Algebraic Algorithms for Finding Patterns in Graphs

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Abstract

I will give a gentle introduction to algebraic graph algorithms by showing how to determine if a given graph contains a simple path of length k. This is a famous problem admitting a beautiful and widely-known algorithm, namely the colour-coding method of Alon, Yuster and Zwick (1995). Starting from this entirely combinatorial approach, I will carefully develop an algebraic perspective on the same problem. First, I will explain how the colour-coding algorithm can be understood as the evaluation of a well-known expression (sometimes called the "walk-sum" of the graph) in a commutative algebra called the zeon algebra. From there, I will introduce the exterior algebra and present the algebraic framework recently developed with Brand and Dell (2018).

The presentation is aimed at a combinatorially-minded audience largely innocent of abstract algebra.

References

1   Noga Alon, Raphael Yuster, and Uri Zwick. Color-coding. *J. ACM*, 42(4):844–856, 1995. doi:10.1145/210332.210337.

2   Cornelius Brand, Holger Dell, and Thore Husfeldt. Extensor-coding. In *Proceedings of the 50th Annual ACM SIGACT Symposium on Theory of Computing*, STOC 2018, page 151–164, New York, NY, USA, 2018. Association for Computing Machinery.