The Spiroplot App

Casper van Dommelen
Utrecht University, The Netherlands

Marc van Kreveld
Utrecht University, The Netherlands
m.j.vankreveld@uu.nl

Jérôme Urhausen
Utrecht University, The Netherlands
j.e.urhausen@uu.nl

Abstract

We introduce an app for generating spiroplots, based on a new discrete-time, linear, dynamic system that repeatedly rotates a pair of points, and plots points where they land. The app supports easy definition of the initial situation and has various visualization settings. It can be accessed at `https://spiroplot.sites.uu.nl`.

2012 ACM Subject Classification Applied computing → Media arts; Computing methodologies → Animation

Keywords and phrases generative art, dynamic system, pattern generation tool

Digital Object Identifier 10.4230/LIPIcs.SoCG.2020.71

Category Media Exposition

Supplementary Material `https://spiroplot.sites.uu.nl`

Funding Supported by the Netherlands Organisation for Scientific Research (NWO) under project no. 612.001.651.

1 Spiroplots

Spiroplots are a new type of discrete-time, linear, dynamic system that produces various nice geometric patterns. They are introduced in [2], where a comparison is made with other systems that generate patterns like L-systems, chaotic systems, and fractals [1]. A spiroplot is defined by a finite set V of points in the plane with their initial positions, a finite sequence R of triplets consisting of two points from V and a rotation angle, and a rotation count k. Each triplet in R – called *r-triplet* – defines a repositioning of the two specified points in V by rotating them around their middle over the specified rotation angle (in degrees in this paper). Whenever points of V land somewhere, they plot a small dot. Each point plots in a different color. We refer to both the system and to the resulting pattern as a spiroplot.

The sequence of k rotations is done by following the sequence in R and repeating it from the start, until k rotations in total have been done, and $2k$ dots are plotted. Even simple spiroplots give nice patterns. For example, a spiroplot with just three points $V = \{v_1, v_2, v_3\}$, and just two r-triplets $R = \langle(v_1, v_2, 90), (v_2, v_3, 90)\rangle$ will show a pattern with eight ellipses after a few thousand rotations (Figure 1, left), regardless of the initial coordinates.

The name spiroplot is derived from the spirograph, a drawing tool for kids from the 1960s. A more complete description and various properties of spiroplots are given in [2], including preservation of the center of mass and the existence of cyclic and non-cyclic spiroplots. The latter ones plot infinitely many points when $k \to \infty$.

36th International Symposium on Computational Geometry (SoCG 2020).
Editors: Sergio Cabello and Danny Z. Chen; Article No. 71; pp. 71:1–71:5
Leibniz International Proceedings in Informatics
LIPICS Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

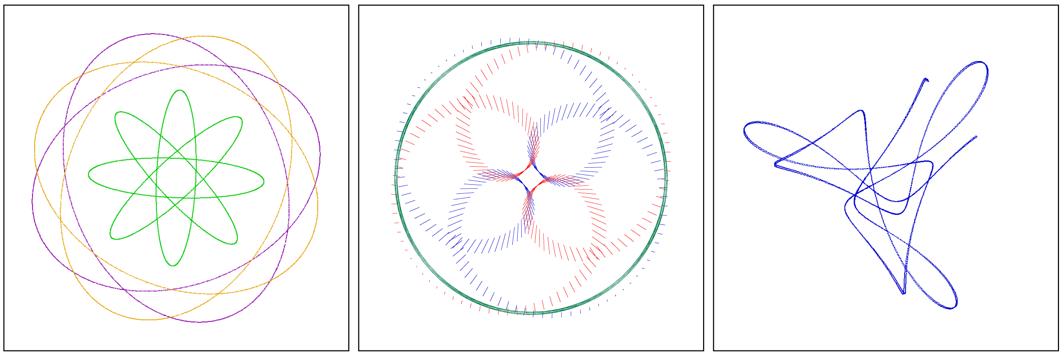

Figure 1 Three spiroplots. Left, with $R = \langle(v_1, v_2, 90), (v_2, v_3, 90)\rangle$, after 5000 rotations. Middle, with $R = \langle(v_1, v_2, 4), (v_2, v_3, 4)\rangle$, after 50,000 rotations. Right, with $R = \langle(v_1, v_2, 1), (v_2, v_3, 1), (v_3, v_4, -1), (v_4, v_5, 1)\rangle$, after 8000 rotations; only v_3 is shown.

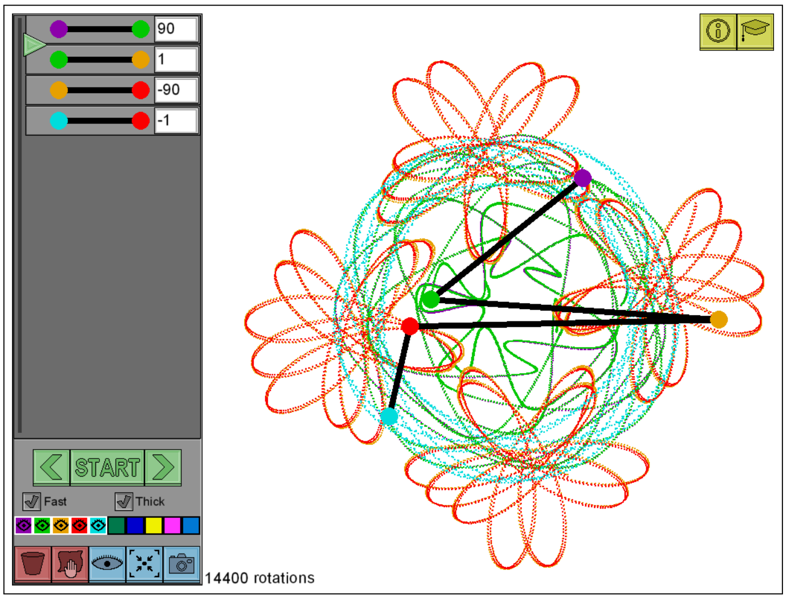

Figure 2 User interface. On the canvas, pairs of rotating points are shown by an edge.

2 A web app for spiroplots

This abstract presents a web app that generates spiroplots with a simple user interface.

`https://spiroplot.sites.uu.nl`

We believe it can serve to make young people enthusiastic about mathematical patterns, dynamic systems, and procedural generation of structures.

The app has a canvas and a control panel, see Figure 2. It allows the user to place points using a mouse left-click, and generate pairs by dragging (mouse-down) from one point to the other point. The default rotation angle is 90 degrees, but other angles can be specified in a text box for that pair. We may repeat a pair of points in the r-triplet sequence. With a right-click on a point, it can be moved, removed, or assigned a different color.

The left panel shows the sequence of r-triplets, each by the colors of the points and its own rotation angle. A green indicator shows what the next rotation will be.

To the bottom in the left panel, options to perform one or many rotations are given, along with visualization options, wiping, resetting, and saving. The center-and-scale option places and scales the point set to ensure that the spiroplot stays inside the canvas and uses most of it. The Fast option performs 100 rotations before the next canvas redraw, doing close to $7,000$ rotations per second (web app, standard laptop, wall clock time). On the desktop app version, it is a few times faster. Simple patterns often start to emerge within seconds; complex patterns may show a pattern only after minutes. Sometimes we do not discern any pattern. The Thick option draws 2×2 pixels when plotting a point. Every color can be toggled on or off in the row of colored eyes, and so can the graph of the spiroplot (bottom eye). Colors are stacked, so there is always a top color. The stacking order can be changed by clicking the colored eyes, which brings the last toggled color to the top. The app has a tutorial built in.

3 Various settings

When trying out different settings in the app, we can make various observations.

Spiroplots with $V = \{v_1, v_2, v_3\}$, $R = \langle (v_1, v_2, 90), (v_2, v_3, 90) \rangle$ always look like ellipses after sufficiently many rotations, regardless of the initial coordinates of v_1, v_2 and v_3. The points plotted for v_2 lie on four ellipses and the points plotted for v_1 or v_3 lie on two ellipses each. If one of the angles is -90 degrees, there are only four ellipses in total, two for v_2 and one for each of v_1 and v_3. When the two angles are 30 or 45 degrees, we see more ellipses.

Spiroplots like $V = \{v_1, v_2, v_3\}$, $R = \langle (v_1, v_2, 90), (v_2, v_3, 90), (v_1, v_3, 90) \rangle$ also show ellipses only, and all points plot on the same eight ellipses.

When spiroplots use small angles only, like 1 degree (or even smaller), the plotted points trace a curve with small steps, essentially drawing it (see Figure 1, right). The types of curves can appear rather complex when there are five or more points in the system.

Certain spiroplots repeat their current points after a few rotations. For example, every spiroplot with $V = \{v_1, v_2, v_3, v_4\}$, $R = \langle (v_1, v_2, 90), (v_2, v_3, 90), (v_3, v_4, 90) \rangle$ will have the same positions for $v_1, \ldots, v_4$ after 36 rotations, regardless of the initial coordinates. Hence, no interesting pattern appears. The three rotations can be represented together by an 8×8 matrix operating on the eight coordinates of the four points. Raising this matrix to the power 12 yields the identity matrix.

The order of r-triplets is very important. A spiroplot with $R = \langle (v_1, v_2, 90), (v_2, v_3, 90), (v_3, v_4, 90), (v_1, v_4, 90) \rangle$ shows complex patterns while $R = \langle (v_1, v_2, 90), (v_3, v_4, 90), (v_2, v_3, 90), (v_1, v_4, 90) \rangle$ yields a cyclic spiroplot.

Many spiroplots with more complex specifications will not show a pattern in the app. This may be due to the limited resolution that is available. Sometimes one point (color) shows a pattern but another point does not.

For simpler situations, patterns show up after a few thousand rotations. For complex spiroplots, it may take hundreds of thousands rotations. Figure 3 shows how more iterations yields a fuller pattern. Figure 4 shows two more examples of spiroplots.

4 Discussion

There are many interesting open problems related to spiroplots.

1. By observation we noticed that spiroplots with three points only show ellipses, and each ellipse has the same center. Can we prove this, or find a counterexample?
2. Do all points of a spiroplot lie on a finite set of algebraic curves? Or are there positive-area regions of the plane that are covered arbitrarily densely by plotted points, if $k \to \infty$?

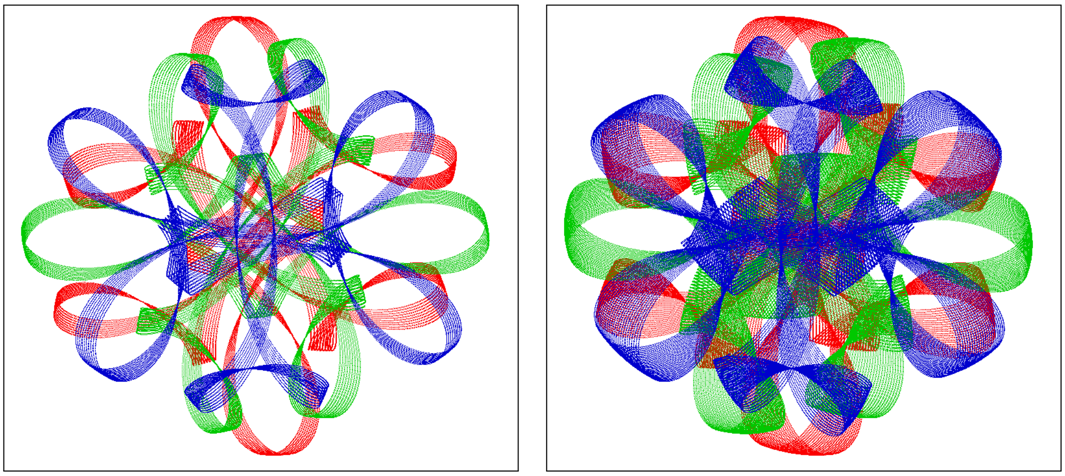

Figure 3 Spiroplots that are the same except for the number of rotations. Left, after 201,000 rotations. Right, after 500,800 rotations. The rotation sequence is $\langle(v_1, v_2, 120), (v_3, v_4, -120), (v_5, v_6, 120), (v_2, v_3, 0.4), (v_4, v_5, 0.4), (v_6, v_1, -0.4)\rangle$. Only three of the six colors are shown: v_1 is red, v_2 is green, and v_5 is blue.

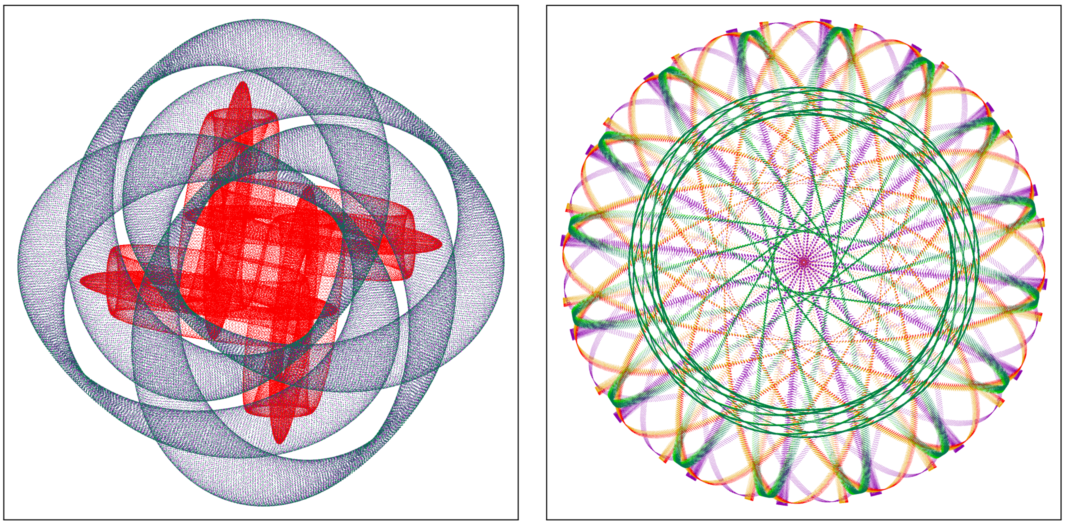

Figure 4 Two spiroplots created with the program. Left, $R = \langle(v_1, v_2, 90), (v_3, v_4, 90), (v_5, v_6, 90), (v_2, v_3, 90), (v_4, v_5, 90)\rangle$. Right, $R = \langle(v_1, v_2, 0.2), (v_1, v_2, 0.2), (v_1, v_2, -0.6), (v_1, v_2, -0.2), (v_1, v_2, 90.4), (v_2, v_3, 0.2), (v_2, v_3, 0.2), (v_2, v_3, -0.6), (v_2, v_3, -0.2), (v_2, v_3, 90.4), (v_1, v_3, 45)\rangle$ (some colors were changed during the run).

3. Can spiroplots be interpreted as projections of higher-dimensional curves?
4. Can we characterize all cyclic spiroplots?
5. How should we define 3D spiroplots? Rotations of two points about their center are no longer unique; we need a rotation axis.

A more practical question is the one of control: How can we provide intuitive control on the spiroplot to be produced? If we generated a spiroplot and want it slightly different, it is unclear how to change the parameters to realize this.

References

1 Heinz-Otto Peitgen, Hartmut Jürgens, and Dietmar Saupe. *Chaos and Fractals: new frontiers of science*. Springer Science & Business Media, 2006.

2 Casper van Dommelen, Marc van Kreveld, and Jérôme Urhausen. Spiroplots: a new discrete-time dynamical system to generate curve patterns, 2020. Submitted.