Coordinated Particle Relocation with Global Signals and Local Friction

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Abstract

In this video, we present theoretical and practical methods for achieving arbitrary reconfiguration of a set of objects, based on the use of external forces, such as a magnetic field or gravity: Upon actuation, each object is pushed in the same direction. This concept can be used for a wide range of applications in which particles do not have their own energy supply or in which they are subject to the same global control commands.

A crucial challenge for achieving any desired target configuration is breaking global symmetry in a controlled fashion. Previous work (some of which was presented during SoCG 2015) made use of specifically placed barriers; however, introducing precisely located obstacles into the workspace is impractical for many scenarios. In this paper, we present a different, less intrusive method: making use of the interplay between static friction with a boundary and the external force to achieve arbitrary reconfiguration. Our key contributions are *theoretical* characterizations of the critical coefficient of friction that is sufficient for rearranging two particles in triangles, convex polygons, and regular polygons; a method for reconfiguring multiple particles in rectangular workspaces, and deriving *practical* algorithms for these rearrangements. Hardware experiments show the efficacy of these procedures, demonstrating the usefulness of this novel approach.

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1 Introduction

Reconfiguring a large set of objects in a prespecified manner is a fundamental task for a large spectrum of applications, including swarm robotics, smart materials and advanced manufacturing. In many of these scenarios, the involved items are not equipped with individual motors or energy supplies, so actuation must be performed from the outside. Moreover, reaching into the workspace to manipulate individual particles of an arrangement is often impractical or even impossible; instead, global external forces (such as gravity or a magnetic force) may have to be employed, targeting each object in the same, uniform manner. These limitations of individual navigation apply even in scenarios of swarm robotics, e.g.

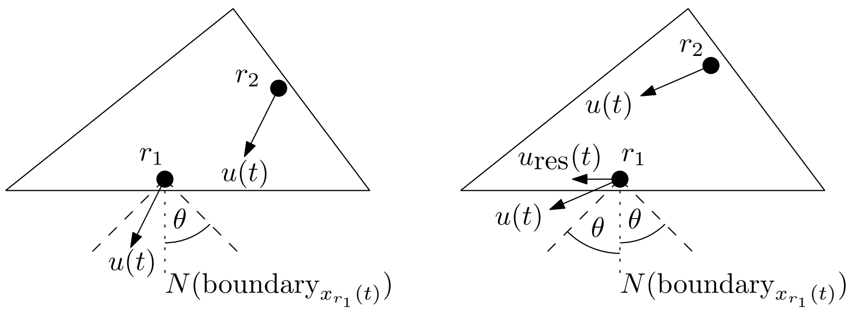


Figure 1 Left: An input force command $u(t)$ within the cone $\pm\theta$ about the normal to the boundary results in no motion of r_1. Right: An input force command $u(t)$ outside the cone results in a motion of both particles. Observe that r_1 slides along the boundary with a resulting force $u_{\text{res}}(t)$.

for the well-known kilobots [12] that can be directed by switching on a light beacon, which works just like activating an external force. This concept of global control has also been studied for using biological cells as reactive robots controlled by magnetic fields, see Arbuckle and Requicha [3] and Kim et al. [9]. Global control also has applications in assembling nano- and micro-structures. Related work shows how to assemble shapes by adding one particle at a time [8, 4], or combining multiple pairs of subassemblies in parallel in one time step [14].

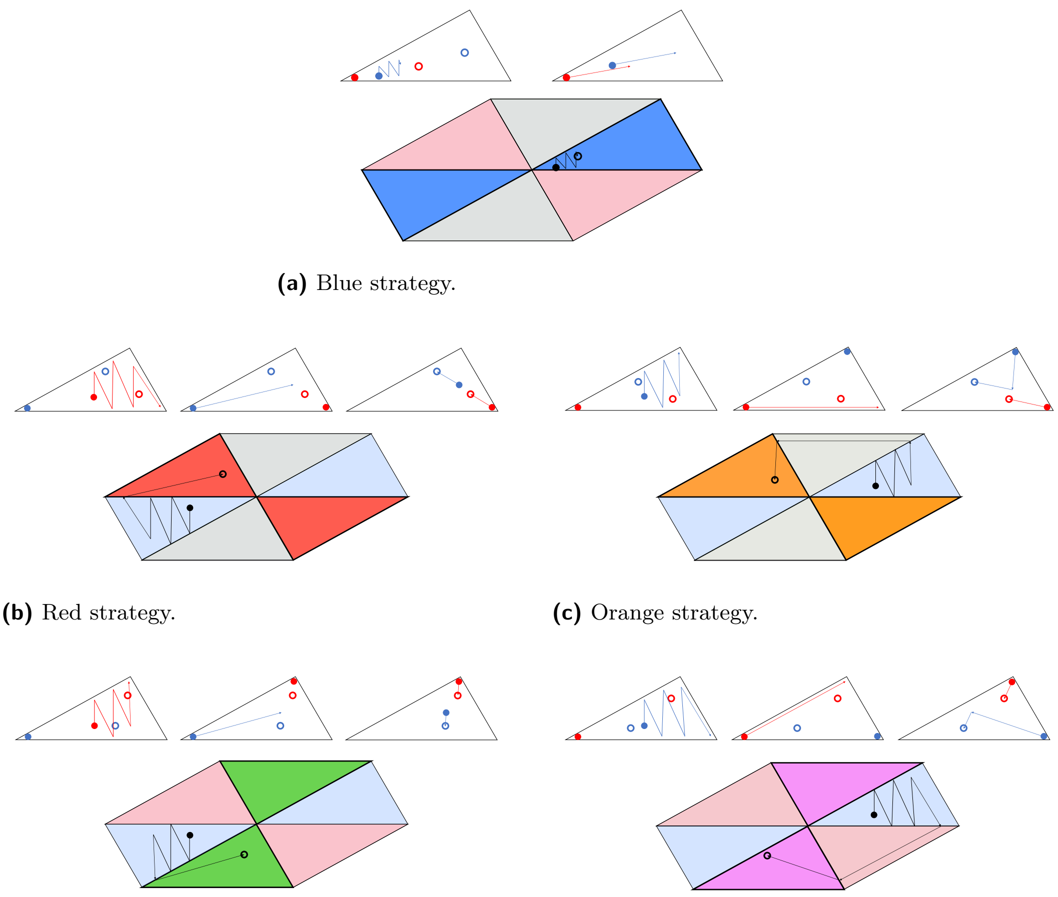
Considering this approach of navigation by a global external force gives rise to a number of problems, including navigation of one particle from a start to a goal position [10], particle computation [6, 7], or emptying a polygon [2]. Zhang et al. [15, 16] show how to rearrange a rectangle of agents in a workspace that is only constant times larger than the number of agents. Akella et al. [1] consider the problem of reconfiguring an object on a conveyor belt with a simple robot, and Lynch et al. [11] use a mobile robot with a flat pusher plate as the gripper to manipulate objects. A crucial issue for all these tasks is how to combine the use of a uniform force (which is the same for all involved items) with the individual requirements of object relocation (which may be distinct for different particles): How can we achieve an *arbitrary* arrangement of particles if all of them are subjected to the same external force? Previous work (such as [7]) has shown how arbitrary reconfiguration of an ensemble is possible with the help of specifically placed barriers; this has also been the subject of a previous multimedia contribution to SoCG [5]. However, introducing precisely located obstacles into the workspace is impractical for many scenarios.

In this contribution, we present a different, less intrusive method: making use of the interplay between static friction with a boundary of the workspace and the external force to achieve any desired configuration. For more details, see our journal paper [13].

2 Using friction for reconfiguration

The *coefficient of friction* μ is the ratio between the strength of an orthogonal force against a surface and the resistance parallel to the surface. Geometrically, this corresponds to a (static) *angle of friction* θ: which is the critical angle when sliding commences, satisfying $\mu := \tan\theta$. See Fig. 1 for an illustration.

Just like in the context of sorting algorithms in computer science or discrete mathematics, a critical component for achieving arbitrary reconfiguration of larger ensembles is the ability to rearrange two specific particles. The idea is to completely cover the Δ configuration, which is the set of all differences between all pairs of possible particle locations. To this end, we employ a number of different strategies (shown in Fig. 2 and visualized in the video). As shown in Fig. 3, these can be combined to yield an overall lower bound for θ, as follows.



(a) Blue strategy.

(b) Red strategy.

(c) Orange strategy.

(d) Green strategy.

(e) Violet strategy.

Figure 2 Illustration of the five strategies for dealing with different portions of Δ space. In each case, shown are two particles in actual space (top), and in Δ space (bottom).

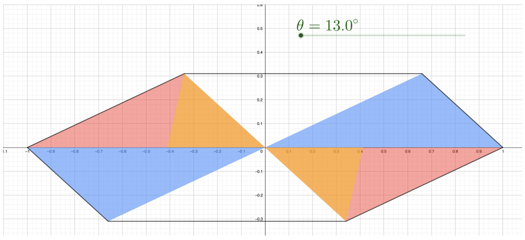
▶ **Theorem 1.** *Let T be a triangle with angles $\alpha \leq \beta \leq \gamma$. If $\theta > \frac{\pi}{2} - \beta$, then we can guarantee any reconfiguration of two particles, i.e., Δ_T is completely covered by our strategies.*

This can be generalized to other environments, as follows.

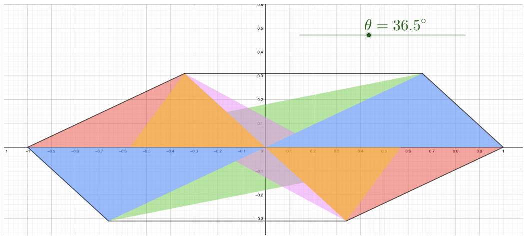
▶ **Theorem 2.** *Let P be a convex polygon with vertices $C_0, \ldots, C_{n-1}$ and angles $\gamma_0, \ldots, \gamma_{n-1}$. If $\theta > \max_{0 \leq i < n} \left(\min_{j \in P_i} \left(\frac{\gamma_i}{2}, \max\left(\frac{\gamma_j}{2}, \eta^+_{i,j} - \frac{\pi}{2}, \eta^-_{i,j} - \frac{\pi}{2} \right) \right) \right)$, where $\eta^+_{i,j} := \sum_{C_k \in P^+_{i+1,j-1}} \delta_k$ and $\eta^-_{i,j} := \sum_{C_k \in P^-_{i-1,j+1}} \delta_k$, then every configuration of two particles can be reached.*

▶ **Theorem 3.** *If P is a regular polygon with n vertices and if $\mu > \cot(\pi/n)$, then every reconfiguration is possible.*

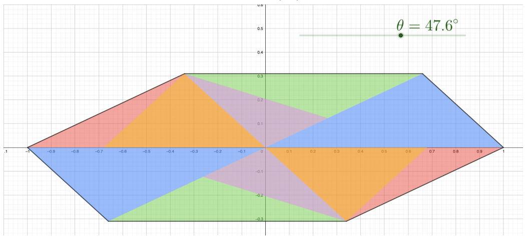
These results for static friction can be extended to rearranging multiple particles; this is visualized in the video, and demonstrated for a real-world application.



(a) Small angle of friction.



(b) Medium angle of friction.



(c) Large angle of friction.

Figure 3 Combining the different strategies for covering Δ space. **(a)** For small θ, a portion remains uncovered, while the rest is covered by the blue, orange and red strategies. **(b)** For growing θ, green and violet strategies cover an increasing portion of the remaining sections. **(c)** For large enough θ, the whole Δ space is covered.

▶ **Theorem 4.** *Consider the class $\mathcal{C}$ of configurations of three particles in a square, where one of the particles lies within the bounding rectangle of the other two particles. If $\theta > \frac{\pi}{4}$, then we can reconfigure any configuration to any configuration of $\mathcal{C}$.*

Using induction, we can achieve arbitrary reconfiguration of a set of collinear particles, as demonstrated in the video for an example with six particles.

▶ **Theorem 5.** *For $\theta > \frac{\pi}{4}$, we can sort any permuted set of collinear particles.*

3 The video

The video starts with an introduction of controlling a swarm of particles or robots by a uniform global force, and the problem of controlled reconfiguration. After a brief review of previous work (which employed obstacles), we introduce the approach of using local differences in boundary friction for breaking symmetry between different particles. This is followed by a description of involved parameters, the concept of Δ space and our five different, "colored" strategies for using static friction to achieve arbitrary reconfiguration of two particles. This ultimately leads to Theorem 1 and can be extended to Theorems 2 and 3; it also can be extended to multiple particles, which yields Theorems 4 and 5. We conclude with practical demonstrations with real particles that are rearranged by a robot controller, and a pair of particles of size 1 mm that are relocated in the stomach of a cow.

References

1 Srinivas Akella, Wesley H Huang, Kevin M Lynch, and Matthew T Mason. Parts feeding on a conveyor with a one joint robot. *Algorithmica*, 26(3-4):313–344, 2000.

2 Greg Aloupis, Jean Cardinal, Sébastien Collette, Ferran Hurtado, Stefan Langerman, and Joseph O'Rourke. Draining a polygon – or – rolling a ball out of a polygon. *Computational Geometry*, 47(2):316–328, 2014.

3 DJ Arbuckle and Aristides AG Requicha. Self-assembly and self-repair of arbitrary shapes by a swarm of reactive robots: algorithms and simulations. *Autonomous Robots*, 28(2):197–211, 2010.

4 Jose Balanza-Martinez, Austin Luchsinger, David Caballero, Rene Reyes, Angel A Cantu, Robert Schweller, Luis Angel Garcia, and Tim Wylie. Full tilt: Universal constructors for general shapes with uniform external forces. In *ACM-SIAM Symposium on Discrete Algorithms (SODA)*, pages 2689–2708, 2019.

5 A. Becker, Erik D. Demaine, Sándor P. Fekete, S. H. Mohtasham Shad, and R. Morris-Wright. Tilt: The video. designing worlds to control robot swarms with only global signals. In *Symposium on Computational Geometry (SoCG)*, pages 16–18, 2015.

6 Aaron Becker, Erik D Demaine, Sándor P Fekete, and James McLurkin. Particle computation: Designing worlds to control robot swarms with only global signals. In *IEEE International Conference on Robotics and Automation (ICRA)*, pages 6751–6756, 2014.

7 Aaron T. Becker, Erik D. Demaine, Sándor P. Fekete, Jarrett Lonsford, and Rose Morris-Wright. Particle computation: complexity, algorithms, and logic. *Natural Computing*, 18(1):181–201, 2019.

8 Aaron T Becker, Sándor P Fekete, Phillip Keldenich, Dominik Krupke, Christian Rieck, Christian Scheffer, and Arne Schmidt. Tilt assembly: algorithms for micro-factories that build objects with uniform external forces. *Algorithmica*, pages 1–23, 2017.

9 Paul Seung Soo Kim, Aaron T. Becker, Yan Ou, Anak Agung Julius, and Min Jun Kim. Imparting magnetic dipole heterogeneity to internalized iron oxide nanoparticles for microorganism swarm control. *Journal of Nanoparticle Research*, 17(3):1–15, 2015.

10 Jeremy S. Lewis and Jason M. O'Kane. Planning for provably reliable navigation using an unreliable, nearly sensorless robot. *The International Journal of Robotics Research*, 32(11):1342–1357, 2013.

11 Kevin M Lynch and Matthew T Mason. Stable pushing: Mechanics, controllability, and planning. *The International Journal of Robotics Research*, 15(6):533–556, 1996.

12 Michael Rubenstein, Christian Ahler, Nick Hoff, Adrian Cabrera, and Radhika Nagpal. Kilobot: A low cost robot with scalable operations designed for collective behaviors. *Robotics and Autonomous Systems*, 62(7):966–975, 2014.

13 Arne Schmidt, Victor M. Baez, Aaron T. Becker, and Sándor P. Fekete. Coordinated particle relocation using finite static friction with boundary walls. *Robotics and Automation Letters*, 2:985–992, 2020.

14 Arne Schmidt, Sheryl Manzoor, Li Huang, Aaron T Becker, and Sándor P Fekete. Efficient parallel self-assembly under uniform control inputs. *IEEE Robotics and Automation Letters*, 3(4):3521–3528, 2018.

15 Y. Zhang, X. Chen, H. Qi, and D. Balkcom. Rearranging agents in a small space using global controls. In *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pages 3576–3582, 2017. `doi:10.1109/IROS.2017.8206202`.

16 Yinan Zhang, Emily Whiting, and Devin Balkcom. Assembling and disassembling planar structures with divisible and atomic components. *Transactions on Automation Science and Engineering*, 15(3):945–954, 2018.