal with uncontrollable durations. We highlight uncontrollable parts in red.

encompass all previous formalisms. All these networks extend STNs by adding at least an uncontrollable part. In this work, we do not address any uncontrollable part. STNDs derive from [4] by removing uncontrollable conditionals and from [29, 33] by removing uncontrollable conditionals and uncontrollable durations. This work extends [4] by providing `STND-HSCC2` both to speed up the consistency checking phase and to allow for the synthesis of all consistent scenarios. This work is, however, incomparable with [29, 33] as that work employs timed game automata. Figure 5 provides a hierarchy of simple temporal networks.

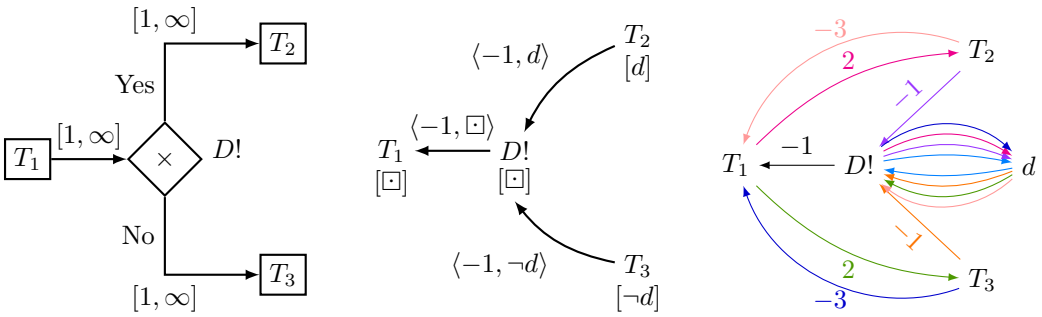
There also have been attempts to consider time and resources together, e.g., *Access Controlled Temporal Networks* (*ACTNs*, [6]) and *Conditional Simple Temporal Networks with Uncertainty and Resources* (*CSTNURs*, [7]), which were preceded by an initial proposal in [8]. However, neither ACTNs nor CSTNURs employ decision time points. Research on temporal networks has inspired a recent line of work in which controllability analysis focused on resource allocation under uncertainty employing a qualitative temporal approach instead of a quantitative one. This is the case of access controlled workflows investigated in [31, 36] and of extensions of constraint networks proposed in [34, 35], where *Constraint Networks Under Conditional Uncertainty* (*CNCUs*) are introduced (see also [32]). As we said, temporal relations are only qualitative (specifically, "before/after") and these proposals do not employ decision time points. A short summary of temporal and resource controllability based on constraint networks and considered either in isolation or simultaneously can be found in [30].

Planning as satisfiability was formally introduced in [19, 20] and relies on a set of axioms where any model corresponds to a valid plan. Before that, planning was based on deduction. Recently, more performant SAT encodings have been provided (e.g., [14]). However, none of these approaches is incremental and thus they are incomparable with ours.

Gocht and Balyo [13] provide an incremental SAT solving approach for SAT-based planning and prove that incremental SAT solving outperforms the non-incremental one but they do not address temporal constraints. Our work does not model "transitions" but applies shortest path algorithms, incrementally, on STN-projections.

Temporal induction is an incremental technique to check safety properties on finite state machines and it is strongly related to bounded model checking [12]. It is similar to SAT-based planning and allows for the detection of the unreachability of a goal. Our analysis is not bounded with respect to the "depth".

Satisfiability modulo theory (*SMT*, [2]) can describe STNDs by using a fragment of Linear Real Arithmetic called Difference Logic. However, SMT-solvers do not guarantee to find early schedules. A run of an HSCC algorithm and a run of an SMT-solver are not guaranteed to return the same consistent scenarios (Boolean part). A fairer comparison is when both HSCC-algorithms and SMT-solvers seek all consistent scenarios, but then we should make sure that the SMT-solver does not return more than one schedule for each consistent scenario.



(a) Temporal Workflow with controllable conditional paths.

(b) Simple Temporal Network with Decisions.

(c) DTN. Edges $D! \to d$ are labeled by 0, $d \to D!$ by -1.

Figure 6 Possible encodings of a temporal workflow (a) into STNDs (b) and into DTNs (c). Note that (c) is the DTN equivalent to (b) streamlined. That is, (b) without labels on time points and plus the constraints $(T_2 - T_1 \leq 2, d)$, $(T_1 - T_2 \leq -3, \neg d)$, $(T_3 - T_1 \leq 2, \neg d)$ and $(T_1 - T_3 \leq -3, d)$. There, we chose 2 as an horizon (see [5] for more details on how to streamline a temporal network).

Incremental task planning adopts incremental features of SMT-solvers to extend a constraint-based task planning to motion domains [10]. Our approach is not probabilistic and does not consider a motion domain.

7 Conclusions and Future Work

We provided `STND-HSCC2`, a novel hybrid SAT-based consistency checking algorithm for STNDs. This new version of the algorithm still relies on a SAT solver but differently from the previous one, it exploits partial truth value assignments to hunt down negative cycles in STN-projections as early as possible. The previous algorithm tested STN-projections for negative cycles by iterating on *complete* models returned by the SAT solver. When the SAT solver makes an assumption, we project the STND over the current truth value assignment of the propositions (i.e., partial model) extended with this new assumption. If the projected STN is inconsistent, we add a clause to the SAT solver to exclude that scenario, else we let the solver go. We implemented our approach and provided Kappa, a tool for STNDs that implements `STND-HSCC1` and `STND-HSCC2` both supporting the synthesis of single or all consistent scenarios[2] and we compared the results. The more inconsistent STN-projections an STND admits, the better `STND-HSCC2` performs. The solutions saved to file allow for an offline planning in which all decisions are made before starting and all time points have already been scheduled to execute as soon as possible. Finally, we proved that STNDs and DTNs are equivalent. Considering this equivalence result, one could fairly wonder why use STNDs instead of DTNs. Here is a possible reason: STNDs offer a more "structured" language, which, exploiting labels, allows for an easier modeling of temporal workflows with controllable conditional paths (see Figure 6 for a comparison of workflow modeling methods).

As future work, we plan to investigate optimizations to reduce the size of the CNF clauses added on the fly. We also plan to give a metric suggesting the best algorithm to use depending on the form of the STND in input.

[2] Another minor contribution is the extension of `STND-HSCC1` in our tool to support the synthesis of all consistent scenarios.

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