Report from Dagstuhl Seminar 19512

Interactive Design and Simulation

Edited by

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 19512 “"Interactive Design and Simulation”. After the executive summary, the collection of abstracts of the presentations forms the core of this report, complemented by an example of working group results that highlights the diversity of backgrounds and approaches.

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Edited in cooperation with Angelos Mantzaflaris

1 Executive Summary

Jorg Peters (University of FLorida, USA)

Dagstuhl Seminar 19512 presented and debated a rich set of techniques for improving algorithms and interfaces for interactive physical simulation, based on geometric and physical models and governed by partial differential equations. The techniques originate in geometry processing, computational geometry, geometric design, and the use of splines in meshing-less and iso-geometry approaches.

Thanks to its diverse roster of participants, with expertise spanning computer science, applied mathematics and engineering, the seminar enabled rare new interactions between academia, industrial and government-sponsored labs and fostered new insights apart from technical considerations. For example, one of the ad hoc discussions centered around the mechanisms and person-to-person considerations that enable transfer of new techniques from academia to industry. Another discussion focused on bridging the divide between geometric modeling and engineering analysis. A third focused on the usage (or lack thereof) of academic open-source libraries. And a fourth elucidated the different error measures that allow or prevent model reduction techniques for non-linear models (e.g. of elasticity) for given applications ranging from animation to product design.

The seminar was well-paced, avoiding densely-packed presentations. An emphasis was placed on time to formulate both specific and long range challenges. The benchmark problem in Section 4 is an example of specific problems that clarify and contrast the competing approaches and objectives and advertised the different strengths and the synergy of the areas: responses that permit two-sided error bounds, responses based on mathematical reformulation, applying advanced computational geometry and new software packages that leverage hierarchical spline software.

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3 Overview of Talks

3.1 Animation and Simulation of Realistic Virtual Humans

Mario Botsch (Universität Bielefeld, DE)

Joint work of Jascha Achenbach, Mario Botsch, Dominik Gall, Martin Komaritzan, Marc Latoschik, Daniel Roth, Thomas Waltemate
Main reference Martin Komaritzan, Mario Botsch: "Projective Skinning", Proc. ACM Comput. Graph. Interact. Tech., Vol. 1(1), The Association for Computers in Mathematics and Science Teaching, 2018.
URL http://dx.doi.org/10.1145/3203203

Realistic virtual humans have many interesting applications in several fields–ranging from characters in computer games, to avatars representing ourselves in virtual environments, and digital models for medical diagnosis and intervention planning.

Over the last five years we built a fast avatar generation pipeline that produces an accurate virtual clone of a real person in less than 10 minutes [2]. We first scan the real person by taking photos in a multi-camera rig consisting of 48 synchronized DSLR cameras. A multi-view photogrammetric reconstruction computes a dense point cloud from these images. In order to robustly cope with noise, outliers, and missing data we fit a statistical human body template to the high-resolution point cloud, producing an accurate geometric reconstruction. An anisotropic discrete curvature model allows to faithfully reconstruct folds and wrinkles even in the presence of noise [1]. A high-resolution texture is finally generated from the reconstructed geometry and the input camera images. The resulting avatars were evaluated in a virtual mirror scenario and were shown in significantly improve perceived body ownership and perceived presence [3].

Many real-time character animation approaches, such as Linear Blend Skinning or Dual Quaternion Skinning, are known to produce artifacts in the vicinity of strongly bent joints. Our *Projective Skinning* yields physically plausible animations by combining an articulated embedded volumetric skeleton with a real-time projective-dynamics simulation of the elastic tissue enclosed between skin and bone [4]. This approach has recently been accelerated through a custom sparse matrix format, a GPU-based conjugate gradients solver, and an MLS-based quadratic upsampling technique [5]. The resulting *Fast Projective Skinning* features plausible tissue deformation, dynamic motion effects (jiggling), as well as global collision handling–while still being fast enough for real-time applications.

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4 Martin Komaritzan, Mario Botsch. *Projective Skinning*. Proceedings of ACM i3D 2018.
5 Martin Komaritzan, Mario Botsch. *Fast Projective Skinning*. Proceedings of ACM MIG 2019

3.2 Mollified finite element approximants of arbitrary order and smoothness

Fehmi Cirak (University of Cambridge, GB)

Main reference Eky Febrianto, Michael Ortiz, Fehmi Cirak: "Mollified finite element approximants of arbitrary order and smoothness", CoRR, Vol. abs/1910.04002, 2019.
URL https://arxiv.org/abs/1910.04002

The approximation properties of the finite element method can often be substantially improved by choosing smooth high-order basis functions. It is extremely difficult to devise such basis functions for partitions consisting of arbitrarily shaped polytopes. We propose the mollified basis functions of arbitrary order and smoothness for partitions consisting of convex polytopes. On each polytope an independent local polynomial approximant of arbitrary order is assumed. The basis functions are defined as the convolutions of the local approximants with a mollifier. The mollifier is chosen to be smooth, to have a compact support and a unit volume. The approximation properties of the obtained basis functions are governed by the local polynomial approximation order and mollifier smoothness. The convolution integrals are evaluated numerically first by computing the boolean intersection between the mollifier and the polytope and then applying the divergence theorem to reduce the dimension of the integrals. The support of a basis function is given as the Minkowski sum of the respective polytope and the mollifier. The breakpoints of the basis functions, i.e. locations with non-infinite smoothness, are not necessarily aligned with polytope boundaries. Furthermore, the basis functions are not boundary interpolating so that we apply boundary conditions with the non-symmetric Nitsche method as in immersed/embedded finite elements. The presented numerical examples confirm the optimal convergence of the proposed approximation scheme for Poisson and elasticity problems.

3.3 Adaptive isogeometric methods with (T)HB-splines: suitably graded meshes and BPX preconditioners

Carlotta Giannelli (University of Firenze, IT)

Joint work of Carlotta Giannelli, Cesare Bracco, Durkbin Cho, Rafael Vázquez

We consider adaptive isogeometric methods with (truncated) hierarchical B-splines, usually denoted as (T)HB-splines. In virtue of a simple multilevel construction, the hierarchical spline model represents one of the most successful choice for performing adaptivity in the framework of isogeometric analysis. In this talk I will present recent results towards the efficient use of (T)HB-splines. In particular, the construction of an additive multilevel preconditioner, also known as BPX preconditioner, will be discussed. By exploiting the locality of hierarchical spline functions, efficient multilevel decompositions can be identified to prove that, when suitably graded hierarchical meshes are considered, the condition number of the preconditioned system is bounded independently of the number of levels. To highlight the performance of the preconditioner, the theoretical results will be presented together with a selection of numerical examples.

3.4 Grasping and manipulation: Representation of shape and dynamics

Cindy Marie Grimm (Oregon State University – Corvallis, US)

Robotic manipulation is a complex combination of four different elements: 1) The kinematics of the manipulator, 2) The sensors, 3) The actuators, and 4) the control strategy that maps sensors to actuators. In order to optimize in this space, i.e., successfully complete a manipulation task, we have to optimize over all 4 elements. To make this manageable, we propose a general approach that allows us to optimize element-wise. Specifically, we use humans as ideal actuators and controllers, freeing us to explore the kinematic structure. Essentially, we ask humans to "puppeteer" the robotic manipulators through structured tasks. By instrumenting the robotic hands and the objects that are manipulated, we can build up data on how well specific kinematic structures perform. Simultaneously, we also capture actuation, sensor, and control information. Given this, we can now systematically add in actuators and controllers.

Within this learning structure, there are still several open-ended questions that require both shape and dynamic models. Specifically, what are effective models for representing shape and dynamics that are amenable to real-world sensors? We also need statistically accurate physical simulators that allow us to optimize designs for manipulation. Typical manipulators consist of compliant materials connected with both rigid body and soft joints, with cable actuators. Contact is extremely important; and it is important to not only model contact but model it in a statistical manner – how might an object be moved as a result of the contact, not just a single sample.

3.5 Real-Time Simulation Of Elastic Solids and Fluid-Solid Interaction

Klaus Hildebrandt (TU Delft, NL)

Joint work of Klaus Hildebrandt, Christopher Brandt, Leonardo Scandolo, Elmar Eisemann
Main reference Christopher Brandt, Leonardo Scandolo, Elmar Eisemann, Klaus Hildebrandt: "The reduced immersed method for real-time fluid-elastic solid interaction and contact simulation", ACM Trans. Graph., Vol. 38(6), pp. 191:1–191:16, 2019.
URL http://dx.doi.org/10.1145/3355089.3356496
Main reference Christopher Brandt, Elmar Eisemann, Klaus Hildebrandt: "Hyper-reduced projective dynamics", ACM Trans. Graph., Vol. 37(4), pp. 80:1–80:13, 2018.
URL http://dx.doi.org/10.1145/3197517.3201387

This talk splits in two parts. In the first part, we discuss Hyper-Reduced Projective Dynamics (HRPD) an approach for the real-time simulation of deformable objects that combines the robustness, generality, and high performance of Projective Dynamics with the efficiency and scalability offered by model reduction techniques. The method decouples the cost for time integration from the mesh resolution and can simulate large meshes in real-time. In the second part, we look at a method for the real-time simulation of two-way coupled incompressible fluids and elastic solids called the Reduced Immersed Method (RIM). It is based on a novel discretization of the immersed boundary equations of motion that combines HRPD with a PIC/FLIP fluid solver. Crucial for the performance of RIM is the efficient transfer of information between the elasticity and the fluid solver and the synchronization of the Lagrangian and Eulerian settings. We introduce the concept of twin subspaces that enables an efficient reduced-order modeling of the transfer.

3.6 Adaptive spline projectors via restricted hierarchical spline fitting

Bert Jüttler (Johannes Kepler Universität Linz, AT)

Joint work of Bert Jüttler, Alessandro Giust, Angelos Mantzaflaris

This talk is devoted to techniques for adaptive spline projection via quasi-interpolation, enabling the efficient approximation of given sample data or functions. We employ local least-squares fitting in restricted hierarchical spline spaces to establish novel projection operators for hierarchical splines of degree p. This leads to efficient spline projectors that require $\mathcal{O}(p^d)$ floating point operations and $\mathcal{O}(1)$ evaluations of the given function per degree of freedom, while providing essentially the same accuracy as global approximation. The spline projectors, which are based on a unifying framework for quasi-interpolation in hierarchical spline spaces, are shown to compare favorably with other constructions that have been described in the rich literature on this topic.

3.7 Computational Modeling for Cardiac Biomechanics

Adarsh Krishnamurthy (Iowa State University, US)

Joint work of Adarsh Krishnamurthy, Arian Jafari, Edward Pszczolkowski, Aditya Balu, Ming-Chen Hsu, Soumik Sarkar
Main reference Arian Jafari, Edward Pszczolkowski, Adarsh Krishnamurthy: "A framework for biomechanics simulations using four-chamber cardiac models", Journal of Biomechanics, Vol. 91, pp. 92–101, 2019.
URL https://doi.org/10.1016/j.jbiomech.2019.05.019

Cardiovascular diseases, such as heart failure, are one of the leading causes of death in the world and pose a severe burden to the healthcare system. Computational models of the cardiovascular system, developed from patient-specific clinical data, can help refine the diagnosis and personalize the treatment. We have been working on developing an integrative framework for cardiac biomechanics with simulation, analysis, and visualization tools that will significantly advance the state-of-the-art in personalized medicine. In this talk, I presented recent advances in computational modeling that enables simulation of a four-chamber cardiac model [1]. We have developed tools to generate a patient-specific cubic-Hermite four-chamber cardiac mesh and use isogeometric analysis methods to simulate a full cardiac cycle [2].

The second part of the talk focused on novel machine-learning algorithms to optimize the design of bioprosthetic heart valves. Machine-learning tools can significantly accelerate biomechanics simulations, leading to the development of a high-fidelity surrogate model, which can then be used for optimizing the geometry of valves. This surrogate model can then be used for optimizing the geometry of bioprosthetic valves, leading to patient-specific valves with better fit and performance, reducing the need for premature valve replacements [3]. The tools and methods developed as part of this research will help improve patient care and treatment outcomes, ultimately benefiting society with improved healthcare.

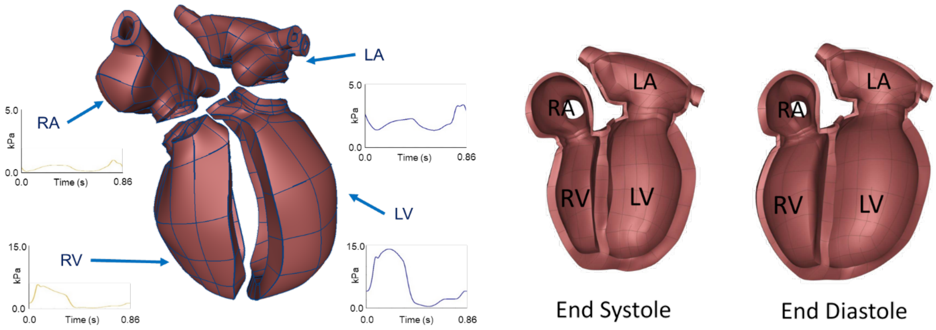

Figure 1 Four chamber cardiac model. The cavity pressures are applied to the inner surfaces of the four chambers to simulate a complete cardiac cycle. Two deformed geometries at end systole and end diastole are shown on the right.

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3.8 Robust Cut-Cells for Triangle Meshes

David I. W. Levin (University of Toronto, CA)

Joint work of David I. W. Levin, Michael Tao, Christopher Batty, Eugene Fiume
Main reference Michael Tao, Christopher Batty, Eugene Fiume, David I. W. Levin: "Mandoline: robust cut-cell generation for arbitrary triangle meshes", ACM Trans. Graph., Vol. 38(6), pp. 179:1–179:17, 2019.
URL https://doi.org/10.1145/3355089.3356543

Although geometry arising "in the wild" most often comes in the form of a surface representation, a plethora of geometrical and physical applications require the construction of volumetric embeddings either of the geometry itself or the domain surrounding it. Cartesian cut-cell-based mesh generation provides an attractive solution in which volumetric elements are constructed from the intersection of the input surface geometry with a uniform or adaptive hexahedral grid. This choice, especially common in computational fluid dynamics, has the potential to efficiently generate accurate, surface-conforming cells; unfortunately, current solutions are often slow, fragile, or cannot handle many common topological situations. We therefore propose a novel, robust cut-cell construction technique for triangle surface meshes that explicitly computes the precise geometry of the intersection cells, even on meshes that

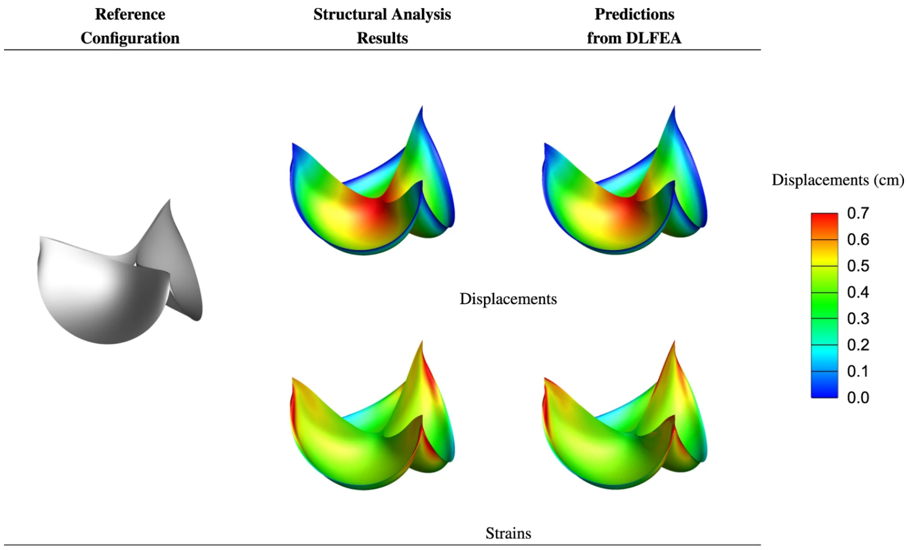


Figure 2 Illustrative example of valve deformations and their corresponding maximum in-plane principal Green-Lagrange strains computed from isogeometric simulations and predicted using deep learning.

are open or non-manifold. Its fundamental geometric primitive is the intersection of an arbitrary segment with an axis-aligned plane. Beginning from the set of intersection points between triangle mesh edges and grid planes, our bottom-up approach robustly determines cut-edges, cut-faces, and finally cut-cells, in a manner designed to guarantee topological correctness. We demonstrate its effectiveness and speed on a wide range of input meshes and grid resolutions, and make the code available as open source.

3.9 Fast algorithms for hierarchical bases over tensor-product splines

Angelos Mantzaflaris (INRIA – Valbonne, FR)

In this talk we present methods to improve the efficiency of computations with hierarchical splines that are constructed over tensor-product spline spaces. We focus on the problem of computing the Gram matrix of the basis. Typically, computations involve numerical integration using tensor-product Gauss quadrature. However, it is known that an element-wise assembly of the Gramian of tensor product B-splines is sub-optimal in dimension bigger than one. We present efficient algorithms for this computation using the background tensor structure. In particular, we extend the Kronecker formula that is known for the Gramian of tensor-product spaces to a Hadamard formula for the Gramian of hierarchical bases. This implies an efficient algorithm for the computation of the matrix, that does not involve a multivariate quadrature over the elements.

3.10 Current challenges in industrial surface reconstruction at MTU

Dominik Mokriš (MTU Aero Engines – München, DE)

Joint work of Dominik Mokriš, Johannes Barner, David Großmann, Urška Zore

The aim of the talk is to contribute to the exchange of ideas and experience between academia and industry. This is achieved both by explaining some of the specifics of implementing geometric algorithms in industrial setting and by showing several hand-picked problems where we expect the attending scientists to be able to suggest effective remedies.

In the first part, I give a broad idea of how our geometry generator (the purpose and operation of which are covered in the talk of Urška Zore) is organised and what are its interfaces. Although seemingly irrelevant to the scientific contents, such interfaces are crucial in the industry, as they reflect how the tasks are distributed between various specialists. The ability of a newly proposed method to conform to pre-existing interfaces can be one of the deciding factors of its acceptance at a particular company.

To demonstrate the principle I show several examples of how we use the scientific libraries G+Smo and CGAL that are well-known and co-developed by several of the participants.

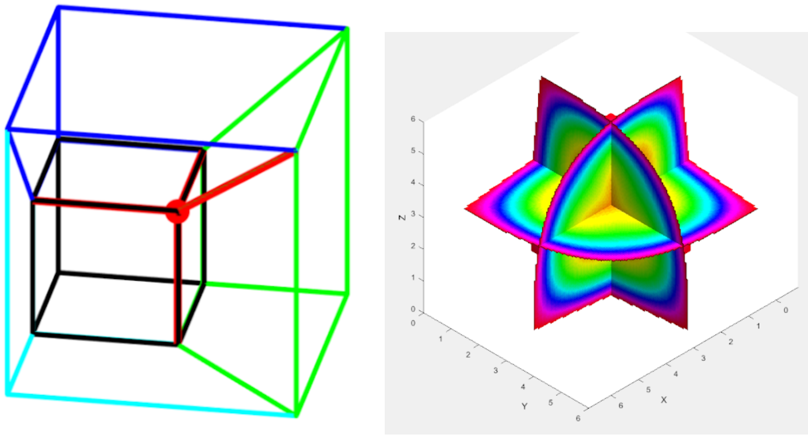
In the final part, I concentrate on explaining several practical geometric problems for which the other participants might have suggestions. These problems come mainly from the area of surface reconstruction (which despite its already big number of practial applications keeps finding new ones), where improvements in surface fitting and (re-)parametrisations of triangulations allow for higher level of precision without introducing artifacts. Another briefly mentioned problem is the possibility to use isogeometric analysis (or other PDE-related methods) in geometric modeling.

3.11 Refinable tri-variate C^1 splines for box-complexes including irregular points and irregular edges

Jörg Peters (University of Florida – Gainesville, US)

The grid points of a regular partition of 3-space into boxes can be interpreted as the control points of a tri-variate tensor-product spline with one polynomial piece per cube. The theory of such splines is well-understood. By contrast, for box-complexes where the tensor-grid gives way to an irregular arrangement of boxes including irregular points and irregular edges, there is to date no simple prescription to join the corresponding polynomial pieces with more than C^0 continuity. Efficiently modeling C^1 fields over general box-complexes is of interest in areas ranging from scientific data visualization to solving higher-order differential equations. For example, to visualize a flow computed by the Discontinuous Galerkin approach currently requires substantial post-processing to extract stream lines that the theory predicts to be smooth.

Already in two variables – where the box-complex is a quad mesh and the only irregularities are points where more or fewer than four quadrilaterals meet – associating one or more bi-cubic polynomial pieces with each quad and joining them to form a C^1 space is far from trivial: (i) Geometric continuity requires increased polynomial degree near irregularities



(a) box-complex with irregular point ($n = 4$) and irregular edges ($n_e = 3$) **(b)** Solution of $\Delta(u \circ \mathbf{x}) = 1$

Figure 3 Modeling and computing with refinable tri-variate C^1 splines for box-complexes including (a) four irregular edges of valence 3 and one irregular vertex of valence 4. (b) The four domains map to curved boxes partitioning an octant of a ball and Poisson's equation is solved on the octant, by collocation.

and careful book-keeping to adjust reparameterizations under refinement; (ii) Subdivision creates an infinite sequence of nested piecewise polynomial rings that complicate engineering analysis, e.g. integration, near irregularities; and (iii) vertex-singular parameterizations can have poor shape deficient and must ensure that the singularity is locally removable.

In three variables, tri-variate Catmull-Clark subdivision lacks a guarantee of smoothness and approximation order; and geometric continuity, although well-understood in principle, is in practice barely explored.

This talk introduced a trivariate C^1 space with singular parameterization. Wherever possible, the vertices of the box-complex are interpreted as B-spline coefficients. Then, at each irregularity, a well-behaved linear function is determined and composed with a local singular expansion $\mathbf{x}$ that is based on the intersection of edge-dual planes within each box. All first derivatives of the expansion $\mathbf{x}$ are continuous, albeit zero across irregularities. Apart from the irregularities, its Jacobian is positive definite so that $\mathbf{x}^{-1}$ is well defined. Evaluating the local expansion of the linear function composed with $\mathbf{x}$ at $\mathbf{x}^{-1}$ removes the singularity. The polynomial pieces of the spline space therefore join not just nominally C^1, but smoothly over the whole box-complex. The spline space has a basis of $2 \times 2 \times 2$ independent functions per hexahedral input box (one per sub-box after a dyadic split in each dimension), can reproduce linear functions and is refinable.

Fig. 3a shows a box complex with irregular point ($n = 4$) and four irregular edges ($n_e = 3$). The corresponding piecewise tri-cubic map is smooth across the irregularities and parameterizes an octant of a ball. To test the construction as physical domain, the Poisson equation is solved on the octant, with zero boundary conditions. Fig. 3b shows slices colored by the resulting scalar field satisfying the Poisson equation in the sense of collocation.

3.12 From Delaunay Triangulations to Curved Optimal Delaunay Triangulations

Pierre Alliez (INRIA – Valbonne, FR)

Joint work of Leman Fen, Laurent Busé, Hervé Delingette, Mathieu Desbrun, Pierre Alliez
Main reference Leman Feng, Pierre Alliez, Laurent Busé, Hervé Delingette, Mathieu Desbrun: "Curved optimal delaunay triangulation", ACM Trans. Graph., Vol. 37(4), pp. 61:1–61:16, 2018.
URL http://dx.doi.org/10.1145/3197517.3201358

Meshes with curvilinear elements hold the appealing promise of enhanced geometric flexibility and higher-order numerical accuracy compared to their commonly-used straight-edge counterparts. However, the generation of curved meshes remains a computationally expensive endeavor with current meshing approaches: high-order parametric elements are notoriously difficult to conform to a given boundary geometry, and enforcing a smooth and non-degenerate Jacobian everywhere brings additional numerical difficulties to the meshing of complex domains. In this talk I will present an extension of Optimal Delaunay Triangulations (ODT) to curved and graded isotropic meshes. By exploiting a continuum mechanics interpretation of ODT instead of the usual approximation theoretical foundations, we formulate a very robust geometry and topology optimization of Bézier meshes based on a new simple functional promoting isotropic and uniform Jacobians throughout the domain. The resulting curved meshes can adapt to complex domains with high precision even for a small count of elements thanks to the added flexibility afforded by higher order basis functions.

3.13 Quasi-interpolants and the solution of fractional differential problems

Francesca Pitolli (Sapienza University of Rome, IT)

Nowadays, Fractional Calculus is a well-established tool to model a great variety of real-world phenomena, from viscoelasticity to population growth, from anomalous diffusion to wave propagation. The growing popularity of differential problems having derivatives of fractional, i.e. noninteger, order is due to their ability to model nonlocality in space or memory effects in time.

Unfortunately, the analytical solution of fractional differential problems is known just in some special cases and it is often expressed as a series expansion. Thus, there is a great effort in constructing efficient numerical methods to solve this kind of problems.

To this end, collocation methods have proved to be particularly effective since their are global methods that can easily take into account the nonlocal behavior of the fractional derivative. Here, we construct a collocation method based on spline quasi-interpolants and use it to solve boundary differential problems having space derivative of fractional order. The main advantage of this method is that the fractional derivative of the spline basis functions can be evaluated explicitly by a differentiation rule that involves the finite difference operator. We analyze the approximation properties of the method and show some numerical results.

3.14 A Few Thoughts about Design and Simulation

Ulrich Reif (TU Darmstadt, DE)

Trimmed NURBS are the standard of industrial surface modeling, today. This talk addresses two main challenges in this context: First, composite trimmed NURBS surfaces typically reveal gaps along boundary curves. Second, simulation on domains bounded by trimmed NURBS surfaces requires a time-consuming and sometimes very complicated meshing process. The latter statement is true even for Isogeometric Analysis, which requests a volumetric parametrization of the domain, which cannot be derived easily from a given boundary representation.

We suggest a new class of trimmed NURBS surfaces with accurate boundary control, called ABC-surfaces, which provably solves the first problem and offers a promising new idea to deal with the second one. Given a tensor product spline surface b describing the interior shape of the desired surface, called the base, and auxiliary spline surfaces r_ℓ representing the shape near the boundary, called ribbons, the corresponding ABC-surface a is defined by

$$a = \frac{wb + \sum_\ell w_\ell r_\ell \circ \kappa_\ell}{w + \sum_\ell w_\ell}.$$

Here, w is a function vanishing at the boundary, and the w_ℓ are functionis vanishing at all boundary segments except for that with index ℓ. Further, κ_ℓ is a reparametrization used to adapt the ribbons to the base, characterized by $b \approx r_\ell \circ \kappa_\ell$. With this setting, the surface a inherits its shape at the boundary from the ribbons and thus can be deigned easily to match the geometry of neighboring spline surfaces. In principle, it is possible to ensure a G^k-contact for any order k.

Just as it is possible in this way to parameterize surfaces with a prescribed boundary curve, it is possible to parameterize volumes with a prescribed boundary surface. This idea, which can be regarded as a combination of web-splines and Isogeometric Analysis, might have some potential to solve the notorious meshing problem in simulation, but this line of research is still in its infancy.

3.15 Quadrature schemes based on quasi-interpolation for Isogeometric Boundary Element Methods

Maria Lucia Sampoli (University of Siena, IT)

Joint work of Maria Lucia Sampoli, Alessandra Aimi, Francesco Calabrò, Tadej Kanduč, Antonella Falini, Carlotta Giannelli Alessandra Sestini

Boundary Element Methods (BEMs) are schemes studied since the mid ’80s, for the numerical solution of those Boundary Valued Problems, which can be reformulated as a system of integral equations defined only on the boundary of the domain. These methods have two main advantages, the dimension reduction of the computational domain and the simplicity for treating external problems. One of the important challenges in this topic is to accurately and efficiently solve singular integrals that arise from the boundary integral equations so formulated. Therefore, designing suitable quadrature schemes is one of the main active research topic in BEM, [1].

Recently new quasi-interpolation (QI) based quadrature rules have been introduced specifically for IgA-BEM setting, [2]. Such quadrature schemes are tailored for B-splines and provide very good accuracy and optimal convergence rate. In this talk we show how weakly, strongly and hyper-singular integrals related to the 2D integral formulation of the Laplace equation with different types of boundary conditions can be approximated by using these new rules, exhibiting promising results. Moreover local refinability of the approximated solution of the problem is also addressed by using hierarchical B-spline spaces. It can be seen that the local nature of the QI perfectly fits with hierarchical spline constructions and leads to an efficient and accurate numerical scheme, [3].

The research is part of a collaboration with A. Aimi, F. Calabrò, T. Kanduč, A. Falini, C. Giannelli and A. Sestini.

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3.16 Multi-patch discretizations for isogeometric analysis

Thomas Takacs (Johannes Kepler Universität Linz, AT)

Joint work of Thomas Takacs, Mario Kapl, Giancarlo Sangalli, Somayeh Kargaran, Bert Jüttler, Stefan K. Kleiss, Angelos Mantzaflaris

In this talk we discuss different approaches to represent geometrically difficult domains using an isogeometric discretization. Isogeometric analysis (IGA) is based on the geometry representation of CAD models, which are usually composed of (possibly trimmed) B-spline or NURBS patches. This talk is covers two topics. In the first part we discuss the construction of C^1 isogeometric spaces [1], in the second part we present a method that can handle overlapping multi-patch domains [2].

In IGA globally C^1 smooth spaces over unstructured meshes can be used to discretize and solve fourth order partial differential equations using a Galerkin approach. We focus on C^0-conforming multi-patch domains. We present the construction of a specific C^1 isogeometric spline space for the class of so-called analysis-suitable G^1 (AS-G^1) multi-patch parametrizations. We analyze the properties of the space and present an approach how to handle non-AS-G^1 parametrizations, by relaxing the smoothness criteria.

Finally we present a method to solve second order PDEs on overlapping multi-patch domains. We define a coupled problem, where the solution on every patch is given as a combination of local solutions on the patch together with contributions from the neighboring patches. The resulting system can be solved directly, in contrast to iterative methods such as multiplicative/additive Schwarz.

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3.17 Wave Packets on Surfaces

Chris Wojtan (IST Austria – Klosterneuburg, AT)

Joint work of Stefan Jeschke, Camille Schreck, Christian Hafner, Tomas Skrivan, Ken Museth, Andreas Soderstrom, Christoph Sprenger, John Johansson, Matthias Mueller-Fischer, Nuttapong Chentanez, Miles Macklin, Chris Wojtan
Main reference Stefan Jeschke, Chris Wojtan: "Water wave packets", ACM Trans. Graph., Vol. 36(4), pp. 103:1–103:12, 2017.
URL https://doi.org/10.1145/3072959.3073678
Main reference Stefan Jeschke, Tomás Skrivan, Matthias Müller-Fischer, Nuttapong Chentanez, Miles Macklin, Chris Wojtan: "Water surface wavelets", ACM Trans. Graph., Vol. 37(4), pp. 94:1–94:13, 2018.
URL https://doi.org/10.1145/3197517.3201336
Main reference Camille Schreck, Christian Hafner, Chris Wojtan: "Fundamental solutions for water wave animation", ACM Trans. Graph., Vol. 38(4), pp. 130:1–130:14, 2019.
URL https://doi.org/10.1145/3306346.3323002

Computational fluid simulations can be used to solve for the motion of water surface waves. These waves are especially important in computer animations applications (real-time or off-line virtual worlds). Qualitative effects like wave reflection, refraction, diffraction, and dispersion are especially visually salient and important to capture. Unfortunately, typical simulation approaches like finite elements, finite differences, and spectral/Fourier methods either fail to capture boundary effects or provide insufficient accuracy/detail due to signal processing and frequency limitations.

We propose using wave packets, essentially a wavelet basis, for animating water surface waves. The discretization explicitly models dispersion and wave group effects, and the frequency is independent of the computational degrees of freedom. We present analytic techniques for evolving wave packets, Eulerian wavelets on a flat surface, and radially symmetric Greens functions for boundary integral problems. Finally, we introduce a new technique for seeding and evolving wave packets on moving manifolds, deriving new equations of motion depending on the surface normal, acceleration, and deformation. The technique appears useful for enhancing the visual detail of an existing 3D computational fluid dynamics simulation.

3.18 Industrial practices in geometric modelling at MTU Aero Engines

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Main reference Gábor Kiss, Carlotta Giannelli, Urška Zore, Bert Jüttler, David Großmann, Johannes Barner: "Adaptive CAD model (re-)construction with THB-splines", Graphical Models, Vol. 76(5), pp. 273–288, 2014.
URL https://doi.org/10.1016/j.gmod.2014.03.017

MTU Aero Engines, as one of the leading aircraft engine manufacturers, is continuously extending and improving the state-of-the-art in engineering design, in order to increase its own long-term competitiveness. This requires highly efficient and flexible geometric methods and technologies, combining newest research results, real-world product design and repair inspection processes directly used by the specialists. MTU invested heavily into the use of adaptive splines, such as truncated hierarchical B-Splines (THB-Splines) and patchwork B-Splines (PB-Splines), as generalizations of standard tensor-product splines, in order to tackle demanding industrial practices. We show two concrete examples of our recent applications: model reconstruction of measured data for analysis as part of reverse engineering process, and lofting of airfoil profiles for flexible geometric modelling and design of crucial engine components. We show how we use these promising concepts to extend the design space for an enhanced product performance, thereby improving the geometric construction for a tight CAD-CAE-CAM integration.

4 The L-shape challenge

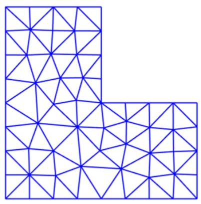

Figure 4 triangulation for classical linear finite elements.

To establish the state of the art, the working group challenged the participants to use their techniques and software packages to solve the following problem posed originally by Abinand Gopal and Lloyd N. Trefethen, see https://arxiv.org/abs/1902.00374: Compute at $(0.99, 0.99)$ the solution to the Laplace equation with zero boundary conditions on an L-shape with re-entrant corner at $(1, 1)$. The answer is known to be $u = 1.02679192610\ldots$ but is hard to compute due to the rapid change of the solution.

Earlier responses to the problem were collected in the NA Digest mailing list (Nov. 2018) and received ca 20 replies, most of which generated 2–4 correct digits, with only one close to 8 digits.

As a specific example, one response using $158, 997$ degrees of freedom (DoFs) of 5th-order triangular finite elements generated 6 digits. Some of the interesting results generated by the workshop participants include:

- Least squares collocation (U. Reif) using 10,000 quadratic Hierarchical B-splines (HBs) reproduced just two digits, but unlike other approaches provides a two-sided error bound: 1.007±0.02.

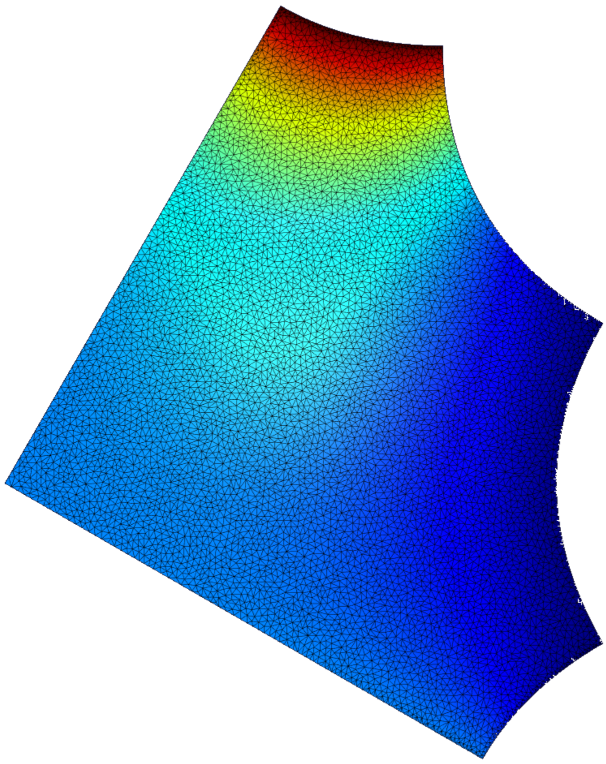

Figure 5 Conformal mapping prior to computation.

- Linear elements (cf. Fig. 4) on a uniform Delaunay mesh using the software package CGAL (P. Alliez:) and 800,000 DoFs reproduced 4 digits: 1.0264 . . . .

- Linear elements applied after *a priori* resolving the singularity at the reentrant corner via a conformal mapping, followed by uniform Delaunay triangulation with 13,699 DoFs (E. Vouga) reproduced 1.026792. . . (see Fig. 5).
- Adaptively refined HBs, ∼10 levels, deg. 5 by the software G+Smo (A. Mantzaflaris) using 30,793 DoFs reproduced 7 digits: 1.0267915. . . (see Fig. 6).

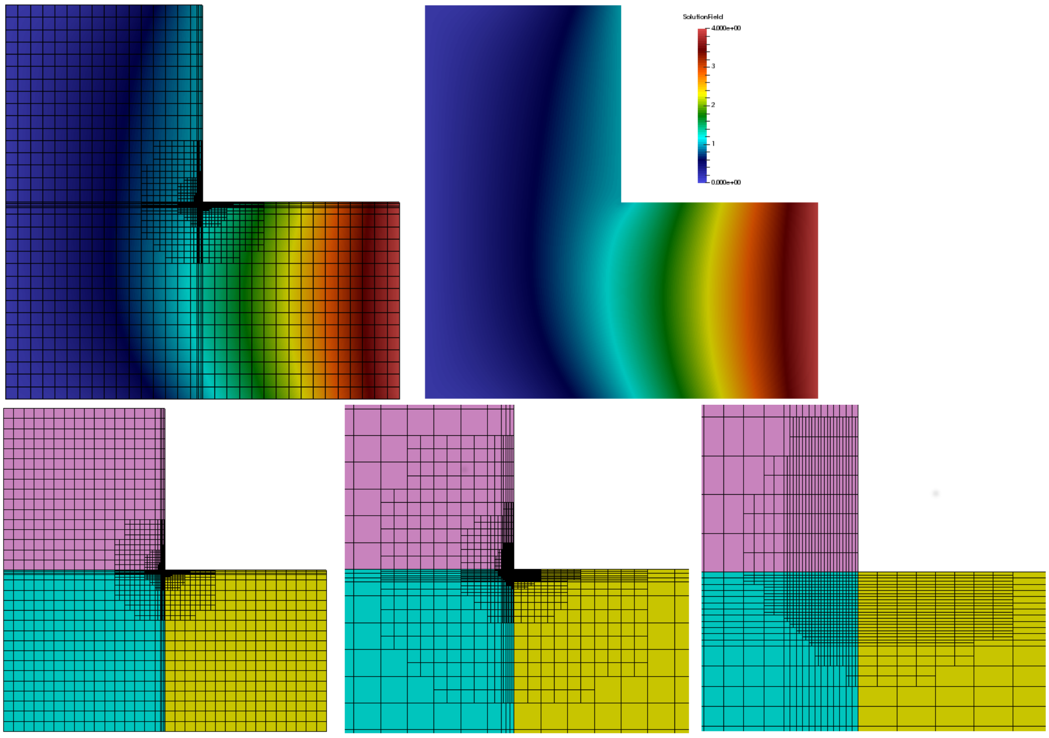

Figure 6 Hierarchical Splines.

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