On Laplacians

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Abstract

The graph Laplacian is a fundamental tool in computational geometry and data science; its geometric and spectral properties combine to yield a bridge between analytic methods, geometric intuition, and topological properties.

This talk will survey recent generalizations of the graph Laplacian inspired by algebraic geometry and algebraic topology. Given a sheaf over a graph – a functor from the vertex-edge poset to a category of algebraic data – one can assign a type of Hodge Laplacian which, in the case of a constant sheaf of 1-dimensional vector spaces, recapitulates the classic graph Laplacian. This Hodge (or *sheaf*) Laplacian is the basis for diffusion and distributed algorithms over network sheaves. This talk has very few prerequisites and will introduce the concepts carefully in the context of applications ranging from distributed optimization, to learning, opinion dynamics, neural networks, and more.

From a mathematical perspective, the most interesting challenge is the extension of Laplacians to systems – or sheaves – taking values in non-abelian categories, such as categories of lattices. Such extensions will be a particular focus of the talk.

This talk surveys joint works with Jakob Hansen, Paige Randall North, and Hans Riess.

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